

5 ELECTRICITY AND MAGNETISM

Introduction

Modern society uses a whole range of electrical devices from the simplest heated metal filaments that provide light, through to the most sophisticated medical instruments and computers. Devices of increasing technical complexity are developed every day.

In this topic we look at the phenomenon of electricity, and what is meant by charge and electric current. We consider the three effects that can be observed when charge flows in an electric circuit.

5.1 Electric fields

Understanding

- Charge
- Electric field
- Coulomb's law
- Electric current
- Direct current (dc)
- Potential difference (pd)

Nature of science

Electrical theory resembles the kinetic theory of gases in that a theory of the microscopic was developed to explain the macroscopic observations that had been made over centuries. The development of this subject and some of the byways that were taken make this a fascinating study. We should remember the many scientists who were involved. It is a tribute to them that they could make so much progress when the details of the microscopic nature of electronic charge were unknown to them.

Applications and skills

- Identifying two species of charge and the direction of the forces between them
- Solving problems involving electric fields and Coulomb's law
- Calculating work done when charge moves in an electric field in both joules and electronvolts
- Identifying sign and nature of charge carriers in a metal
- Identifying drift speed of charge carriers
- Solving problems using the drift-speed equation
- Solving problems involving current, potential difference, and charge

Equations

- current-charge relationship: $I = \frac{\Delta q}{\Delta t}$
- Coulomb's law: $F = k \frac{q_1 q_2}{r^2}$
- the coulomb constant: $k = \frac{1}{4\pi\epsilon_0}$
- potential difference definition: $V = \frac{W}{q}$
- conversion of energy in joule to electron-volt:
 $W[\text{J}] \equiv \frac{W[\text{eV}]}{e}$
- electric field strength: $E = \frac{F}{q}$
- drift speed: $I = nAvq$

Charge and field

Simple beginnings

Take a plastic comb and pull it through your hair. Afterwards, the comb may be able to pick up small pieces of paper. Look closely and you may see the paper being thrown off shortly after touching the comb.

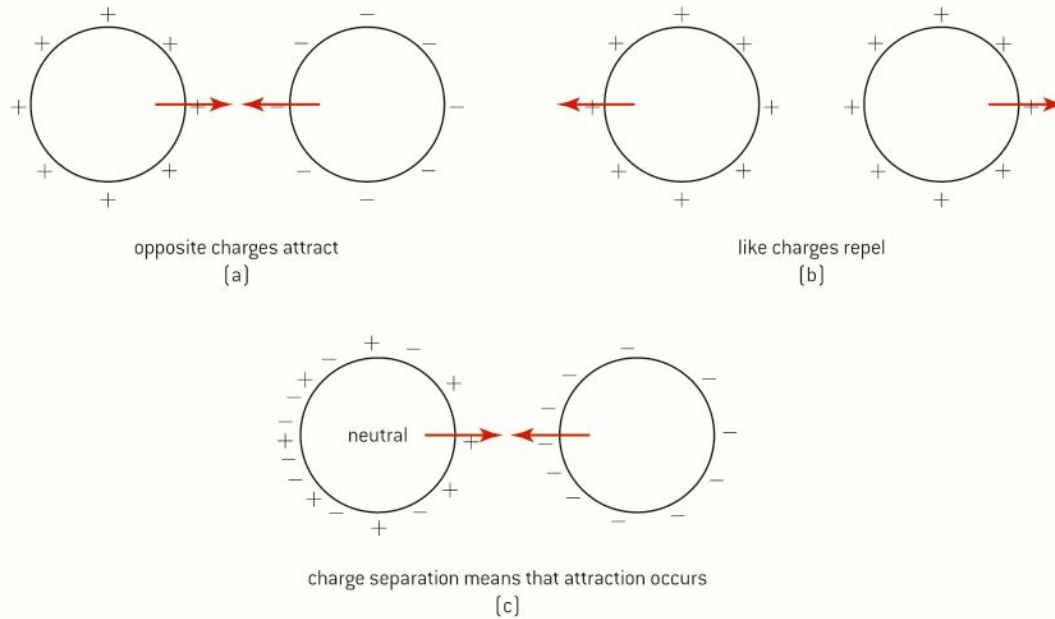
Similar observations were made early in the history of science. The discovery that objects can be charged by friction (you were doing this when you drew the comb through your hair) is attributed to the Greek scientist Thales who lived about 2600 years ago. In those days, silk was spun on amber spindles and as the amber rotated and rubbed against its bearings, the silk was attracted to the amber. The ancient Greek word for amber is $\eta\lambda\epsilon\kappa\tau\rho\nu$ (electron).

In the 1700s, du Fay found that both conductors and insulators could be “electrified” (the term used then) and that there were two opposite kinds of “electrification”. However, he was unable to provide any explanation for these effects. Gradually, scientists developed the idea that there were two separate types of charge: positive and negative. The American physicist, Benjamin Franklin, carrying out a famous series of experiments flying kites during thunderstorms, named the charge on a glass rod rubbed with silk as “positive electricity”. The charge on materials similar to ebonite (a very hard form of rubber) rubbed with animal fur was called “negative”. One of Franklin’s other discoveries was that a charged conducting sphere has no electric field inside it (the field and the charges always being outside the sphere). Joseph Priestley was able to deduce from this that the force between two charges is inversely proportional to the square of the distance between the charges.

At the end of the nineteenth century J. J. Thomson detected the presence of a small particle that he called the electron. Experiments showed that all electrons have the same small quantity of charge and that electrons are present in all atoms. Atoms were found to have protons that have the same magnitude of electronic charge as, but have opposite charge to, the electron. In a neutral atom or material the number of electrons and the number of protons are equal. We now assign a negative charge to the electron and positive to the proton. There are only these two species of charge.

Explaining electrostatics

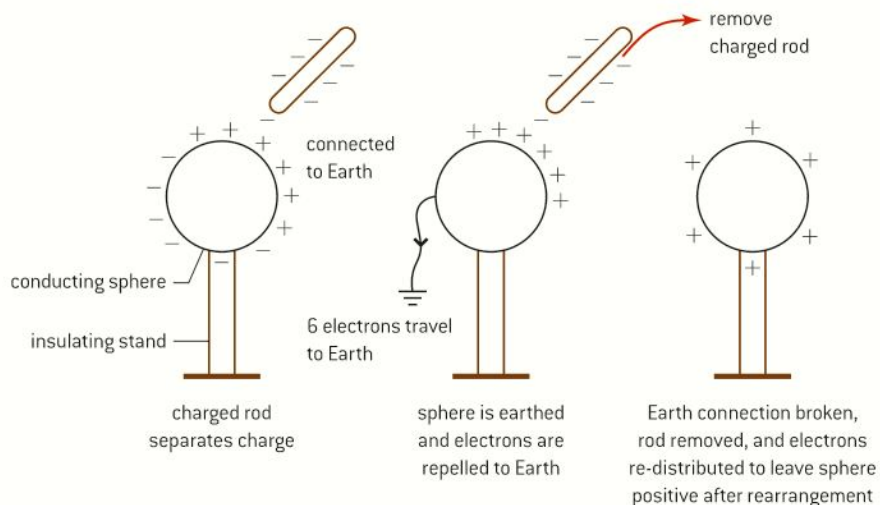
Experiments also show that positively charged objects are attracted to negatively charged objects but repelled by any other positively charged object. The possible cases are summed up in figure 1. There can also be an attraction between charged and uncharged objects due to the separation of charge in the uncharged object. In figure 1(c) the electrons in the uncharged sphere are attracted to the positively charged sphere (sphere A) and move towards it. The electrons in sphere B are now closer to the positives in sphere A than the fixed positive charges on B. So the overall force is towards sphere A as the force between two charges increases as the distance between them decreases.



▲ Figure 1 Attractions between charges.

We now know that the simple electrostatic effects early scientists observed are due only to the movement of the negatively charged electrons. An object with no *observed* charge has an exact balance between the electrons and the positively charged protons; it is said to be **neutral**. Some electrons in conducting materials are loosely attached to their respective atoms and can leave the atoms to move from one object to another. This leaves the object that lost electrons with an overall positive charge. The electrons transferred give the second object an overall negative charge. Notice that electrons are not lost in these transfers. If 1000 electrons are removed from a rod when the rod is charged by a cloth, the cloth will be left with 1000 extra electrons at the end of the process. **Charge is conserved**; the law of conservation of charge states that in a closed system the amount of charge is constant.

When explaining the effects described here, always use the idea of surplus of electrons for a negative charge, and describe positive charge in terms of a lack (or deficit) of electrons. Figure 2 shows how an experiment to charge a metal sphere by induction is explained.



▲ Figure 2 Charging by induction.

TOK

Inverse-square laws

Forces between charged objects is one of several examples of inverse-square laws that you meet in this course. They are of great importance in physics. Inverse-square laws model a characteristic property of some fields, which is that as distance doubles, observed effects go down by one quarter.

Mathematics helps you to learn and conceptualize your ideas about the subject. When you have learnt the physics of one situation (here, electrostatics) then you will be able to apply the same rules to new situations (for example, gravitation, in the next topic).

Is the idea of field a human construct or does it reflect the reality of the universe?

Measuring and defining charge

The unit of charge is the coulomb (abbreviated to C). Charge is a scalar quantity.

The coulomb is defined as the charge transported by a current of one ampere in one second.

Measurements show that all electrons are identical, with each one having a charge equal to -1.6×10^{-19} C; this fundamental amount of charge is known as the **electronic (or elementary) charge** and given the symbol e .

Charges smaller than the electronic charge are not observed in nature. (Quarks have fractional charges that appear as $\pm\frac{1}{3}e$ or $\pm\frac{2}{3}e$; however, they are never observed outside their nucleons.)

In terms of the experiments described here, the coulomb is a very large unit. When a comb runs through your hair, there might be a charge of somewhere between 1 pC and 1 nC transferred to it.

Forces between charged objects

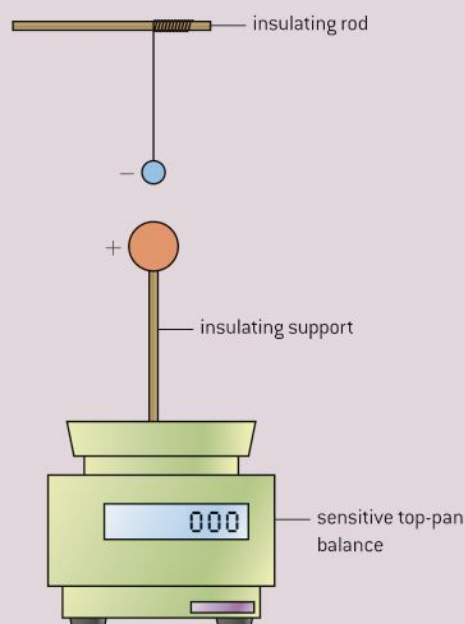
In 1785, Coulomb published the first of several *Mémoires* in which he reported a series of experiments that he had carried out to investigate the effects of forces arising from charges.

He found, experimentally, that the force between two point charges separated by distance r is proportional to $\frac{1}{r^2}$ thus confirming the earlier theory of Priestley. Such a relationship is known as an inverse-square law.

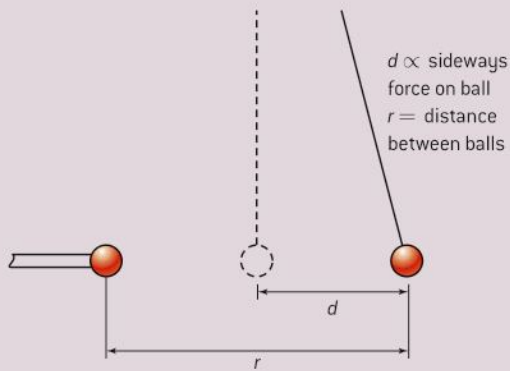
Investigate!

Forces between charges

- These are sensitive experiments that need care and a dry atmosphere to achieve a result.
- Take two small polystyrene spheres and paint them with a metal paint or colloidal graphite, or cover with aluminum foil. Suspend one from an insulating rod using an insulating (perhaps nylon) thread. Mount the other on top of a sensitive top-pan balance, again using an insulating rod.
- Charge both spheres by induction when they are apart from each other. An alternative charging method is to use a laboratory high voltage power supply. Take care, your teacher will want to give you instructions about this.
- Bring the spheres together as shown in figure 3 and observe changes in the reading on the balance.



▲ Figure 3



▲ Figure 4

- Another method is to bring both charged spheres together as shown in figure 4.

- The distance d moved by the sphere depends on the force between the charged spheres. The distance r is the distance between the centres of the spheres.
- Vary d and r making careful measurements of them both.
- Plot a graph of d against $\frac{1}{r^2}$. An experiment performed with care can give a straight-line graph.
- For small deflections, d is a measure of the force between the spheres (the larger the force the greater the distance that the sphere is moved) whereas r is the distance between sphere centres.



Nature of science

Scientists in Coulomb's day published their work in a very different way from scientists today. Coulomb wrote his results in a series of books called *Mémoires*. Part of Coulomb's original *Mémoire* in which he states the result is shown in figure 5.

Loi fondamentale de l'Électricité

La force répulsive de deux petits globes électrisés de la même nature d'électricité, est en raison inverse du carré de la distance du centre des deux globes.

▲ Figure 5

Later experiments confirmed that the force is proportional to the product of the size of the point charges q_1 and q_2 .

Combining Coulomb's results together with these gives

$$F \propto \frac{q_1 q_2}{r^2}$$

where the symbol \propto means "is proportional to".

The magnitude of the force F between two point charges of charge q_1 and q_2 separated by distance r in a vacuum is given by

$$F = \frac{kq_1 q_2}{r^2}$$

where k is the constant of proportionality and is known as Coulomb's constant.

In fact we do not always quote the law in quite this mathematical form. The constant is frequently quoted differently as

$$k = \frac{1}{4\pi\epsilon_0}$$

The new constant ϵ_0 is called the permittivity of free space (free space is an older term for "vacuum"). The 4π is added to rationalize electric and

magnetic equations – in other words, to give them a similar shape and to retain an important relationship between them (see the TOK section on page 177).

So the equation becomes

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

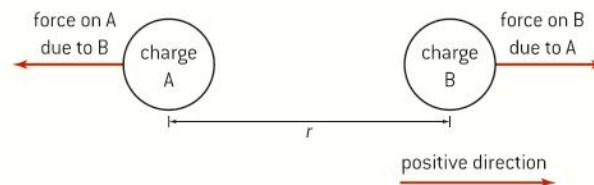
When using charge measured in coulombs and distance measured in metres, the value of k is $9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$.

This means that ϵ_0 takes a value of $8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ or, in fundamental units, $\text{m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2$.

The equation as it stands applies only for charges that are in a vacuum. If the charges are immersed in a different medium (say, air or water) then the value of the permittivity is different. It is usual to amend the equation slightly too, k becomes $\frac{1}{4\pi\epsilon}$ as the “0” subscript in ϵ_0 should only be used for the vacuum case. For example, the permittivity of water is $7.8 \times 10^{-10} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ and the permittivity of air is $8.8549 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$. The value for air is so close to the free-space value that we normally use $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ for both. The table gives a number of permittivity values for different materials.

Material	Permittivity / $10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$
paper	34
rubber	62
water	779
graphite	106
diamond	71

This equation appears to say nothing about the direction of the force between the charged objects. Forces are vectors, but charge and $(\text{distance})^2$ are scalars. There are mathematical ways to cope with this, but for point charges the equation gives an excellent clue when the signs of the charges are included.



▲ Figure 6 Force directions.

We take the positive direction to be from charge A to charge B; in figure 6 that is from left to right. Let's begin with both charge A and charge B being positive. When two positive charges are multiplied together in $\frac{q_1 q_2}{r^2}$, the resulting sign of the force acting on charge B due to charge A is also positive. This means that the direction of the force will be assigned the positive direction (from charge A to charge B): in other words, left to right. This agrees with the physics because the charges are repelled. If both charges are negative then the answer is the same, the charges are repelled and the force is to the right.



If, however, one of the charges is positive and the other negative, then the product of the charges is negative and the force direction will be opposite to the left-to-right positive direction. So the force on charge 2 is now to the left. Again, this agrees with what we expect, that the charges attract because they have opposite signs.

Worked examples

- 1** Two point charges of $+10\text{ nC}$ and -10 nC in air are separated by a distance of 15 mm .
- Calculate the force acting between the two charges.
 - Comment on whether this force can lift a small piece of paper about $2\text{ mm} \times 2\text{ mm}$ in area.

Solution

- a)** It is important to take great care with the prefixes and the powers of ten in electrostatic calculations.

The charges are: $+10 \times 10^{-9}\text{ C}$ and $-10 \times 10^{-9}\text{ C}$. The separation distance is $1.5 \times 10^{-2}\text{ m}$ (notice how the distance is converted right at the outset into consistent units).

$$\begin{aligned} \text{So } F &= \frac{(+1 \times 10^{-8}) \times (-1 \times 10^{-8})}{4\pi\epsilon_0 (1.5 \times 10^{-2})^2} \\ &= 4.0 \times 10^{-3}\text{ N} \end{aligned}$$

The charges are attracted along the line joining them. (Do not forget that force is a vector and needs both magnitude and direction for a complete answer.)

- b)** A sheet of thin A4 paper of dimensions 210 mm by 297 mm has a mass of about 2 g . So the small area of paper has a mass of about $1.3 \times 10^{-7}\text{ kg}$ and therefore a weight of $1.3 \times 10^{-6}\text{ N}$. The electrostatic force could lift this paper easily.
- 2** Two point charges of magnitude $+5\text{ }\mu\text{C}$ and $+3\text{ }\mu\text{C}$ are 1.5 m apart in a liquid that has a permittivity of $2.3 \times 10^{-11}\text{ C}^2\text{ N}^{-1}\text{ m}^{-2}$.

Calculate the force between the point charges.

Solution

$$F = \frac{(+5 \times 10^{-6}) \times (+3 \times 10^{-6})}{4\pi \times 2.3 \times 10^{-11} \times (1.5)^2} = 23\text{ mN};$$

a repulsive force acting along the line joining the charges.

Electric fields

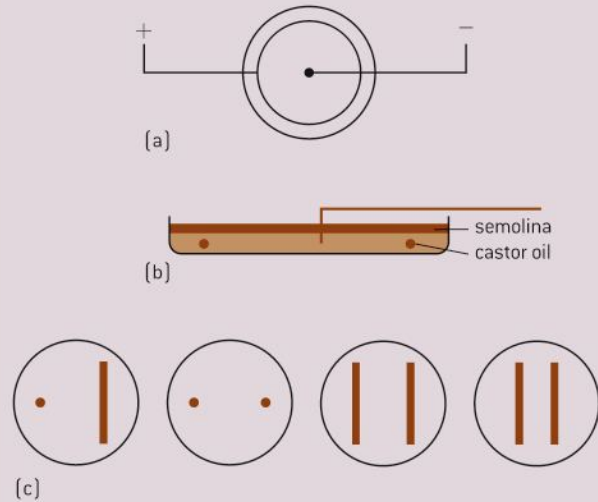
Sometimes the origin of a force between two objects is obvious, an example is the friction pad in a brake rubbing on the rim of a bicycle wheel to slow the cycle down. In other cases there is no physical contact between two objects yet a force exists between them. Examples of this include the magnetic force between two magnets and the electrostatic force between two charged objects. Such forces are said to “act at a distance”.

The term **field** is used in physics for cases where two separated objects exert forces on each other. We say that in the case of the comb picking up the paper, the paper is sitting in the **electric field** due to the comb. The concept of the field is an extremely powerful one in physics not least because there are many ideas common to all fields. As well as the magnetic and electrostatic fields already mentioned, gravity fields also obey the same rules. Learn the underlying ideas for one type of field and you have learnt them all.

Mapping fields

 **Investigate!**

Plotting electric fields



▲ Figure 7

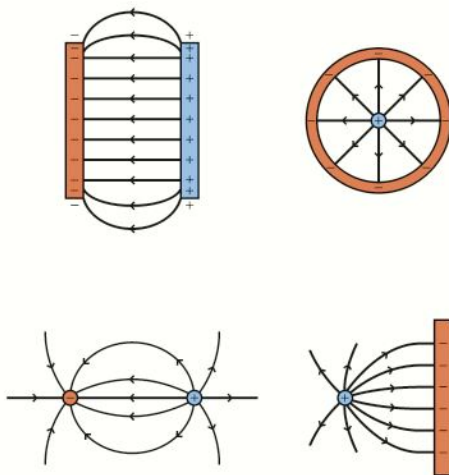
- At an earlier stage in your school career you may have plotted magnetic field patterns using iron filings (if you have not done this there is an Investigate! in Sub-topic 5.4 to illustrate

the method). This experiment allows patterns to be observed for electric fields.

- Put some castor oil in a Petri dish and sprinkle some grains of semolina (or grits) onto the oil. Alternatives for the semolina include grass seed and hairs cut about 1 mm long from an artist paint brush.
- Take two copper wires and bend one of them to form a circle just a little smaller than the internal diameter of the Petri dish. Place the end of the other wire in the centre of the Petri dish.
- Connect a 5 kV power supply to the wires – take care with the power supply!
- Observe the grains slowly lining up in the electric field.
- Sketch the pattern of the grains that is produced.
- Repeat with other wire shapes such as the four shown in figure 7(c).

In the plotting experiment, the grains line up in the field that is produced between the wires. The patterns observed resemble those in figure 8. The experiment cannot easily show the patterns for charges with the same sign.

The idea of field lines was first introduced by Michael Faraday (his original idea was of an elastic tube that repelled other tubes). Although field lines are imaginary, they are useful for illustrating and understanding the nature of a particular field. There are some conventions for drawing these electric field patterns.



▲ Figure 8 Electric field patterns.

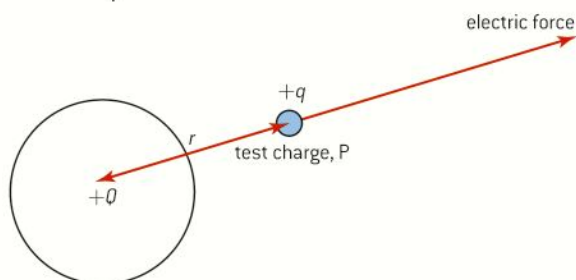
- The lines start and end on charges of opposite sign.
- An arrow is essential to show the direction in which a positive charge would move (i.e. away from the positive charge and towards the negative charge).
- Where the field is strong the lines are close together. The lines act to repel each other.
- The lines never cross.
- The lines meet a conducting surface at 90° .



Electric field strength

As well as understanding the field pattern, we need to be able to measure the strength of the electric field. The **electric field strength** is defined using the concept of a **positive test charge**. Imagine an isolated charge Q sitting in space. We wish to know what the strength of the field is at a point P, a distance r away from the isolated charge. We put another charge, a positive test charge of size q , at P and measure the force F that acts on the test charge due to Q . Then the magnitude of the electric field strength is defined to be

$$E = \frac{F}{q}$$



▲ Figure 9 Definition of electric field strength.

The units of electric field strength are N C^{-1} . (Alternative units, that have the same meaning, are V m^{-1} and will be discussed in Topic 11.) Electric field strength is a vector, it has the same direction as the force F (this is because the charge is a scalar which only “scales” the value of F up or down). **A formal definition for electric field strength at a point is the force per unit charge experienced by a small positive point charge placed at that point.**

Coulomb’s law can be used to find how the electric field strength varies with distance for a point charge.

Q is the isolated point charge and q is the test charge, so

$$F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2}$$

$$E = \frac{F}{q}$$

Therefore

$$E = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2} \times \frac{1}{q}$$

so

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

The electric field strength of the charge at a point is proportional to the charge and inversely proportional to the square of the distance from the charge.

If Q is a positive charge then E is also positive. Applying the rule that r is measured from the charge to the test charge, then if E is positive it acts outwards away from the charge. This is what we expect as both charge and test charge are positive. When Q is negative, E acts towards the charge Q .

The field shape for a point charge is known as a **radial field**. The field lines radiate away (positive) or towards (negative) the point charge as shown in figure 10.

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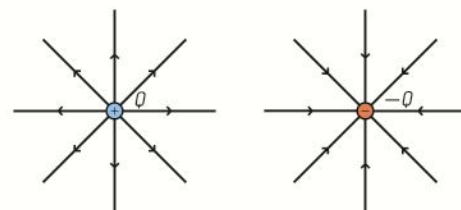
So why not use k ?

James Maxwell, working in the middle of the nineteenth century, realized that there was an important connection between electricity, magnetism and the speed of light. In particular he was able to show that the permittivity of free space ϵ_0 (which relates to electrostatics) and the permeability of free space μ_0 (which relates to electromagnetism) are themselves connected to the speed of light c :

$$\frac{1}{\epsilon_0\mu_0} = c^2$$

This proves to be an important equation. So much so that, in the set of equations that arise from the SI units we use, we choose to use ϵ_0 in all the electrostatics equations and μ_0 in all the magnetic equations.

However, not all unit systems choose to do this. There is another common system, the cgs system (based on the centimetre, the gram and the second, rather than the metre, kilogram and second). In cgs, the value of the constant k in Coulomb’s law is chosen to be 1 and the equation appears as $F = \frac{q_1q_2}{r^2}$. If the numbers are different, is the physics the same?



▲ Figure 10 Radial fields for positive and negative point charges.

Worked examples

1 Calculate the electric field strengths in a vacuum

- 1.5 cm from a $+10 \mu\text{C}$ charge
- 2.5 m from a -0.85 mC charge

Solution

a) Begin by putting the quantities into consistent units: $r = 1.5 \times 10^{-2} \text{ m}$ and $Q = 1.0 \times 10^{-5} \text{ C}$.

$$\text{Then } E = \frac{1.0 \times 10^{-5}}{4\pi\epsilon_0 (1.5 \times 10^{-2})^2} = +4.0 \times 10^8 \text{ N C}^{-1}.$$

The field direction is away from the positive charge.

$$\text{b) } E = \frac{-8.5 \times 10^{-4}}{4\pi\epsilon_0 (2.5)^2} = -1.2 \times 10^6 \text{ N C}^{-1}$$

The field direction is towards the negative charge.

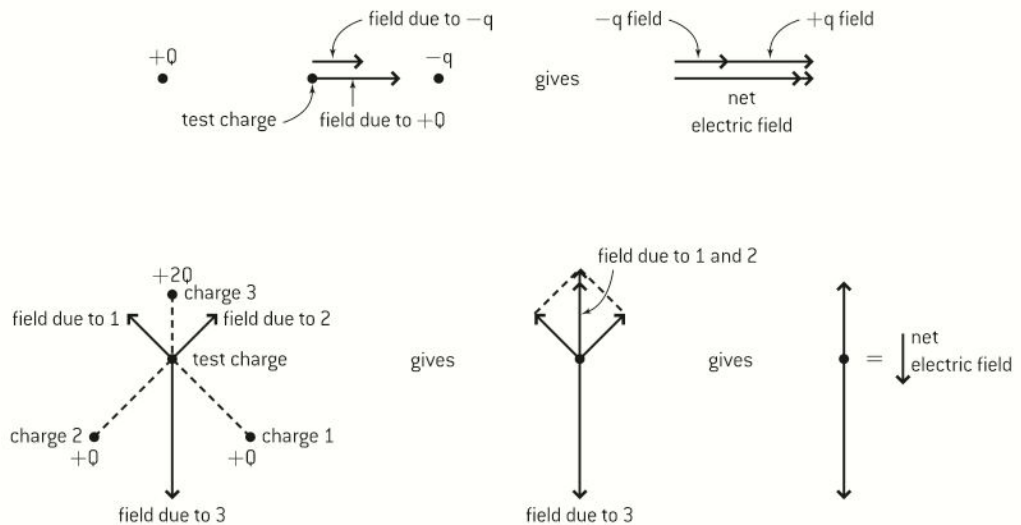
2 An oxygen nucleus has a charge of $+8e$. Calculate the electric field strength at a distance of 0.68 nm from the nucleus.

Solution

The charge on the oxygen nucleus is $8 \times 1.6 \times 10^{-19} \text{ C}$; the distance is $6.8 \times 10^{-10} \text{ m}$.

$$E = \frac{1.3 \times 10^{-18}}{4\pi\epsilon_0 \times (6.8 \times 10^{-10})^2} = -2.5 \times 10^{10} \text{ N C}^{-1} \text{ away from the nucleus.}$$

The electric field strengths can be added using either a calculation or a scale diagram as outlined in Topic 1.



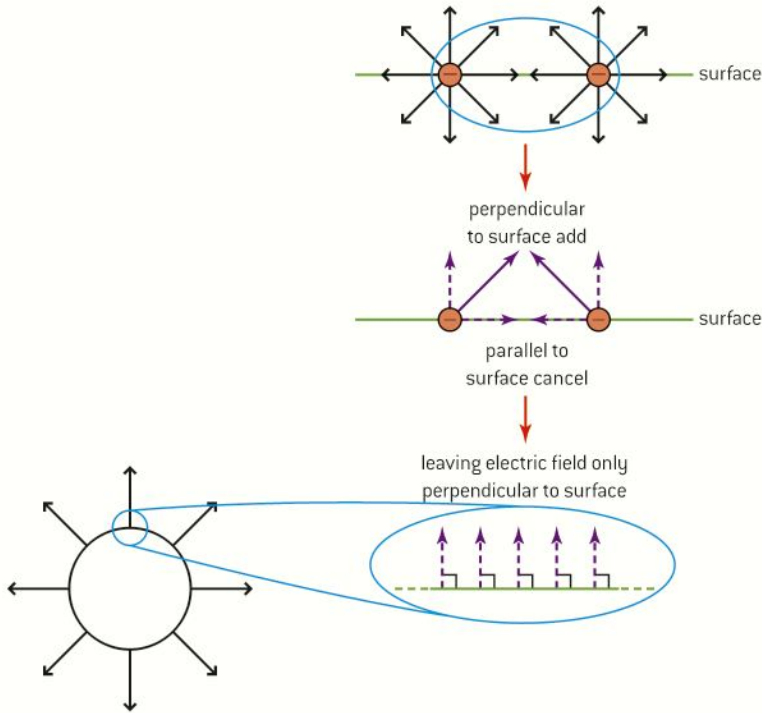
▲ Figure 11 Vector addition of electric fields.

This vector addition of field strengths (figure 11) can give us an insight into electric fields that arise from charge configurations that are more complex than a single point charge.

Close to a conductor

Imagine going very close to the surface of a conductor. Figure 12 shows what you might see. First of all, if we are close enough then the surface will appear flat (in just the same way that we are not aware of the curvature of the Earth until we go up in an aircraft). Secondly we would see that all the free electrons are equally spaced. There is a good reason for this: any one electron has forces acting on it from the other electrons. The electron will accelerate until all these forces balance out and it is in equilibrium, for this to happen they must be equally spaced.

Now look at the field strength vectors radiating out from these individual electrons. Parallel to the surface, these all cancel out with each other so there is no electric field in this direction. (This is the same as saying that any one electron will not accelerate as the horizontal field strength is



▲ Figure 12 Close to conducting surfaces.

zero). Perpendicular to the surface, however, things are different. Now the field vectors all add up, and because there is no field component parallel to the surface, the local field must act at 90° to it.

So, close to a conducting surface, the electric field is at 90° to the surface.

Conducting sphere

This can be taken a step further for a conducting sphere, whether it is hollow or solid. Again, the free electrons at the surface are equally spaced and all the field lines at the surface of the sphere are at 90° to it. The consequence is that the field must be radial, just like the field of an isolated point charge. So, to a test charge outside the sphere, the field of the sphere appears exactly the same as that of the point charge. Mathematical analysis shows that outside a sphere the field indeed behaves as though it came from a point charge placed at the centre of the sphere with a charge equal to the total charge spread over the sphere.

(Inside the sphere is a different matter, it turns out that there is no electric field inside a sphere, hollow or solid, a result that was experimentally determined by Franklin.)

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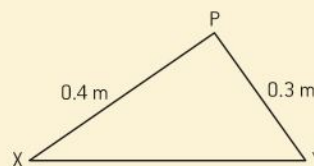
But does the test charge affect the original field?

If we wanted to use the definition of the strength of an electric field practically by using a test charge, we would have to take care. Just as a thermometer alters the temperature of the object it measures, so the test charge will exert a force on the original charge (the one with the field we are trying to measure) and may accelerate it, or disturb the field lines. This means in practice that a test charge is really an imaginary construct that we use to help our understanding. In practice we prefer to measure electric field strengths using the idea that it is the potential gradient, in other words the change in the voltage divided by the change in distance: the larger this value, the greater the field strength. We will leave further discussion of this point to Topic 11 but it is food for thought from a theory of knowledge perspective: a measurement that we can think about but not, in practice, carry out. The German language has a name for it: *gedankenexperiment* "thought experiment". How can a practical subject such as a science have such a thing?

Worked example

Two point charges, a $+25$ nC charge X and a $+15$ nC charge Y are separated by a distance of 0.5 m.

- Calculate the resultant electric field strength at the midpoint between the charges.
- Calculate the distance from X at which the electric field strength is zero.
- Calculate the magnitude of the electric field strength at the point P on the diagram. X and Y are 0.4 m and 0.3 m from P respectively.



Solution

$$\text{a) } E_A = \frac{2.5 \times 10^{-8}}{4\pi\epsilon_0 \times 0.25^2} = 3600 \text{ N C}^{-1}$$

$$E_B = \frac{1.5 \times 10^{-8}}{4\pi\epsilon_0 \times 0.25^2} = 2200 \text{ N C}^{-1}$$

The field strengths act in opposite directions, so the net electric field strength is $(3600 - 2200) = 1400 \text{ N C}^{-1}$; this is directed away from X towards Y.

b) For E to be zero, $E_A = -E_B$ and so

$$\frac{2.5 \times 10^{-8}}{4\pi\epsilon_0 \times d^2} = \frac{1.5 \times 10^{-8}}{4\pi\epsilon_0 \times (0.5 - d)^2}$$

thus

$$\frac{d^2}{(0.5 - d)^2} = \frac{2.5}{1.5}$$

or

$$\frac{d}{(0.5 - d)} = \sqrt{\frac{2.5}{1.5}} = 1.3$$

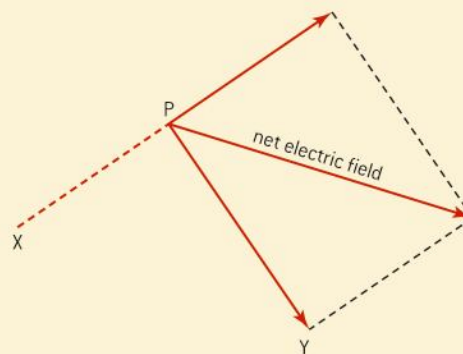
$$d = 0.65 - 1.3d$$

$$2.3d = 0.65$$

$$d = 0.28 \text{ m}$$

c) $PX = 0.4 \text{ m}$ so E_X at P is $\frac{2.5 \times 10^{-8}}{4\pi\epsilon_0 \times 0.4^2}$
 $= 1400 \text{ N C}^{-1}$ along XP in the direction away from X.

$PY = 0.3 \text{ m}$ so E_Y at P is $\frac{1.5 \times 10^{-8}}{4\pi\epsilon_0 \times 0.3^2}$
 $= 1500 \text{ N C}^{-1}$ along PY in the direction towards Y.



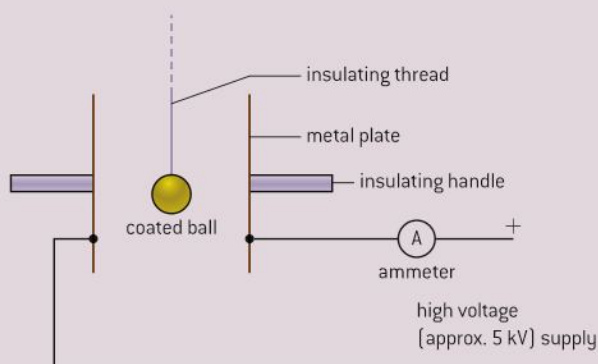
The magnitude of the resultant electric field strength is $\sqrt{1400^2 + 1500^2} = 2100 \text{ N C}^{-1}$

(The calculation of the angles was not required in the question and is left for the reader.)

Moving charge

 Investigate!

Moving charge around



▲ Figure 13

- Arrange two metal plates vertically and about 20 cm apart. Suspend a table tennis (ping-

pong) ball with a painted metal surface from a long insulating thread so that it is midway between the plates.

- Connect the plates to a high-voltage power supply with a sensitive ammeter or light-beam galvanometer in the circuit.
- When the supply is turned on, the ball should “shuttle” between the plates (it may need to be kickstarted by making it touch one of the plates).
- Notice what happens on the scale of the galvanometer. Every time the ball touches a plate there is a deflection on the meter. Look at the direction and size of the deflections – do they vary?



In the previous *Investigate!* the power supply is connected to the plates with conducting leads. Electrons move easily along these leads. When the supply is turned on, the electrons soon distribute themselves so that the plate connected to the negative supply has surplus electrons, and the other plate has a deficit of electrons, becoming positive. As the ball touches one of the plates it loses or gains some of the electrons. The charge gained by the ball will have the same sign as the plate and as a result the ball is almost immediately repelled. A force acts on the ball because it is in the electric field that is acting between the plates. The ball accelerates towards the other plate where it transfers all its charge to the new plate and gains more charge. This time the charge gained has the sign of the new plate. The process repeats itself with the ball transferring charge from plate to plate.

The meter is a sensitive ammeter, so when it deflects it shows that there is current in the wires leading to the plates. Charge is moving along these wires, so this is evidence that:

- an electric current results when charge moves
- the charge is moved by the presence of an electric field.

A mechanism for electric current

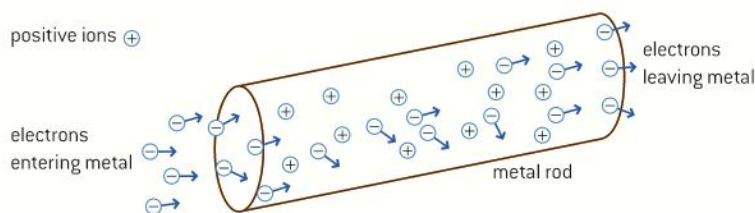
The shuttling ball and its charge show clearly what is moving in the space between the plates. However, the microscopic mechanisms that are operating in wires and cells are not so obvious. This was one of the major historical problems in explaining the physics of electricity.

Electrical conduction is possible in gases, liquids, solids, and a vacuum. Of particular importance to us is the electrical conduction that takes place in metals.

Conduction in metals

The metal atoms in a solid are bound together by the **metallic bond**. The full details of the bonding are complex, but a simple model of what happens is as follows.

When a metal solidifies from a liquid, its atoms form a regular lattice arrangement. The shape of the lattice varies from metal to metal but the common feature of metals is that as the bonding happens, electrons are donated from the outer shells of the atoms to a common sea of electrons that occupies the entire volume of the metal.



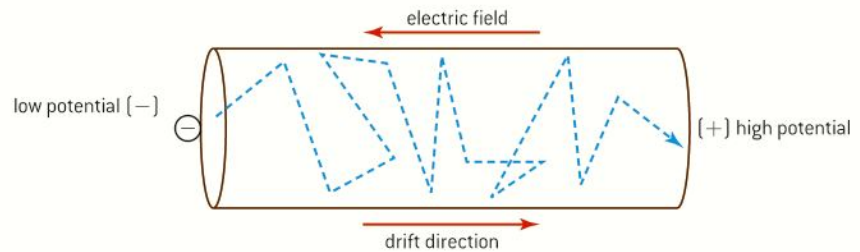
▲ Figure 14 Conduction by free electrons in a metal.

Figure 14 shows the model. The positive ions sit in fixed positions on the lattice. There are ions at each lattice site because each atom has now lost an electron. Of course, at all temperatures above absolute zero

they vibrate in these positions. Most of the electrons are still bound to them but around the ions is the sea of free electrons or **conduction electrons**; these are responsible for the electrical conduction.

Although the conduction electrons have been released from the atoms, this does not mean that there is no interaction between ions and electrons. The electrons interact with the vibrating ions and transfer their kinetic energy to them. It is this transfer of energy from electrons to ions that accounts for the phenomenon that we will call “resistance” in the next sub-topic.

The energy transfer in a conductor arises as follows:



▲ Figure 15

In a metal in the absence of an electric field, the free electrons are moving and interacting with the ions in the lattice, but they do so at random and at average speeds close to the speed of sound in the material. Nothing in the material makes an electron move in any particular direction.

However, when an electric field is present, then an electric force will act on the electrons with their negative charge. The definition of electric field direction reminds us that the electric field is the direction in which a positive charge moves, so the force on the electrons will be in the opposite direction to the electric field in the metal (figure 15).

In the presence of an electric field, the negatively charged electrons drift along the conductor. The electrons are known as **charge carriers**. Their movement is like the random motion of a colony of ants carried along a moving walkway.

Conduction in gases and liquids

Electrical conduction is possible in other materials too. Some gases and liquids contain free ions as a consequence of their chemistry. When an electric field is applied to these materials the ions will move, positive in the direction of the field, negative the opposite way. When this happens an electric current is observed.

If the electric field is strong enough it can, itself, lead to the creation of ions in a gas or liquid. This is known as **electrical breakdown**. It is a common effect during electrical storms when lightning moves between a charged cloud and the Earth. You will have seen such conduction in neon display tubes or fluorescent tubes use for lighting.



Nature of science

Models of conduction

The model here is of a simple flow of free electrons through a solid, a liquid, or a gas. But this is not the end of the story. There are other, more sophisticated models of conduction in solids that can explain the differences between conductors, semiconductors and insulators better than the flow model here. These involve the electronic band theory which arises from the interactions between the electrons within individual atoms and between the atoms themselves.

Essentially, this band model proposes that electrons have to adopt different energies within the substance and that some groups of energy levels (called band gaps) are not permitted to the electrons. Where there are wide band gaps,

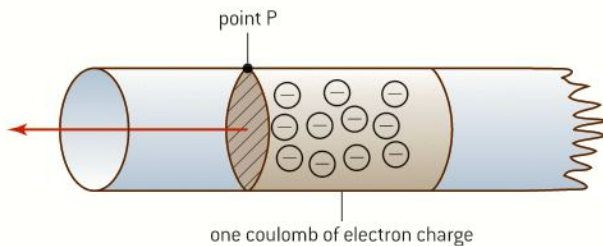
electrons cannot easily move from one set of levels to another and this makes the substance an insulator. Where the band gap is narrow, adding energy to the atomic structure allows electrons to jump across the band gap and conduct more freely – this makes a semiconductor, and you will later see that one of the semiconductor properties is that adding internal energy allows them to conduct better. In conductors the band gap is of less relevance because the electrons have many available energy states and so conduction happens very easily indeed.

Full details of this theory are beyond IB Physics, but if you have an interest in taking this further, you can find many references to the theory on the Internet.

Electric current

When charge flows in a conductor we say that there is an electric current in the conductor. Current is measured in **ampères**, the symbol for the unit is A. Often, in the English speaking world, the accent is omitted.

Current is linked to flow of charge in a simple way.



▲ Figure 16 Charge flow leading to current.

Imagine a block of electrons with a total charge of one coulomb moving along a conductor.

An observer at point P is watching these electrons move along the conductor. If all the electrons in the block move past the point in one second then, the current is one ampere.

If it takes twice as long (2 s) for the block to pass, then the current is half and is 0.5 A.

If the block takes 0.1 s to pass the observer, then the current is 10 A.

Mathematically

$$\text{electric current, } I = \frac{\text{total charge that moved past a point}}{\text{time taken for charge to move past the point}}$$

Tip

It is not good practice to write or say that “current flows”, what is flowing in the circuit is the electric charge. The movement of this charge is what we call current and it is best to write that “there is a current in the circuit, or in a component” as appropriate.

or in symbols

$$I = \frac{\Delta Q}{\Delta t}$$

The ampere is a **fundamental unit** defined as part of the SI. Although it is *explained* here in terms of the flow of charge, this is not how it is *defined*. The SI definition is based on ideas from magnetism and is covered in Sub-topics 1.1 and 5.4.



Nature of science

Another physics link

This link between flowing charge and current is a crucial one. Electrical current is a macroscopic quantity, transfer of charge by electrons is a microscopic phenomenon in every sense of the word. This is another example of a link in physics between macroscopic observations and inferences about what is happening on the smallest scales.

It was the lack of knowledge of what happens inside conductors at the atomic scale that forced scientists, up to the end of the nineteenth century, to develop concepts such as current and field to explain the effects they observed. It also, as we shall see, led to a crucial mistake.

Worked examples

1 In the shuttling ball experiment, the ball moves between the two charged plates at a frequency of 0.67 Hz. The ball carries a charge of magnitude 72 nC each time it crosses from one plate to the other.

Calculate:

- the average current in the circuit
- the number of electrons transferred each time the ball touches one of the plates.

Solution

a) The time between the ball being at the same plate is $\frac{1}{f} = \frac{1}{0.67} = 1.5$ s. The time to transfer 72 nC is therefore 0.75 s.

$$\text{Current} = \frac{7.2 \times 10^{-8}}{0.75} = 96 \text{ nA}$$

b) The charge transferred is 72 nC = 7.2×10^{-8} C

Each electron has a charge of -1.6×10^{-19} C, so the number of electrons involved in the transfer is $\frac{7.2 \times 10^{-8}}{1.6 \times 10^{-19}} = 4.5 \times 10^{11}$

- Calculate the current in a wire through which a charge of 25 C passes in 1500 s.
 - The current in a wire is 36 mA. Calculate the charge that flows along the wire in one minute.

Solution

a) $I = \frac{\Delta Q}{\Delta t}$, so the current = $\frac{25}{1500} = 17$ mA

b) $\Delta Q = I\Delta t$ and $\Delta t = 60$ s. Thus charge that flows = $3.6 \times 10^{-2} \times 60 = 2.2$ C

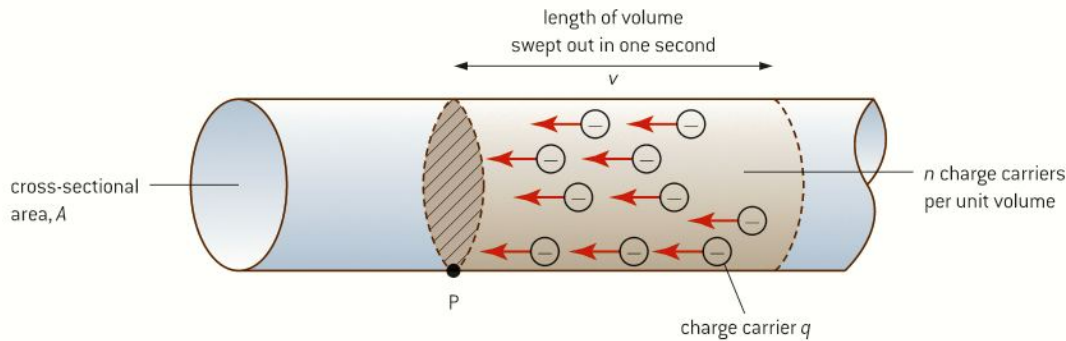
Charge carrier drift speed

Turn on a lighting circuit at home and the lamp lights almost immediately. Does this give us a clue to the speed at which the electrons in the wires move? In the *Investigate!* experiment on page 185, the stain indicating the position of the ions moves at no more than a few millimetres per second. The lower the value of the current, the slower the rate at which the ions move. This slow speed at which the ions move along the conductor is known as the **drift speed**.

We need a mathematical model to confirm this observation.



Imagine a cylindrical conductor that is carrying an electric current I . The cross-sectional area of the conductor is A and it contains charge carriers each with charge q . We assume that each carrier has a speed v and that there are n charge carriers in 1 m^3 of conductor – this quantity is known as the **charge density**.



▲ Figure 18 A model for conduction.

Figure 18 shows charge carriers, each of charge q , moving past point P at a speed v .

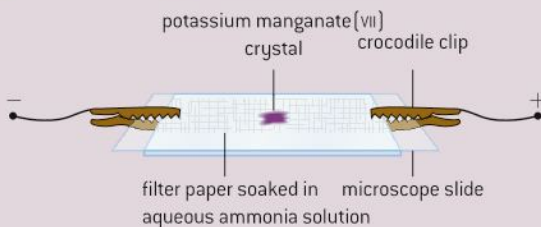
In one second, a volume Av of charge carriers passes P.

The total number of charge carriers in this volume is nAv and therefore the total charge in the volume is $nAvq$.

However, this is the charge that passes point P in one second, which is what we mean by the electric current. So

$$I = nAvq$$

Investigate!



▲ Figure 17

- The speeds with which electrons move in a metal conductor during conduction are difficult to observe, but the progress of conducting ions in a liquid can be inferred by the trace they leave.
- You should wear eye protection during this experiment.
- Fold a piece of filter paper around a microscope slide and fix it with two crocodile clips at the ends of the slide. The crocodile clips should be attached to leads that are connected to a low voltage power supply (no more than 25 V is required in this experiment).
- Wet the filter paper with aqueous ammonia solution.
- Take a small crystal of potassium manganate(VII) and place it in the centre of the filter paper. Ensure that the slide is horizontal.
- Turn on the current and watch the crystal. You should see a stain on the paper moving away from the crystal.
- Reverse the current direction to check that the effect is not due to the slide not being horizontal.
- How fast is the stain moving?

Worked example

- 1 A copper wire of diameter 0.65 mm carries a current of 0.25 A. There are 8.5×10^{28} charge carriers in each cubic metre of copper; the charge on each charge carrier (electron) is 1.6×10^{-19} C. Calculate the drift speed of the charge carriers.

Solution

Rearranging the equation

$$v = \frac{I}{nAQ}$$

and the area A of the wire is $\pi \left(\frac{0.65 \times 10^{-3}}{2}\right)^2$
 $= 3.3 \times 10^{-7} \text{ m}^2$.

$$\text{So } v = \frac{0.25}{8.5 \times 10^{28} \times 3.3 \times 10^{-7} \times 1.6 \times 10^{-19}}$$

$$= 0.055 \text{ mm s}^{-1}$$

The example shows that the drift speed of each charge is less than one-tenth of a millimetre each second. You may well be surprised by this result – but it is probably of the same order as the speed you observed in the experiment with the potassium manganate(VII). Although the electron charge is very small, the speed can also be small because there are very large numbers of free electrons available for conduction in the metal.

To see how sensitive the drift speed is to changes in the charge carrier density, we can compare the drift speed in copper with the drift speed in a semiconductor called germanium. The number of charge carriers in one cubic metre of germanium is about 10^9 less than in copper. So to sustain the same current in a germanium sample would require a drift speed 10^9 times greater than in the copper or a cross-sectional area 10^9 as large.

The slow drift speed in conductors for substantial currents poses the question of how a lamp can turn on when there may be a significant run of cable between switch and lamp. The charge carriers in the cable are drifting slowly around the cable. However, the information that the charge carriers are to begin to move when the switch is closed travels much more quickly – close to the speed of light in fact. The information is transferred when an electromagnetic wave propagates around the cable and produces a drift in all the free electrons virtually simultaneously. So the lamp can turn on almost instantaneously, even though, for direct current, it may take an individual electron many minutes or even hours to reach the lamp itself.

Potential difference

Free electrons move in a conductor when an electric field acts on the conductor. Later we shall see how devices such as electric cells and power supplies provide this electric field. At the same time the power supplies transfer energy to the electrons. As the electrons move through the conductors, they collide with the positive ions in the lattice and transfer the energy gained from the field to the ions.

In situations where fields act, physicists use two quantities called potential and **potential difference** when dealing with energy transfers. Potential difference (often abbreviated to “pd”) is a measure of the electrical potential energy transferred from an electron when it is moving between two points in a circuit. However, given the very small amount of charge possessed by each electron this amount of energy is also very small. It is better to use the much larger quantity represented by one coulomb of charge.



Potential difference between two points is defined as the work done (energy transferred) W when one unit of charge Q moves between the points.

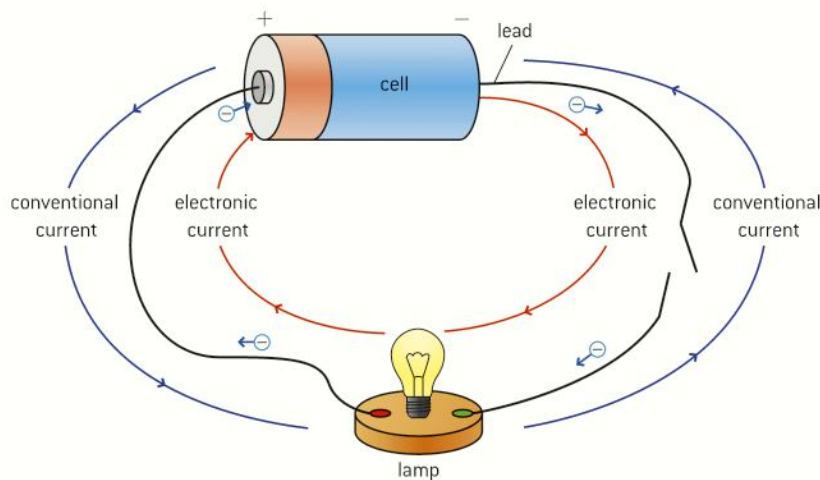
$$\text{potential difference} = \frac{W}{Q}$$

The symbol given to potential difference is V ; its unit is the J C^{-1} and is named the volt (symbol: V) after the Italian scientist Alessandro Volta who was born in the middle of the eighteenth century and who worked on the development of electricity.

The potential difference between two points is one volt if one joule of energy is transferred per coulomb of charge passing between the two points.

A simple circuit will illustrate these ideas:

An electric cell is connected to a lamp via a switch and three leads. Figure 19 shows a picture of the circuit as it would look set up on the bench.



▲ Figure 19 Conventional and electronic current in a circuit.

When the switch is closed, electrons flow round the circuit. Notice the direction in which the electrons move and also that the diagram shows the direction of a **conventional current**. The two directions are opposite; in this case, clockwise for the electron flow and anti-clockwise for the conventional current. The reason for this difference is explained in a later Nature of science section. You need to take care with this difference, particularly when using some of the direction rules that are introduced later in this topic.

What happens to an electron as it goes round the circuit once? The electron gains electric potential energy as it moves through the cell (this will be covered in Sub-topic 5.3). The electron then leaves the cell and begins to move through the connecting lead. Leads are designed so that they do not require much energy transfer to allow the electrons through (we say that they have a low electrical resistance) and so the potential difference from one end of the lead to the other is small. The electron moves through the switch which also gains little energy from the charge carrier.

After moving through another lead the electron reaches the lamp. This component is different from others in the circuit, it is deliberately designed so that it can gain much of the electrical potential energy from

the electrons as they pass through it. The metal lattice in the filament gains energy and as a result the ions vibrate at greater speeds and with greater amplitudes. At these high temperatures the filament in the lamp will glow brightly; the lamp will be lit.

In potential difference terms, the pds across the leads and the switch are small because the passage of one coulomb of charge through them will not result in much energy transfer to the lattice. The pd across the lamp will be large because, for each coulomb going through it, large amounts of energy are transferred from electrons to the lattice ions in the filament raising its temperature.



Nature of science

Conventional and electron currents

In early studies of current electricity, the idea emerged that there was a flow of “electrical fluid” in wires and that this flow was responsible for the observed effects of electricity. At first the suggestion was that there were two types of fluid known as “vitreous” and “resinous”. Benjamin Franklin (the same man who helped draft the US Declaration of Independence) proposed that there was only one fluid but that it behaved differently depending on the circumstances. He was also the first scientist to use the terms “positive” and “negative”.

What then happened was that scientists assigned a positive charge to the “fluid” thought to be moving in the wires. This positive charge was said to flow out of the positive terminal of a power supply (because the charge was repelled) and went around the circuit re-entering the power

supply through the negative terminal. This is what we now term the conventional current.

In fact, we now know that in a metal the charge carriers are electrons and that they move in the opposite direction, leaving a power supply at the negative terminal. This is termed the **electronic current**. You should take care with these two currents and not confuse them.

You may ask: why do we now not simply drop the conventional current and talk only about the electronic current? The answer is that other rules in electricity and magnetism were set up on the assumption that charge carriers are positive. All these rules would need to be reversed to take account of our later knowledge. It is better to leave things as they are.

Worked examples

- 1 A high efficiency LED lamp is lit for 2 hours. Calculate the energy transfer to the lamp when the pd across it is 240 V and the current in it is 50 mA.

Solution

2 hours is $2 \times 60 \times 60 = 7200$ s.

The charge transferred is $I\Delta t$
 $= 7.2 \times 10^3 \times 50 \times 10^{-3} = 360$ C

Work done = charge \times pd = $360 \times 240 = 86\,400$ J

- 2 A cell has a terminal voltage of 1.5 V and can deliver a charge of 460 C before it becomes discharged.

- a) Calculate the maximum energy the cell can deliver.
 b) The current in the cell never exceeds 5 mA. Estimate the lifetime of the cell.

Solution

a) Potential difference, $V = \frac{W}{q}$
 so $W = qV = 460 \times 1.5 = 690$ J

b) The current of 5 mA means that no more than 5 mC flows through the cell at any time. So $\frac{460}{0.005} = 92\,000$ s (which is about 25 hours)



Electromotive force (emf)

Another important term used in electric theory is **electromotive force** (usually written as “emf” for brevity). This term seems to imply that there is a force involved in the movement of charge, but the real meaning of emf is connected to the energy changes in the circuit. When charge flows electrical energy can go *into* another form such as internal energy (through the heating, or Joule, effect), or it can be converted *from* another form (for example, light (radiant energy) in solar (photovoltaic) cells). The term emf will be used in this course when energy is transferred *to* the electrons in, for example, a battery. (Other devices can also convert energy into an electrical form. Examples include microphones and dynamos.)

The term *potential difference* will be used when the energy is transferred *from* the electrical form. So, examples of this would be electrical into heat and light, or electrical into motion energy.

The table shows some of the devices that transfer electrical energy and it gives the term that is most appropriate to use for each one.

Device				pd or emf?	
Cell	<i>converts energy from</i>	chemical	<i>into</i>	electrical	emf
Resistor		electrical		internal	pd
Microphone		sound		electrical	emf
Loudspeaker		electrical		sound	pd
Lamp		electrical		light (and internal)	pd
Photovoltaic cell		light		electrical	emf
Dynamo		kinetic		electrical	emf
Electric motor		electrical		kinetic	pd

Power, current, and pd

We can now answer the question of how much energy is delivered to a conductor by the electrons as they move through it.

Suppose there is a conductor with a potential difference V between its ends when a current I is in the conductor.

In time Δt the charge Q that moves through the conductor is equal to $I\Delta t$.

The energy W transferred to the conductor from the electrons is QV which is $(I\Delta t)V$.

So the energy transferred in time Δt is

$$W = IV\Delta t$$

The electrical power being supplied to the conductor is $\frac{\text{energy}}{\text{time}} = \frac{W}{\Delta t}$ and therefore

$$\text{electrical power } P = IV$$

Alternative forms of this expression that you will find useful are $I = \frac{P}{V}$ and $V = \frac{P}{I}$

The **unit of power is the watt (W)** – 1 watt (1 W) is the power developed when 1 J is converted in 1 s, the same in both mechanics and electricity. So another way to think of the volt is as the power transferred per unit current in a conductor.

Worked examples

1 A 3 V, 1.5 W filament lamp is connected to a 3 V battery. Calculate:

- the current in the lamp
- the energy transferred in 2400 s.

Solution

a) Electrical power, $P = IV$, so $I = \frac{P}{V} = \frac{1.5}{3} = 0.5$ A

b) The energy transferred every second is 1.5 J so in 2400 s, 3600 J.

2 An electric motor that is connected to a 12 V supply is able to raise a 0.10 kg load through

a distance of 1.5 m in 7 s. The motor is 40% efficient. Calculate the average current in the motor while the load is being raised.

Solution

The energy gained is $mg\Delta h$
 $= 0.01 \times g \times 1.5 = 0.147$ J

The power output of the motor must be
 $= \frac{0.147}{7} = 0.021$ W

The current in the motor $= \frac{P}{V} = \frac{0.021}{12} = 1.8$ mA.
 Since the motor is 40% efficient the current will be 4.5 mA.



Nature of science

A word about potential

The use of the term “potential difference” implies that there is something called potential which can differ from point to point. This is indeed the case.

An isolated positive point charge will have field lines that radiate away from it. A small positive test charge in this field will have a force exerted on it in the field line direction and, if free to do so, will accelerate away from the original charge. When the test charge is close, there is energy stored in the system and we say that the system (the two charges interacting) has a high potential. When the test charge is further away, there is still energy stored, but it is smaller because the system has converted energy into the kinetic energy of the charge – it has “done work” on the charge. So to move the test charge away from the original charge transfers some of the original stored

energy; this is described as a loss of potential. Positive charges move from points of high potential to low potential if they are free to do so.

Negative charges on the other hand move from points of low potential to high. You can work this through yourself by imagining a negative test charge near a positive original charge. This time the two charges are attracted and to move the negative charge away we have to do work on the system. This increases the potential of the system. If charges are free they will fall towards each other losing potential energy. In what form does this energy re-appear?

We shall return to a discussion of potential in Topic 10. From now on Topic 5 only refers to potential differences.

The electronvolt

Earlier we said that the energy possessed by individual electrons is very small. If a single electron is moved through a potential difference of equal to 15 V then as $V = \frac{W}{Q}$, so $W = QV$ and the energy gained by this electron is $15 \times 1.6 \times 10^{-19}$ J = 2.4×10^{-18} J. This is a very small amount and involves us in large negative powers of ten. It is more convenient to define a new energy unit called the **electronvolt** (symbol eV).



This is defined as the energy gained by an electron when it moves through a potential difference of one volt. An energy of 1 eV is equivalent to 1.6×10^{-19} J. The electronvolt is used extensively in nuclear and particle physics.

Be careful, although the unit sounds as though it might be connected to potential difference, like the joule it is a unit of energy.

Worked examples

1 An electron, initially at rest, is accelerated through a potential difference of 180 V. Calculate, for the electron:

- the gain in kinetic energy
- the final speed.

Solution

(a) The electron gains 180 eV of energy during its acceleration.

$$1 \text{ eV} \equiv 1.6 \times 10^{-19} \text{ J so } 180 \text{ eV} \equiv 2.9 \times 10^{-17} \text{ J}$$

(b) The kinetic energy of the electron is $\frac{1}{2}mv^2$ and the mass of the electron is 9.1×10^{-31} kg.

$$\text{So } v = \sqrt{\frac{2E_{\text{ke}}}{m_e}} = \sqrt{\frac{2 \times 2.9 \times 10^{-17}}{9.1 \times 10^{-31}}} = 8.0 \times 10^6 \text{ m s}^{-1}.$$

2 In a nuclear accelerator a proton is accelerated from rest gaining an energy of 250 MeV. Estimate the final speed of the particle and comment on the result.

Solution

The energy gained by the proton, in joules, is 4.0×10^{-11} J.

As before, $v = \sqrt{\frac{2E_{\text{ke}}}{m_p}} = \sqrt{\frac{2 \times 4.0 \times 10^{-11}}{1.7 \times 10^{-27}}}$, but using a value for the mass of the proton this time.

The numerical answer for $v = 2.2 \times 10^8 \text{ m s}^{-1}$. This is a large speed, 70% of the speed of light. In fact the speed will be less than this as some of the energy goes into increasing the mass of the proton through relativistic effects rather than into the speed of the proton.