16 OPTION D – ASTROPHYSICS

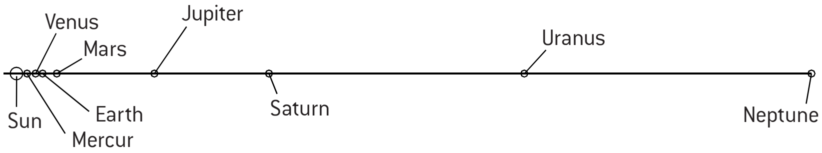
Objects in the universe (1)

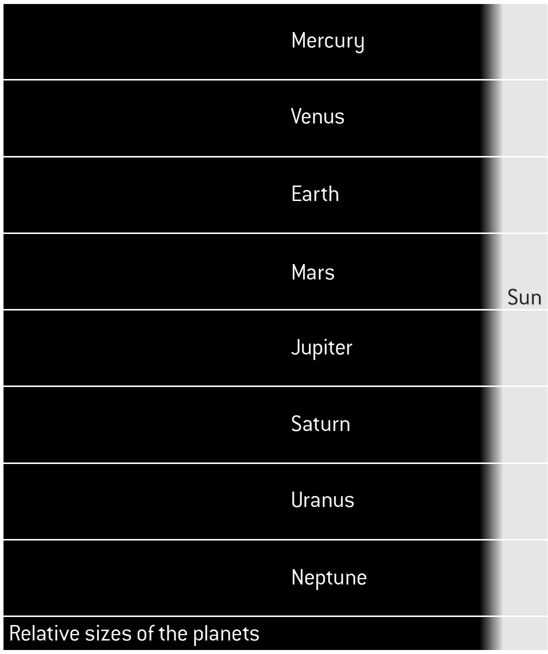
SOLAR SYSTEM

We live on the Earth. This is one of eight planets that orbit the Sun – collectively this system is known as the Solar System. Each planet is kept in its elliptical orbit by the gravitational attraction between the Sun and the planet. Other smaller masses such as **dwarf planets** like Pluto or planetoids also exist.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
diameter / km	4,880	12,104	12,756	6,787	142,800	120,000	51,800	49,500
distance to Sun / × 10^8 m	58	107.5	149.6	228	778	1,427	2,870	4,497

Relative positions of the planets





Some of these planets (including the Earth) have other small objects orbiting around them called moons. Our Moon is 3.8×10^8 m away and its diameter is about 1/4 of the Earth's.

An **asteroid** is a small rocky body that drifts around the Solar System. There are many orbiting the Sun between Mars and Jupiter – the asteroid belt. An asteroid on a collision course with another planet is known as a meteoroid.

Small meteors can be vaporized due to the friction with the atmosphere ('shooting stars') whereas larger ones can land on Earth. The bits that arrive are called **meteorites**.

Comets are mixtures of rock and ice (a 'dirty snowball') in very elliptical orbits around the Sun. Their 'tails' always point away from the Sun.

NEBULAE

In many constellations there are diffuse but relatively large structures which are called nebulae. These are interstellar clouds of dust, hydrogen, helium and other ionized gases. An example is M42 otherwise known as the Orion Nebula.

VIEW FROM ONE PLACE ON EARTH

If we look up at the night sky we see the stars – many of these 'stars' are, in fact, other galaxies but they are very far away. The stars in our own galaxy appear as a band across the sky – the Milky Way.

Patterns of stars have been identified and 88 different regions of the sky have been labelled as the different **constellations**. Stars in a constellation are not necessarily close to one another.

Over the period of a night, the constellations seem to rotate around one star. This apparent rotation is a result of the rotation of the Earth about its own axis.

On top of this nightly rotation, there is a slow change in the stars and constellations that are visible from one night to the next. This variation over the period of one year is due to the rotation of the Earth about the Sun.

Planetary systems have been discovered around many stars.

VIEW FROM PLACE TO PLACE ON EARTH

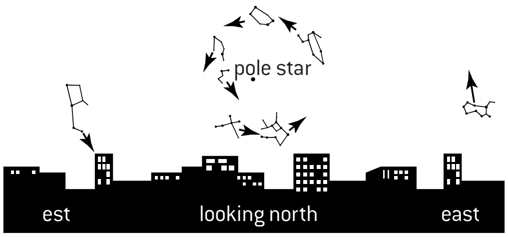
If you move from place to place around the Earth, the section of the night sky that is visible over a year changes with latitude. The total pattern of the constellations is always the same, but you will see different sections of the pattern.

Objects in the universe (2)

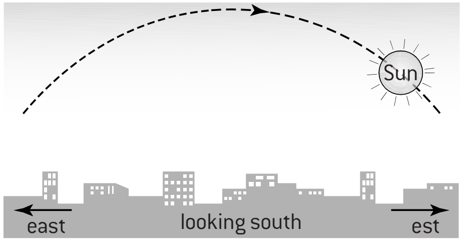
DURING ONE DAY

The most important observation is that the pattern of the stars remains the same from one night to the next. Patterns of stars have been identified and 88 different regions of the sky have been labelled as the different **constellations**. A particular pattern is not always in the same place, however. The constellations appear to move over the period of one night. They appear to rotate around one direction. In the Northern Hemisphere everything seems to rotate about the pole star.

It is common to refer measurements to the 'fixed stars' the patterns of the constellations. The fixed background of stars always appears to rotate around the pole star. During the night, some stars rise above the horizon and some stars set beneath it.



The same movement is continued during the day. The Sun rises in the east and sets in the west, reaching its maximum height at midday. At this time in the Northern Hemisphere the Sun is in a southerly direction.



DURING THE YEAR

Every night, the constellations have the same relative positions to each other, but the location of the pole star (and thus the portion of the night sky that is visible above the horizon) changes slightly from night to night. Over the period of a year this slow change returns back to exactly the same position.

The Sun continues to rise in the east and set in the west, but as the year goes from winter into summer, the arc gets bigger and the Sun climbs higher in the sky.

UNITS

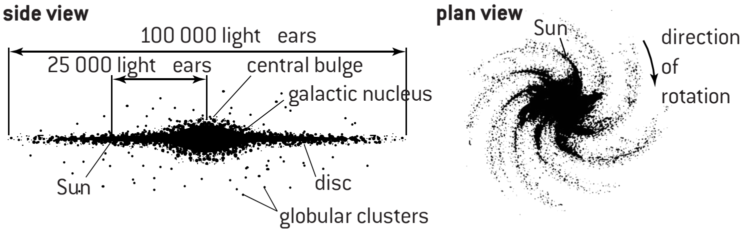
When comparing distances on the astronomical scale, it can be quite unhelpful to remain in SI units. Possible other units include the **astronomical unit** (**AU**), the **parsec** (**pc**) or the **light year** (**ly**). See page 193 for the definition of the first two of these.

The light year is the distance travelled by light in one year (9.5×10^{15} m). The next nearest star to our Sun is about 4 light years away. Our galaxy is about 100,000 light years across. The nearest galaxy is about a million light years away and the observable Universe is 13.7 billion light years in any given direction.

THE MILKY WAY GALAXY

When observing the night sky a faint band of light can be seen crossing the constellations. This 'path' (or 'way') across the night sky became known as the Milky Way. What you are actually seeing is some of the millions of stars that make up our own galaxy but they are too far away to be seen as individual stars. The reason that they appear to be in a band is that our galaxy has a spiral shape.

The centre of our galaxy lies in the direction of the constellation Sagittarius. The galaxy is rotating – all the stars are orbiting the centre of the galaxy as a result of their mutual gravitational attraction. The period of orbit is about 250 million years.

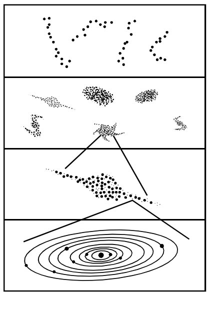


The Milky Way galaxy

THE UNIVERSE

Stars are grouped together in **stellar clusters**. These can be **open** containing 10^3 stars e.g. located in the disc of our galaxy or **globular** containing 10^5 stars. Our Sun is just one of the billions of stars in our **galaxy** (the Milky Way galaxy). The galaxy rotates with a period of about 2.5×10^8 years.

Beyond our galaxy, there are billions of other galaxies. Some of them are grouped together into **clusters** or **super clusters** of galaxies, but the vast majority of space (like the gaps between the planets or between stars) appears to be empty – essentially a vacuum. Everything together is known as the **Universe**.



1.5×10^{26} m (= 15 billion light years)	the visible Universe
5×10^{22} m (= 5 million light years)	local group of galaxies
10^{21} m (= 100,000 light years)	our galaxy
10^{13} m (= 0.001 light years)	our Solar System

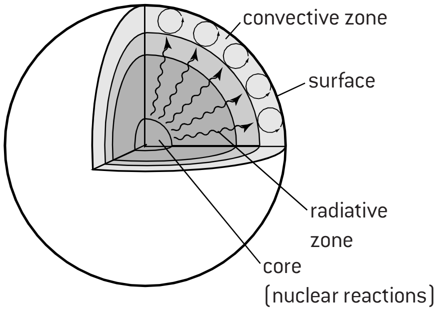
The nature of stars

ENERGY FLOW FOR STARS

The stars are emitting a great deal of energy. The source for all this energy is the fusion of hydrogen into helium. See page 196. Sometimes this is referred to as 'hydrogen burning' but it this is not a precise term. The reaction is a nuclear reaction, not a chemical one (such as combustion). Overall the reaction is

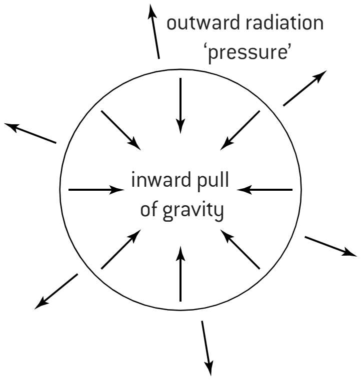
$$4\,{}^{1}_{1}\text{p} \rightarrow {}^{4}_{2}\text{He} + 2\,{}^{0}_{1}\text{e}^{+} + 2\nu$$

The mass of the products is less than the mass of the reactants. Using $\Delta E = \Delta m\, c^2$ we can work out that the Sun is losing mass at a rate of 4×10^{9} kg s^{-1}. This takes place in the core of a star. Eventually all this energy is radiated from the surface – approximately 10^{26} J every second. The structure inside a star does not need to be known in detail.



EQUILIBRIUM

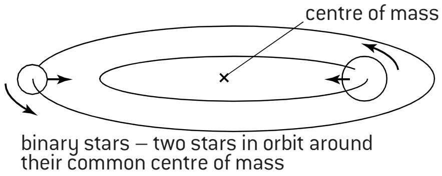
The Sun has been radiating energy for the past 4½ billion years. It might be imagined that the powerful reactions in the core should have forced away the outer layers of the Sun a long time ago. Like other stars, the Sun is stable because there is a **hydrostatic equilibrium** between this outward pressure and the inward gravitational force (see page 164).



A stable star is in equilibrium

BINARY STARS

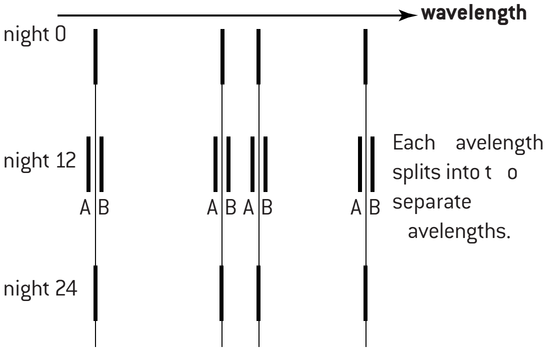
Our Sun is a single star. Many 'stars' actually turn out to be two (or more) stars in orbit around each other. (To be precise they orbit around their common centre of mass.) These are called **binary stars**.



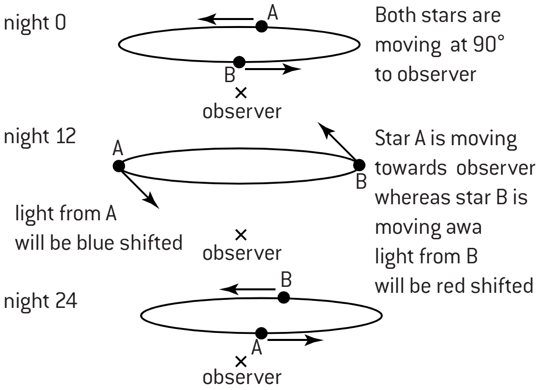
binary stars – two stars in orbit around their common centre of mass

There are different categories of binary star – **visual**, **spectroscopic** and **eclipsing**.

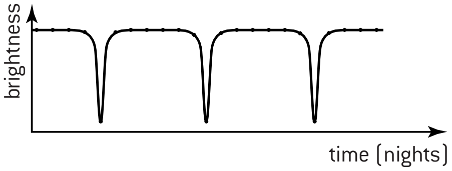
1. A visual binary is one that can be distinguished as two separate stars using a telescope.
2. A spectroscopic binary star is identified from the analysis of the spectrum of light from the 'star'. Over time the wavelengths show a periodic shift or splitting in frequency. An example of this is shown (below).



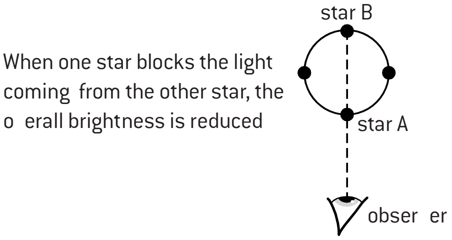
The explanation for the shift in frequencies involves the Doppler effect. As a result of its orbit, the stars are sometimes moving towards the Earth and sometimes they are moving away. When a star is moving towards the Earth, its spectrum will be blue shifted. When it is moving away, it will be red shifted.



3. An eclipsing binary star is identified from the analysis of the brightness of the light from the 'star'. Over time the brightness shows a periodic variation. An example of this is shown below.



The explanation for the 'dip' in brightness is that as a result of its orbit, one star gets in front of the other. If the stars are of equal brightness, then this would cause the total brightness to drop to 50%.

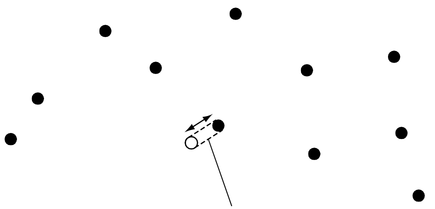


Stellar parallax

PRINCIPLES OF MEASUREMENT

As you move from one position to another objects change their relative positions. As far as you are concerned, near objects appear to move when compared with far objects. Objects that are very far away do not appear to move at all. You can demonstrate this effect by closing one eye and moving your head from side to side. An object that is near to you (for example the tip of your finger) will appear to move when compared with objects that are far away (for example a distant building).

This apparent movement is known as **parallax** and the effect can used to measure the distance to some of the stars in our galaxy. All stars appear to move over the period of a night, but some stars appear to move in relation to other stars over the period of a year.



If carefully observed, over the period of a year some stars can appear to move between two extremes.

The reason for this apparent movement is that the Earth has moved over the period of a year. This change in observing position has meant that a close star will have an apparent movement when compared with a more distant set of stars. The closer a star is to the Earth, the greater will be the parallax shift.

Since all stars are very distant, this effect is a very small one and the parallax angle will be very small. It is usual to quote parallax angles not in degrees, but in seconds. An angle of 1 second of arc ('') is equal to one sixtieth of 1 minute of arc (') and 1 minute of arc is equal to one sixtieth of a degree.

In terms of angles, $3600'' = 1°$

$360° = 1$ full circle.

EXAMPLE

The star alpha Eridani (Achemar) is 1.32×10^{18} m away. Calculate its parallax angle.

$$d = 1.32 \times 10^{18}\ \text{m}$$

$$= \frac{1.32 \times 10^{18}}{3.08 \times 10^{16}}\ \text{pc}$$

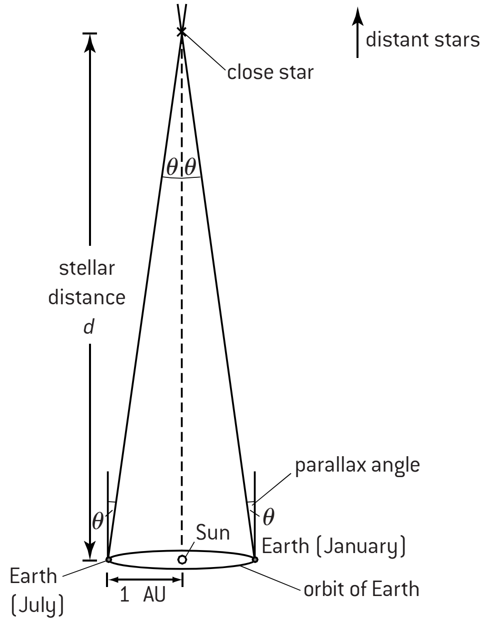
$$= 42.9\ \text{pc}$$

$$\text{parallax angle} = \frac{1}{42.9}$$

$$= 0.02''$$

MATHEMATICS – UNITS

The situation that gives rise to a change in apparent position for close stars is shown below.



The parallax angle, θ, can be measured by observing the changes in a star's position over the period of a year. From trigonometry, if we know the distance from the Earth to the Sun, we can work out the distance from the Earth to the star, since

$$\tan\theta = \frac{(\text{distance from Earth to Sun})}{(\text{distance from Sun to Star})}$$

Since θ is a very small angle, $\tan\theta \approx \sin\theta \approx \theta$ (in radians)

This means that $\theta \propto \dfrac{1}{(\text{distance from Earth to star})}$

In other words, parallax angle and distance away are inversely proportional. If we use the right units we can end up with a very simple relationship. The units are defined as follows.

The distance from the Sun to the Earth is defined to be one **astronomical unit (AU)**. It is 1.5×10^{11} m. Calculations show that a star with a parallax angle of exactly one second of arc must be 3.08×10^{16} m away (3.26 light years). This distance is defined to be one **parsec (pc)**. The name 'parsec' represents '**par**allax angle of one **sec**ond'.

If distance = 1 pc, $\theta = 1$ second

If distance = 2 pc, $\theta = 0.5$ second etc.

Or, distance in pc $= \dfrac{1}{(\text{parallax angle in seconds})}$

$$d = \frac{1}{p}$$

The parallax method can be used to measure stellar distances that are less than **about 100 parsecs**. The parallax angle for stars that are at greater distances becomes too small to measure accurately. It is common, however, to continue to use the unit. The standard SI prefixes can also be used even though it is not strictly an SI unit.

1000 parsecs = 1 kpc

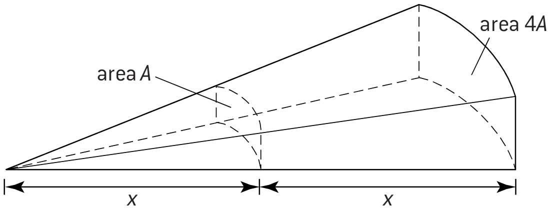
10^6 parsecs = 1 Mpc etc.

Luminosity

LUMINOSITY AND APPARENT BRIGHTNESS

The total power **radiated** by a star is called its **luminosity** (L). The SI units are watts. This is very different to the power **received** by an observer on the Earth. The power received per unit area is called the **apparent brightness** of the star. The SI units are W m^{-2}.

If two stars were at the **same distance** away from the Earth then the one with the greater luminosity would be brighter. Stars are, however, at different distances from the Earth. The brightness is inversely proportional to the (distance)^2.



As distance increases, the brightness decreases since the light is spread over a bigger area.

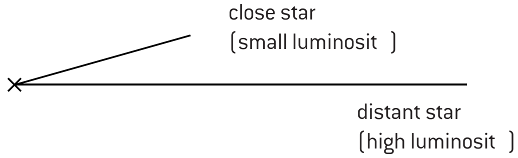
distance	**brightness**
x	b
$2x$	$\frac{b}{4}$
$3x$	$\frac{b}{9}$
$4x$	$\frac{b}{16}$
$5x$	$\frac{b}{25}$

inverse square

and so on

apparent brightness $b = \frac{L}{4\pi r^2}$

It is thus possible for two very different stars to have the same apparent brightness. It all depends on how far away the stars are.



Two stars can have the same apparent brightness even if they have different luminosities

ALTERNATIVE UNITS

The SI units for luminosity and brightness have already been introduced. In practice astronomers often compare the brightness of stars using the **apparent magnitude** scale. A magnitude 1 star is brighter than a magnitude 3 star. This measure of brightness is sometimes shown on star maps.

The magnitude scale can also be used to compare the luminosity of different stars, provided the distance to the star is taken into account. Astronomers quote values of **absolute magnitude** in order to compare luminosities on a familiar scale.

EXAMPLE ON LUMINOSITY

The star Betelgeuse has a parallax angle of 7.7 × 10^{-3} arc seconds and an apparent brightness of 2.0 × 10^{-7} W m^{-2}. Calculate its luminosity.

Distance to Betelgeuse d

$$= \frac{1}{p}$$

$$= \frac{1}{7.7 \times 10^{-3}} \text{ pc}$$

$$= 129.9 \text{ pc}$$

$$= 129.9 \times 3.08 \times 10^{16} \text{ m}$$

$$= 4.0 \times 10^{18} \text{ m}$$

$$L = b \times 4\pi d^2 = 4.0 \times 10^{31} \text{ W}$$

BLACK-BODY RADIATION

Stars can be analysed as perfect emitters, or black bodies. The luminosity of a star is related to its brightness, surface area and temperature according to the Stefan–Boltzmann law. Wien's law can be used to relate the wavelength at which the intensity is a maximum to its temperature. See page 90 for more details.

Example:

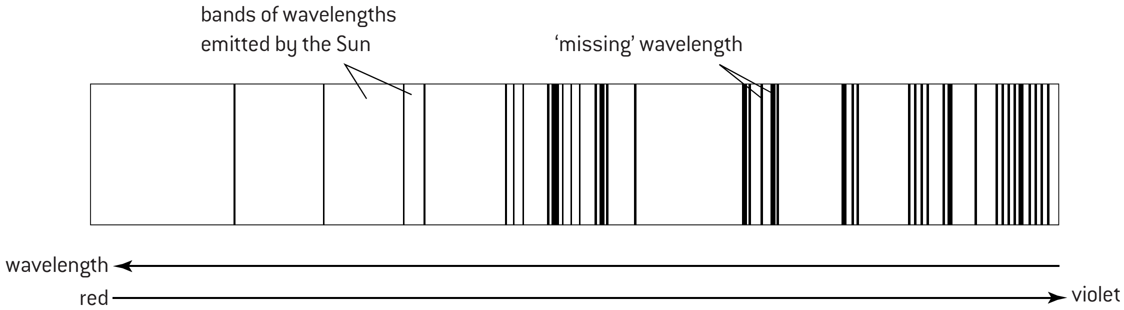
e.g. our sun's temperature is 5,800k

So the wavelength at which the intensity of its radiation is at a maximum is $\lambda_{\max} = \frac{2.9 \times 10^{-3}}{5800} = 500$ nm

Stellar spectra

ABSORPTION LINES

The radiation from stars is not a perfect continuous spectrum – there are particular wavelengths that are 'missing'.



The missing wavelengths correspond to the absorption spectra of a number of elements. Although it seems sensible to assume that the elements concerned are in the Earth's atmosphere, this assumption is incorrect. The wavelengths would still be absent if light from the star was analysed in space.

The absorption is taking place in the outer layers of the star. This means that we have a way of telling what elements exist in the star – at least in its outer layers.

A star that is moving relative to the Earth will show a Doppler shift in its absorption spectrum. Light from stars that are receding will be **red shifted** whereas light from approaching stars will be **blue shifted**.

CLASSIFICATION OF STARS

Different stars give out different spectra of light. This allows us to classify stars by their **spectral class**. Stars that emit the same type of spectrum are allocated to the same spectral class. Historically these were just given a different letter, but we now know that these different letters also correspond to different surface temperatures.

The seven main spectral classes (in order of **decreasing** surface temperature) are O, B, A, F, G, K and M. The main spectral classes can be subdivided.

Class	Effective surface temperature/K	Colour
O	30,000–50,000	blue
B	10,000–30,000	blue-white
A	7,500–10,000	white
F	6,000–7,500	yellow-white
G	5,200–6,000	yellow
K	3,700–5,200	orange
M	2,400–3,700	red

Spectral classes do not need to be mentioned but are used in many text books.

STEFAN–BOLTZMANN LAW

The Stefan–Boltzmann law links the **total** power radiated by a black body (per unit area) to the temperature of the black body. The important relationship is that

Total power radiated $\propto T^4$

In symbols we have,

Total power radiated $= \sigma A T^4$

Where

σ is a constant called the Stefan–Boltzmann constant.

$\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}

A is the surface area of the emitter (in m^2)

T is the absolute temperature of the emitter (in kelvin)

e.g. The radius of the Sun $= 6.96 \times 10^8$ m.

Surface area $= 4\pi r^2 = 6.09 \times 10^{10}$ m^2

If temperature $= 5800$ K

then total power radiated $= \sigma A T^4$

$$= 5.67 \times 10^{-8} \times 6.09 \times 10^{18} \times (5800^4)$$

$$= 3.9 \times 10^{26} \text{ W}$$

The radius of the star r is linked to its surface area, A, using the equation $A = 4\pi r^2$.

SUMMARY

If we know the distance to a star we can analyse the light from the star and work out:

- the chemical composition (by analysing the absorption spectrum)
- the surface temperature (using a measurement of λ_{max} and Wien's law – see page 90)
- the luminosity (using measurements of the brightness and the distance away)
- the surface area of the star (using the luminosity, the surface temperature and the Stefan–Boltzmann law).

Nucleosynthesis

STELLAR TYPES AND BLACK HOLES

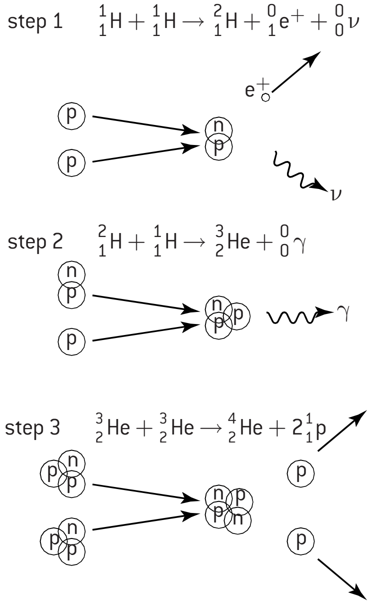
The source of energy for our Sun is the fusion of hydrogen into helium. This is also true for many other stars.
There are however, other types of object that are known to exist in the Universe.

Type of object	Description
Red giant stars	As the name suggests, these stars are large in size and red in colour. Since they are red, they are comparatively cool. They turn out to be one of the later possible stages for a star. The source of energy is the fusion of some elements other than hydrogen. **Red supergiants** are even larger.
White dwarf stars	As the name suggests, these stars are small in size and white in colour. Since they are white, they are comparatively hot. They turn out to be one of the final stages for some smaller mass stars. Fusion is no longer taking place, and a white dwarf is just a hot remnant that is cooling down. Eventually it will cease to give out light when it becomes sufficiently cold. It is then known as a **brown dwarf**.
Cepheid variables	These are stars that are a little unstable. They are observed to have a regular variation in brightness and hence luminosity. This is thought to be due to an oscillation in the size of the star. They are quite rare but are very useful as there is a link between the period of brightness variation and their average luminosity. This means that astronomers can use them to help calculate the distance to some galaxies.
Neutron stars	Neutron stars are the post-supernova remnants of some larger mass stars. The gravitational pressure has forced a total collapse and the mass of a neutron star is not composed of atoms – it is essentially composed of neutrons. The density of a neutron star is enormous. Rotating neutron stars have been identified as **pulsars**.
Black holes	Black holes are the post-supernova remnant of larger mass stars. There is no known mechanism to stop the gravitational collapse. The result is an object whose escape velocity is greater than the speed of light. See page 150.

MAIN SEQUENCE STARS

The general name for the creation of nuclei of different elements as a result of fission reactions is **nucleosynthesis**. Details of how this overall reaction takes place in the Sun do not need to be recalled by SL candidates, but HL candidates do need this information.

One process is known as the **proton–proton cycle** or **p–p cycle**.



the proton–proton cycle (p–p cycle)

In order for any of these reactions to take place, two positively charged particles (hydrogen or helium nuclei) need to come close enough for interactions to take place. Obviously they will repel one another.

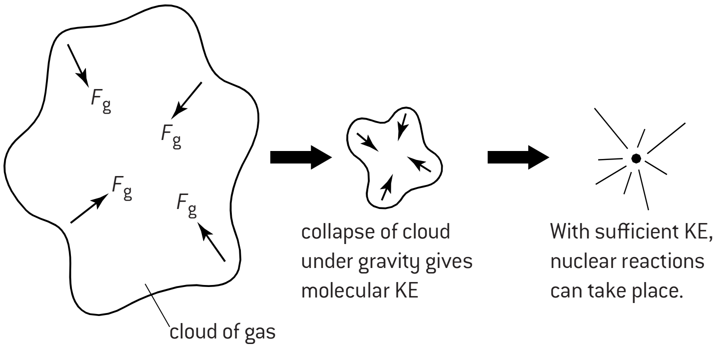
This means that they must be at a high temperature.

If a large cloud of hydrogen is hot enough, then these nuclear reactions can take place spontaneously. The power radiated by the star is balanced by the power released in these reactions – the temperature is effectively constant.

The star remains a stable size because the outward pressure of the radiation is balanced by the inward gravitational pull.

But how did the cloud of gas get to be at a high temperature in the first place? As the cloud comes together, the loss of gravitational potential energy must mean an increase in kinetic energy and hence temperature. In simple terms the gas molecules speed up as they fall in towards the centre to form a proto-star.

Once ignition has taken place, the star can remain stable for billions of years. See page 205 for more details.



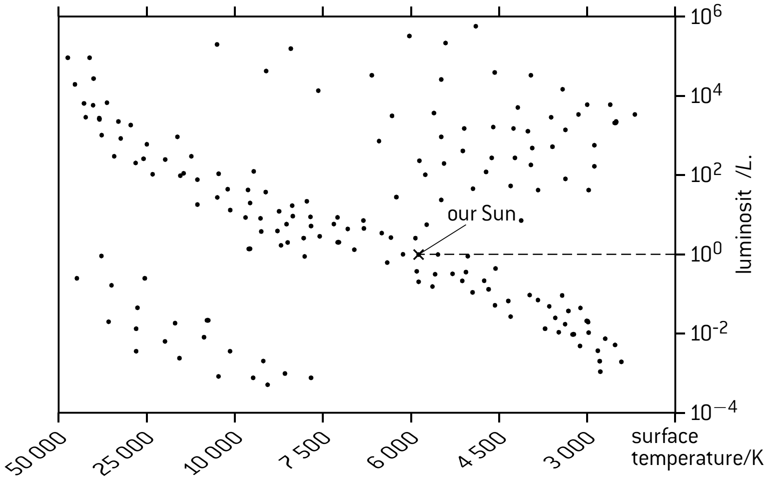
The Hertzsprung–Russell diagram

H–R DIAGRAM

The point of classifying the various types of stars is to see if any patterns exist. A useful way of making this comparison is the **Hertzsprung Russell diagram**. Each dot on the diagram represents a different star. The following axes are used to position the dot.

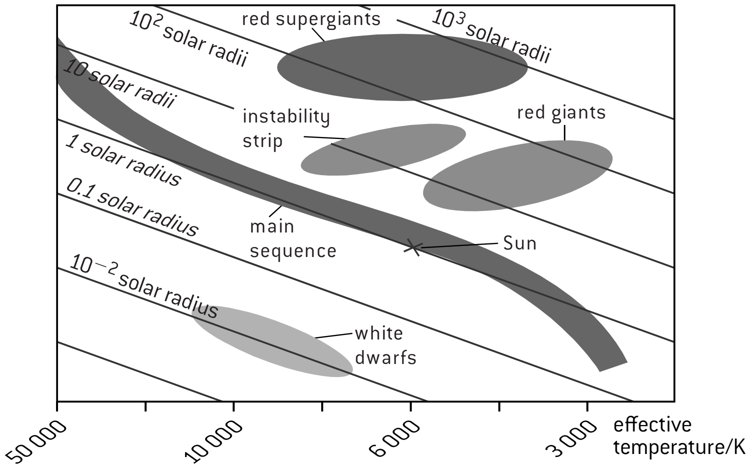
- The vertical axis is the luminosity of the star as compared with the luminosity of the Sun. It should be noted that the scale is logarithmic.
- The horizontal axis a scale of **decreasing** temperature. Once again, the scale is not a linear one. (It is also the spectral class of the star OBAFGKM)

The result of such a plot is shown below.



A large number of stars fall on a line that (roughly) goes from top left to bottom right. This line is known as the **main sequence** and stars that are on it are known as main sequence stars. Our Sun is a main sequence star. These stars are 'normal stable stars – the only difference between them is their mass. They are fusing hydrogen to helium. The stars that are not on the main sequence can also be broadly put into categories.

In addition to the broad regions, lines of constant radius can be added to show the size of stars in comparison to our Sun s radius. These are lines going from top left to bottom right.



MASS-LUMINOSITY RELATION FOR MAIN SEQUENCE STARS

For stars on the main sequence, there is a correlation between the star's mass, M, and its luminosity, L. Stars that are brighter on the main sequence (i.e. higher up) are more massive and the relationship is:

$L \propto M^{3.5}$

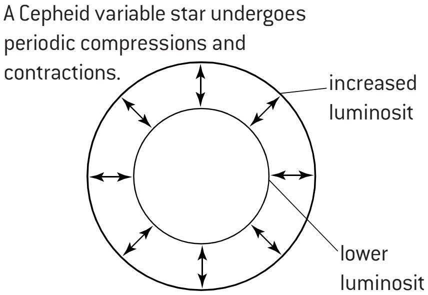
Cepheid variables

PRINCIPLES

Very small parallax angles can be measured using satellite observations (e.g. Gaia mission) but even these measurement are limited to stars that are about 100 kpc away. The essential difficulty is that when we observe the light from a very distant star, we do not know the difference between a bright source that is far away and a dimmer source that is closer. This is the principal problem in the experimental determination of astronomical distances to other galaxies.

When we observe another galaxy, all of the stars in that galaxy are approximately the same distance away from the Earth. What we really need is a light source of known luminosity in the galaxy. If we had this then we could make comparisons with the other stars and judge their luminosities. In other words we need a '**standard candle**' – that is a star of known luminosity. Cepheid variable stars provide such a 'standard candle'.

A Cepheid variable star is quite a rare type of star. Its outer layers undergo a periodic compression and contraction and this produces a periodic variation in its luminosity.

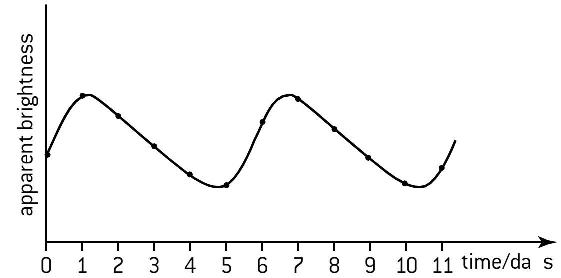


These stars are useful to astronomers because the period of this variation in luminosity turns out to be related to the average absolute magnitude of the Cepheid. Thus the luminosity of a Cepheid can be calculated by observing the variations in brightness.

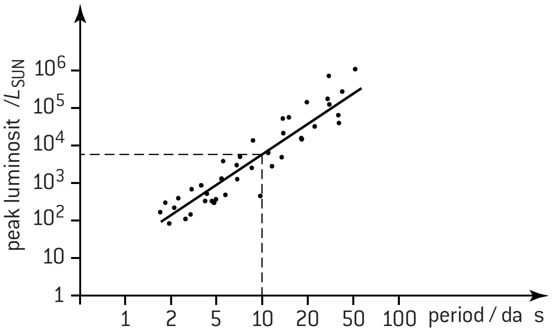
MATHEMATICS

The process of estimating the distance to a galaxy (in which the individual stars can be imaged) might be as follows:

- Locate a Cepheid variable in the galaxy.
- Measure the variation in brightness over a given period of time.
- Use the luminosity–period relationship for Cepheids to estimate the average luminosity.
- Use the average luminosity, the average brightness and the inverse square law to estimate the distance to the star.



Variation of apparent magnitude for a particular Cepheid variable



General luminosity–period graph

EXAMPLE

A Cepheid variable star has a period of 10.0 days and apparent peak brightness of 6.34×10^{-11} W m^{-2}

The luminosity of the Sun is 3.8×10^{26} W. Calculate the distance to the Cepheid variable in pc.

Using the luminosity–period graph (above)

$$\Rightarrow \text{peak luminosity} = 10^{3.7} \times L_{\text{sun}} = 5012 \times 3.8 \times 10^{26} = 1.90 \times 10^{30}\ \text{W}$$

$$L = b \times 4\pi r^2$$

$$r = \sqrt{\frac{L}{4\pi b}}$$

$$= \sqrt{\frac{1.90 \times 10^{30}}{4 \times \pi \times 6.34 \times 10^{-11}}}$$

$$= 4.88 \times 10^{19}\ \text{m}$$

$$= \frac{4.88 \times 10^{19}}{3.08 \times 10^{16}}\ \text{pc}$$

$$= 1590\ \text{pc}$$

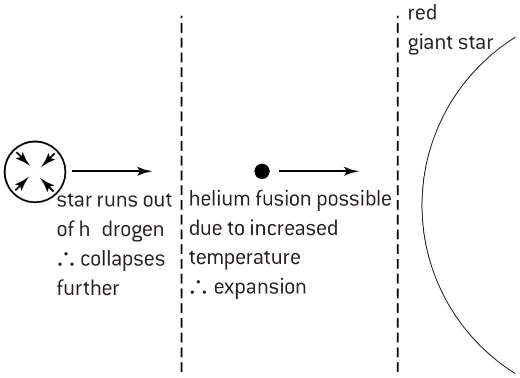
Red giant stars

AFTER THE MAIN SEQUENCE

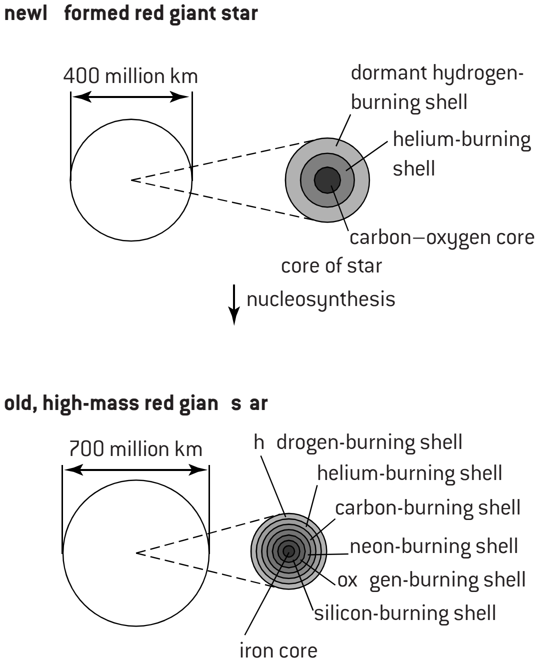
The mass–luminosity relation (page 197) can be used to compare the amount of time different mass stars take before the hydrogen fuel is used. Consider a star that is 10 times more massive than our Sun. This means that the luminosity of the larger star will be $(10)^{3.5} = 3{,}162$ times more luminous that our Sun. Since the source of this luminosity is the mass of hydrogen in the star, then the larger star effectively has 10 times more 'fuel' but is using the fuel at more than 3000 times the rate. The more massive star will finish its fuel in $\frac{1}{300}$ of the time. A star that has more mass exists for a shorter amount of time.

A star cannot continue in its main sequence state forever. It is fusing hydrogen into helium and at some point hydrogen in the core will become rare. The fusion reactions will happen less often. This means that the star is no longer in equilibrium and the gravitational force will, once again, cause the core to collapse.

This collapse increases the temperature of the core still further and helium fusion is now possible. The net result is for the star to increase massively in size – this expansion means that the outer layers are cooler. It becomes a red giant star.



If it has sufficient mass, a red giant can continue to fuse higher and higher elements and the process of nucleosynthesis can continue.



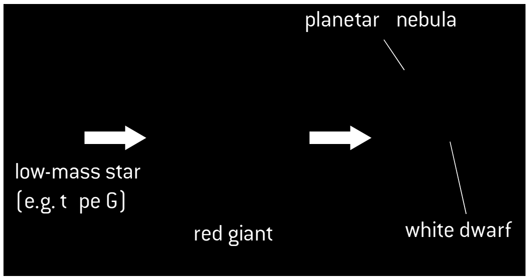
This process of fusion as a source of energy must come to an end with the nucleosynthesis of iron. The iron nucleus has one of the greatest binding energies per nucleon of all nuclei. In other words the fusion of iron to form a higher mass nucleus would need to take in energy rather than release energy. The star cannot continue to shine. What happens next is outlined on the following page.

Stellar evolution

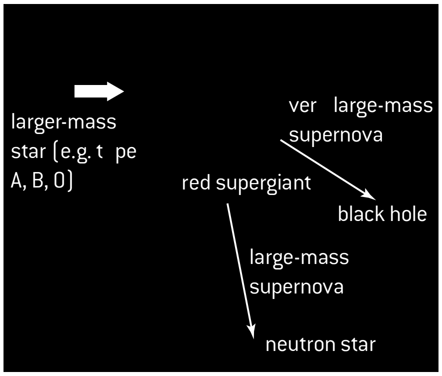
POSSIBLE FATES FOR A STAR (AFTER RED GIANT PHASES)

Page 199 showed that the red giant phase for a star must eventually come to an end. There are essentially two possible routes with different final states. The route that is followed depends on the initial mass of the star and thus the mass of the remnant that the red giant star leaves behind: with no further nuclear reactions taking place gravitational forces continue the collapse of the remnant. An important 'critical' mass is called the **Chandrasekhar limit** and it is equal to approximately 1.4 times the mass of our Sun. Below this limit a process called **electron degeneracy pressure** prevents the further collapse of the remnant.

If a star has a mass less than 4 Solar masses, its remnant will be less than 1.4 Solar masses and so it is below the Chandrasekhar limit. In this case the red giant forms a **planetary nebula** and becomes a **white dwarf** which ultimately becomes invisible. The name 'planetary nebula' is another term that could cause confusion. The ejected material would not be planets in the same sense as the planets in our Solar System.



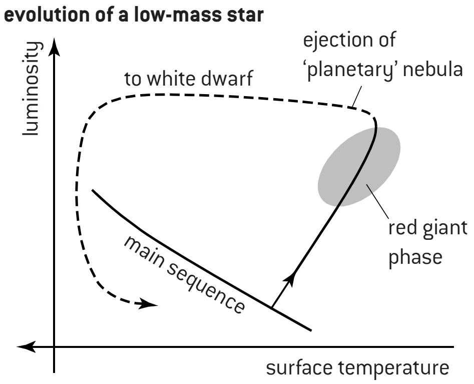
If a star is greater than 4 Solar masses, its remnant will have a mass greater than 1.4 Solar masses. It is above the Chandrasekhar limit and electron degeneracy pressure is not sufficient to prevent collapse. In this case the red supergiant experiences a **supernova**. It then becomes a **neutron star** or collapses to a **black hole**. The final state again depends on mass.

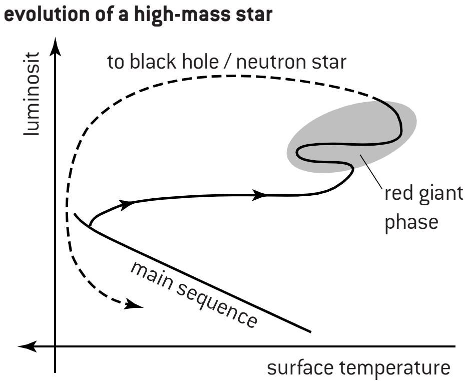


A neutron star is stable due to neutron degeneracy pressure. It should be emphasized that white dwarfs and neutron stars do not have a source of energy to fuel their radiation. They must be losing temperature all the time. The fact that these stars can still exist for many millions of years shows that the temperatures and masses involved are enormous. The largest mass a neutron star can have is called the **Oppenheimer Volkoff limit** and is 2–3 Solar masses. Remnants above this limit will form black holes.

H – R DIAGRAM INTERPRETATION

All of the possible evolutionary paths for stars that have been described here can be represented on a H – R diagram. A common mistake in examinations is for candidates to imply that a star somehow moves along the line that represents the main sequence. It does not. Once formed it stays at a stable luminosity and spectral class – i.e. it is represented by one fixed point in the H – R diagram.





PULSARS AND QUASARS

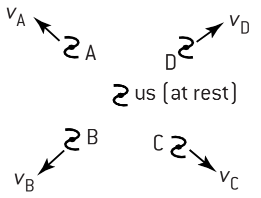
Pulsars are cosmic sources of very weak radio wave energy that pulsate at a very rapid and precise frequency. These have now been theoretically linked to rotating neutron stars. A rotating neutron star would be expected to emit an intense beam of radio waves in a specific direction. As a result of the star's rotation, this beam moves around and causes the pulsation that we receive on Earth.

Quasi-stellar objects or quasars appear to be point-like sources of light and radio waves that are very far away. Their red shifts are very large indeed, which places them at the limits of our observations of the Universe. If they are indeed at this distance they must be emitting a great deal of power for their size (approximately 10^{40} W!). The process by which this energy is released is not well understood, but some theoretical models have been developed that rely on the existence of super-massive black holes. The energy radiated is as a result of whole stars 'falling' into the black hole.

The Big Bang model

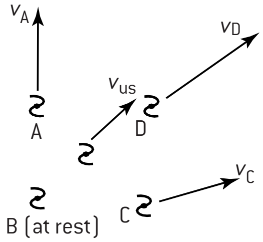
EXPANSION OF THE UNIVERSE

If a galaxy is moving away from the Earth, the light from it will be red shifted. The surprising fact is that light from almost all galaxies shows red shifts – almost all of them are moving away from us. The Universe is expanding.



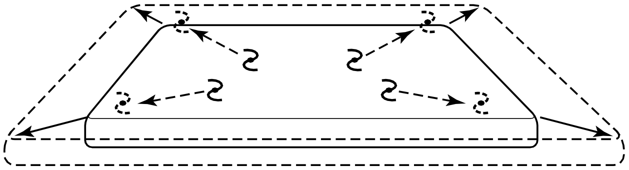
As far as we are concerned, most galaxies are moving away from us.

At first sight, this expansion seems to suggest that we are in the middle of the Universe, but this is a mistake. We only seem to be in the middle because it was we who worked out the velocities of the other galaxies. If we imagine being in a different galaxy, we would get exactly the same picture of the Universe.



Any galaxy would see all the other galaxies moving away from it.

A good way to picture this expansion is to think of the Universe as a sheet of rubber stretching off into the distance. The galaxies are placed on this huge sheet. If the tension in the rubber is increased, everything on the sheet moves away from everything else.



As the section of rubber sheet expands, everything moves away from everything else.

THE UNIVERSE IN THE PAST – THE BIG BANG

If the Universe is currently expanding, at some time in the past all the galaxies would have been closer together. If we examine the current expansion in detail we find that all the matter in the observable universe would have been together at the SAME point approximately 15 billion years ago.

This point, the creation of the Universe, is known as the **Big Bang**. It pictures all the matter in the Universe being crushed together (very high density) and being very hot indeed. Since the Big Bang, the Universe has been expanding – which means that, on average, the temperature and density of the Universe have been decreasing. The rate of expansion would be expected to decrease as a result of the gravitational attraction between all the masses in the Universe.

Note that this model does not attempt to explain how the Universe was created, or by Whom. All it does is analyse what happened after this creation took place. The best way to imagine the expansion is to think of the expansion of space itself rather than the galaxies expanding into a void. The Big Bang was the creation of space and time. Einstein's theory of relativity links the measurements of space and time so properly we need to imagine the Big Bang as the creation of space **and time**. It does not make sense to ask about what happened before the Big Bang, because the notion of before and after (i.e. time itself) was created in the Big Bang.

COSMIC MICROWAVE BACKGROUND RADIATION

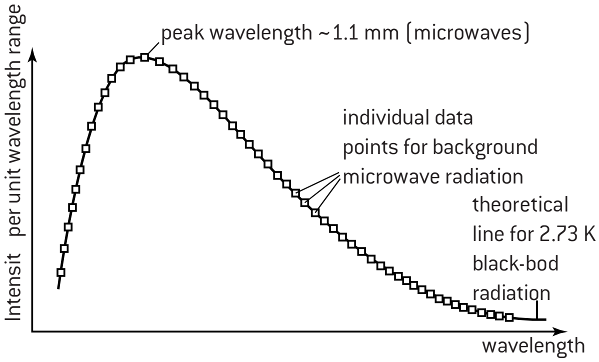
A further piece of evidence for the Big Bang model came with the discovery of the **Cosmic microwave background (CMB) radiation** by Penzias and Wilson.

They discovered that microwave radiation was coming towards us from all directions in space. The strange thing was that the radiation was the same in all directions (**isotopic**) and did not seem to be linked to a source. Further analysis showed that this radiation was a very good match to theoretical black-body radiation produced by an extremely cold object – a temperature of just 2.73 K.

This is in perfect agreement with the predictions of Big Bang. There are two ways of understanding this.

1. All objects give out electromagnetic radiation. The frequencies can be predicted using the theoretical model of black-body radiation. The background radiation is the radiation from the Universe itself which has now cooled down to an average temperature of 2.73 K.
2. Some time after the Big Bang, radiation became able to travel through the Universe (see page 210 for details).

   It has been travelling towards us all this time. During this time the Universe has expanded – this means that the wavelength of this radiation will have increased (space has stretched). See page 210 for anisotropies in the CMB.



Galactic motion

DISTRIBUTIONS OF GALAXIES

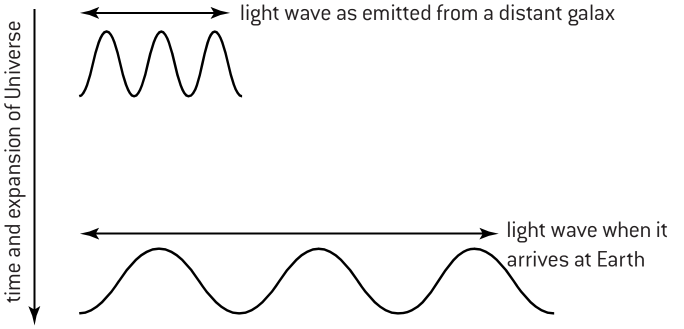
Galaxies are not distributed randomly throughout space. They tend to be found clustered together. For example, in the region of the Milky Way there are twenty or so galaxies in less than 2.5 million light years.

The Virgo galactic cluster (50 million light years away from us) has over 1,000 galaxies in a region 7 million light years across. On an even larger scale, the galactic clusters are grouped into huge **superclusters** of galaxies. In general, these superclusters often involve galaxies arranged together in joined 'filaments' (or bands) that are arranged as though randomly throughout empty space.

MOTION OF GALAXIES

As has been seen on page 201 it is a surprising observational fact that the vast majority of galaxies are moving away from us. The general trend is that the more distant galaxies are moving away at a greater speed as the Universe expands. This does not, however, mean that we are at the centre of the Universe – this would be observed wherever we are located in the Universe.

As explained on page 201, a good way to imagine this expansion is to think of space itself expanding. It is the expansion of space (as opposed to the motion of the galaxies through space) that results in the galaxies' relative velocities. In this model, the red shift of light can be thought of as the expansion of the wavelength due to the 'stretching' of space.



MATHEMATICS

If a star or a galaxy moves away from us, then the wavelength of the light will be altered as predicted by the Doppler effect (see page 102). If a galaxy is going away from the Earth, the speed of the galaxy with respect to an observer on the Earth can be calculated from the red shift of the light from the galaxy. As long as the velocity is small when compared with the velocity of light, a simplified red shift equation can be used.

$$Z = \frac{\Delta\lambda}{\lambda_0} \approx \frac{v}{c}$$

Where

$\Delta\lambda$ = change in wavelength of observed light (positive if wavelength is increased)

λ_0 = wavelength of light emitted

v = relative velocity of source of light

c = speed of light

Z = red shift.

Example

A characteristic absorption line often seen in stars is due to ionized helium. It occurs at 468.6 nm. If the spectrum of a star has this line at a measured wavelength of 499.3 nm, what is the recession speed of the star?

$$Z = \frac{\Delta\lambda}{\lambda_0} = \frac{(499.3 - 468.6)}{468.6}$$

$$= 6.55 \times 10^{-2}$$

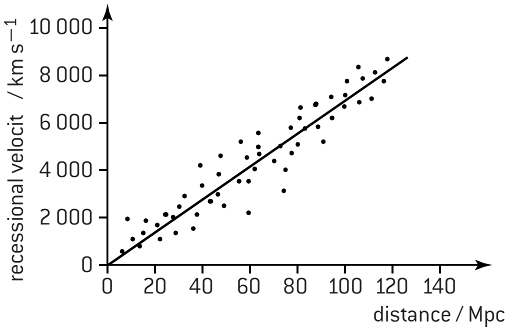
$$\therefore \quad v = 6.55 \times 10^{-2} \times 3 \times 10^{8} \text{ m s}^{-1}$$

$$= 1.97 \times 10^{7} \text{ m s}^{-1}$$

Hubble's law and cosmic scale factor

EXPERIMENTAL OBSERVATIONS

Although the uncertainties are large, the general trend for galaxies is that the recessional velocity is proportional to the distance away from Earth. This is Hubble's law.



Mathematically this is expressed as

$$v \propto d$$

or

$$v = H_0\, d$$

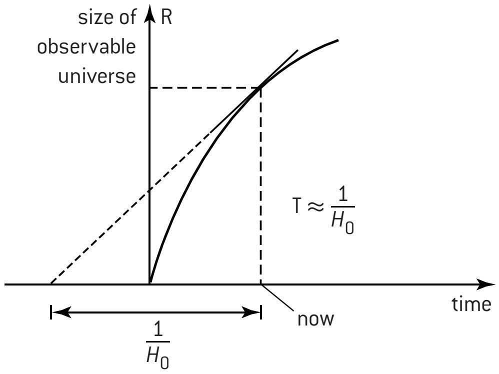
where H_0 is a constant known as the **Hubble constant**. The uncertainties in the data mean that the value of H_0 is not known to any degree of precision. The SI units of the Hubble constant are s^{-1}, but the unit of km s^{-1} Mpc^{-1} is often used.

HISTORY OF THE UNIVERSE

If a galaxy is at a distance x, then Hubble's law predicts its velocity to be H_0x. If it has been travelling at this constant speed since the beginning of the Universe, then the time that has elapsed can be calculated from

$$\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{x}{H_0x} = \frac{1}{H_0}$$

This is an upper limit for the age of the Universe. The gravitational attraction between galaxies predicts that the speed of recession decreases all the time.



THE COSMIC SCALE FACTOR (*R*)

Page 202 shows how the Doppler red shift equation, $= \frac{\Delta\lambda}{\lambda_0} \approx \frac{v}{c}$, can be used to calculate the recessional velocity, v, of certain galaxies. This equation can only be used when $v \ll c$ or in other words, the recessional velocity, v, has to be small in comparison to the speed of light, c. There are however plenty of objects in the night sky (e.g. quasars) for which the observed red shift, z, is greater than 1.0. This implies that their speed of recession is greater than the speed of light. In these situations it is helpful to consider a quantity called the cosmic scale factor (R).

As introduced on page 201, the expansion of the Universe is best pictured as the expansion of space itself. The expansion of the Universe means that a measurement undertaken at some time in the distant past, for example the wavelength of light emitted by an object 10 million years ago, will be stretched and will be recorded as a larger value when measured now. All measurement will be stretched over time and this can be considered as a rescaling of the Universe (the Universe getting bigger).

The cosmic scale factor, R, is a way of quantifying the expansion that has taken place. In the above example, if the wavelength was emitted 10 million years ago with wavelength λ_0 when the scale factor was R_0, the wavelength measured today would have increased by $\Delta\lambda$ to a larger value λ ($\lambda = \lambda_0 + \Delta\lambda$). This is because the cosmic scale factor has increased by ΔR (to the larger value $R = R_0 + \Delta R$). All measurements will have increased by the ratio, $\frac{R}{R_0}$. The ratio of the measured wavelengths, $\frac{\lambda}{\lambda_0}$, is equal to the ratio of the cosmic scale factors, $\frac{R}{R_0}$, so the red shift ratio, z is given by:

$$z = \frac{\Delta\lambda}{\lambda_0} = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\lambda}{\lambda_0} - 1 = \frac{R}{R_0} - 1$$

or $z = \frac{R}{R_0} - 1$

So a measured red shift of 4 means that $\frac{R}{R_0} = 5$. If we consider R to be the present 'size' of the observable Universe, then the light must have been emitted when the Universe was one fifth of its current size.

The accelerating universe

SUPERNOVAE AND THE ACCELERATING UNIVERSE

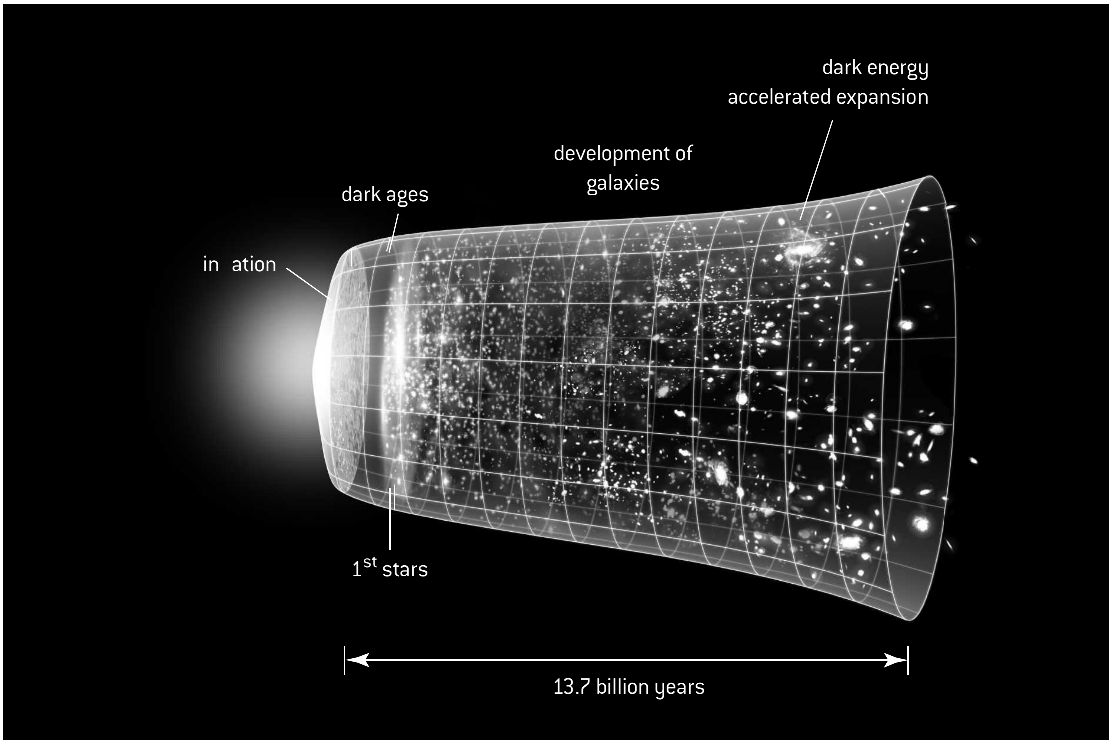
Supernovae are catastrophic explosions that can occur in the development of some stars (see page 200). Supernovae are rare events (the last one to occur in our galaxy took place in 1604) but the large number of stars in the Universe means that many have been observed. An observer on the Earth sees a rapid increase in brightness (hence the word 'nova' = new star) which then diminishes over a period of some weeks or months. Huge amounts of radiated energy are emitted in a short period of time and, at its peak, the apparent brightness of a single supernova often exceeds many local stars or individual galaxies.

Supernovae have been categorized into two different main types (see page 207 for more details) according to a spectral analysis of the light that they emit. The light from a type II supernova indicates the presence of hydrogen (from the absorption spectra) whereas there is no hydrogen in a type I supernova. There are further subdivisions of these types (Ia, Ib, etc.) based on different aspects of the light spectrum.

Type Ia supernovae are explosions involving white dwarf stars. When these events take place, the amount of energy released can be predicted accurately and these supernovae can be used as 'standard candles'. By comparing the known luminosity of a type Ia supernova and its apparent brightness as observed in a given galaxy, a distance measurement to that galaxy can be calcuated. This technique can be used with galaxies up to approximately 1,000 Mpc away.

The expanding Universe (which is consistent with the Big Bang model) means that that the cosmic scale factor, R, is increasing. As a result of gravitational attraction, we might expect the rate at which R increases to be slowing down. Analysis of a large number of type Ia supernovae has, however, provided strong evidence that not only is the cosmic scale factor, R, increasing but the rate at which it increases is getting larger as time passes. In other words the expansion of the Universe is accelerating. The evidence from type Ia supernovae identifies this effect from a time when the universe was approximately $\frac{2}{3}$ of its current size. Note that this acceleration is different to the very rapid period of expansion of the early Universe which is called inflation.

The mechanisms that cause an accelerating Universe are not fully understood but must involve an outward accelerating force to counteract the inward gravitational pull. There must also be a source of energy which has been given the name **dark energy** (see page 212).



Source: NASA/WMAP Science Team

HL Nuclear fusion – the Jeans criterion

THE JEANS CRITERION

As seen on page 196, stars form out of interstellar clouds of hydrogen, helium and other materials. Such clouds can exist in stable equilibrium for many years until an external event (e.g. a collision with another cloud or the influence of another incident such as a supernova) starts the collapse. At any given point in time, the total energy associated with the gas cloud can be thought of as a combination of:

- The negative gravitational potential energy, E_P, which the cloud possesses as a result of its mass and how it is distributed in space. Important factors are thus the mass and the density of the cloud.
- The positive random kinetic energy, E_K, that the particles in the cloud possess. An important factor is thus the temperature of the cloud.

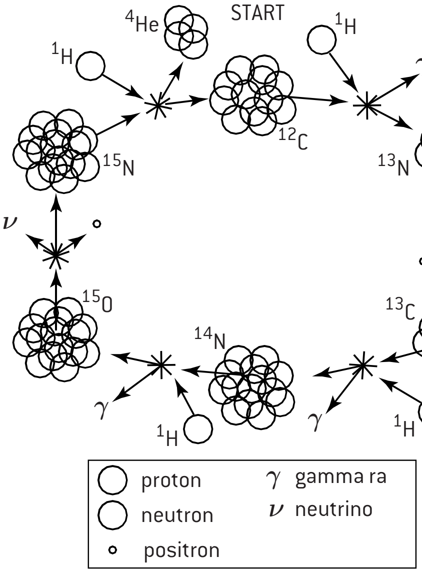
The cloud will remain gravitationally bound together if $E_P + E_K <$ zero. Using this information allows us to predict that the collapse of an interstellar cloud may begin if its mass is greater than a certain critical mass, M_J. This is the **Jeans criterion**. For a given cloud of gas, M_J is dependent on the cloud's density and temperature and the cloud is more likely to collapse if it has:

- large mass
- small size
- low temperature.

In symbols, the Jeans criterion is that collapse can start if $M > M_J$

NUCLEAR FUSION

A star on the main sequence is fusing hydrogen nuclei to produce helium nuclei. One process by which this is achieved is the proton–proton chain as outlined on page 196. This is the predominant method for nuclear fusion to take place in small mass stars (up to just above the mass of our Sun). An alternative process, called the CNO (carbon–nitrogen–oxygen) process takes place at higher temperatures in larger mass stars. In this reaction, carbon, nitrogen and oxygen are used as catalysts to aid the fusion of protons into helium nuclei. One possible cycle is shown below:



TIME SPENT ON THE MAIN SEQUENCE

For so long as a star remains on the main sequence, hydrogen 'burning' is the source of energy that allows the star to remain in hydrostatic equilibrium (see page 192) and have a constant luminosity L. A star that exists on the main sequence for a time T_{MS} must in total radiate an energy E given by:

$$E = L \times T_{MS}$$

This energy release comes from the nuclear synthesis that has taken place over its lifetime. A certain fraction f of the mass of the star M has been converted into energy according to Einstein's famous relationship:

$$E = f \times Mc^2$$

$$L \times T_{MS} = f \times Mc^2$$

$$T_{MS} = \frac{f \times Mc^2}{L}$$

But the mass–luminosity relationship applies, $L \propto M^{3.5}$

$$T_{MS} \propto \frac{M}{M^{3.5}}$$

$$T_{MS} \propto \mathrm{M}^{-2.5}$$

Thus the higher the mass of a star, the shorter the lifetime that it spends on the main sequence

$$\frac{\textit{Time on main sequence for star A}}{\textit{Time on main sequence for star B}} = \left(\frac{\textit{Mass of star A}}{\textit{Mass of star B}}\right)^{-2.5} = \left(\frac{\textit{Mass of star B}}{\textit{Mass of star A}}\right)^{2.5}$$

For example our Sun is expected to have a main sequence lifetime of approximate 10^{10} years. How long would a star with 100 times its mass be expected to last?

Time on MS for 100 solar mass star $= 10^{10} \times \left(\frac{1}{100}\right)^{2.5} = 10^5$ years

Nucleosynthesis off the main sequence

NUCLEOSYNTHESIS OFF THE MAIN SEQUENCE

For so long as a star remains on the main sequence, hydrogen 'burning' is the source of energy that allows the star to continue emitting energy whilst remaining in a stable state. More and more helium exists in the core. A nuclear synthesis involving helium (helium 'burning') does release energy (since the binding energy per nucleon of the products is greater than that of the reactants) but can only take place at high temperatures.

For high mass stars, the helium burning process can begin gradually and spread throughout the core whereas in small mass stars this process starts suddenly. Whatever the mass of the star, a new equilibrium state is created: the red giant or red supergiant phase (see page 200).

A common process by which helium is converted is a series of nuclear reactions called the **triple alpha process** in which carbon is produced.

1. Two helium nuclei fuse into a beryllium nucleus (and a gamma ray), releasing energy.

   $^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{8}_{4}\mathrm{Be} + \gamma$

2. The beryllium nucleus fuses with another helium nucleus to produce a carbon nucleus (and a gamma ray), releasing energy.

   $^{8}_{4}\mathrm{Be} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{12}_{6}\mathrm{C} + \gamma$

3. Some of the carbon produced in the triple alpha process can go on to fuse with another helium nucleus to produce oxygen. Again this process releases energy:

   $^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{16}_{8}\mathrm{O} + \gamma$

In high and very high mass stars, gravitational contraction means that the temperature of the core can continue to rise and more massive nuclei can continue to be produced. These reactions all involve the release of energy. Typical reactions include:

Production of neon: $^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{20}_{10}\mathrm{Ne} + {}^{4}_{2}\mathrm{He}$

Production of magnesium: $^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{24}_{12}\mathrm{Mg} + \gamma$

Production of oxygen: $^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{16}_{8}\mathrm{O} + 2\,{}^{4}_{2}\mathrm{He}$

In addition if the temperatures are high enough, neon and oxygen burning can occur:

$^{20}_{10}\mathrm{Ne} + \gamma \rightarrow {}^{16}_{8}\mathrm{O} + {}^{4}_{2}\mathrm{He}$

$^{20}_{10}\mathrm{Ne} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{24}_{12}\mathrm{Mg} + \gamma$

Production of sulfur: $^{16}_{8}\mathrm{O} + {}^{16}_{8}\mathrm{O} \rightarrow {}^{32}_{16}\mathrm{S} + \gamma$

Many reactions are possible and other heavy nuclei such as silicon and phosphorus are also produced. Some of these alternative nuclear reactions also produce neutrons, which can easily be captured by other nuclei to form new isotopes. This process of **neutron capture** is explored further below.

In very high mass stars, silicon burning can also take place which results in the formation of iron, $^{56}_{26}\mathrm{Fe}$. As explained on page 199, iron has one of the highest binding energies per nucleon and represents the largest nucleus that can be created in a fusion process that releases energy. Heavier nuclei can be acquired, but the reactions require an energy input.

NUCLEAR SYNTHESIS OF HEAVY ELEMENTS – NEUTRON CAPTURE

Many of the reactions that take place in the core of stars also involve the release of neutrons. Since neutrons are without any charge, it is easy for them to interact with other nuclei that are present in the star. When a nucleus captures a neutron, the resulting nucleus is said to be **neutron rich**. Given enough time, most of these neutron-rich nuclei would undergo beta decay. In this process, the neutron changes into a proton, emitting an electron and an antineutrino:

$^{1}_{0}\mathrm{n} \rightarrow {}^{1}_{1}\mathrm{p} + {}^{0}_{-1}\beta + \bar{\nu}$

$^{A}_{Z}\mathrm{X} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{A+1}_{Z}\mathrm{X} \rightarrow {}^{A+1}_{Z+1}\mathrm{Y} + {}^{0}_{-1}\beta + \bar{\nu}$

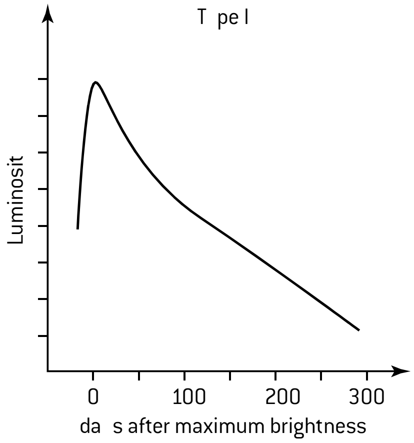
This is known as **slow neutron capture** or the **s-process**. The overall result of the s-process is a new element. Typically the s-process takes place during the helium burning stage of a red giant star. Typically this means that elements that are heavier than helium but lighter than iron are able to be created.

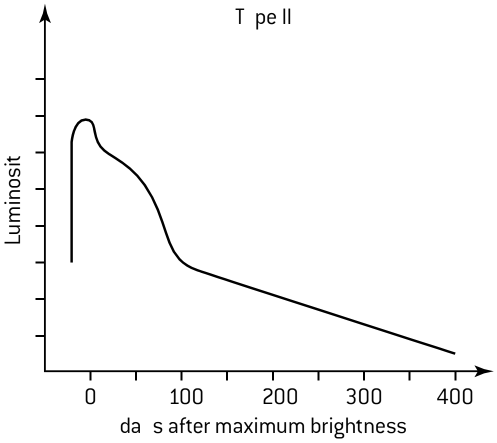
The alternative process, **rapid neutron capture** or the **r-process**, takes place when the neutrons are present in such vast numbers that there is not sufficient time for the neutron-rich nuclei to undergo beta decay before several more neutrons are captured. The result is for very heavy nuclei to be created. Typically the r-process takes place during the catastrophic explosion that is a supernova. Elements that are heavier than iron, such as uranium and thorium, can only be created in this way at very high temperatures and densities.

Types of supernovae

SUPERNOVAE

Supernovae are among the most gigantic explosions in the Universe (see page 200). The two categories of supernova are based on their **light curves** – a plot of how their brightness varies with time and a spectral analysis of the light that they emit. Type I supernovae quickly reach a maximum brightness (and an equivalent luminosity of 10^{10} Suns) which then gradually decreases over time. Type II supernovae often have lower peak luminosities (equivalent to, say, 10^9 Suns).





Supernovae types are distinguish by analysis of their light spectra. All type I supernovae do not include the hydrogen spectrum in the elements identified and the different subdivisions (Ia, Ib and Ic) are based on a more detailed spectral analysis:

- Type Ia shows the presence of singly ionized silicon.
- Type Ib shows the presence of non-ionized helium.
- Type Ic does not show the presence of helium.

All type II supernovae show the presence of hydrogen. The different subdivisions (IIP, IIL, IIn and IIb) again depend on the presence, or not, of different elements.

The reasons for these differences are the different mechanisms that are taking place:

	Supernova Type Ia	**Supernova Type II**
Spectra	Does not show hydrogen but does show singly ionized silicon.	Shows hydrogen.
Cause	White dwarf exploding.	Large mass red giant star collapsing.
Context	Binary star system with white dwarf and red giant orbiting each other.	Large star (greater than 8 Solar masses) at the end of its lifetime, fusing lighter elements up to the production of iron.
Process	The gravity field of the white dwarf star attracts material from the red giant star, thus increasing the mass of the white dwarf.	When the star runs out of fuel, the iron centre core cannot release any further energy by nuclear fusion. The star collapses under its own gravity forming a neutron star.
Explosion	The extra mass gained by the white dwarf takes the total mass of the star beyond the Chandrasekhar limit (1.4 Solar masses) for a white dwarf. Electron degeneracy pressure is no longer sufficient to halt the gravitational collapse. Nuclear fusion of heavier elements (up to iron) starts and the resulting sudden release of energy causes the star to explode with the matter being distributed throughout space.	Electron degeneracy pressure is not sufficient to halt the gravitational collapse of the core, but neutron degeneracy pressure is and the core becomes a stable and rigid neutron star. The rest of the infalling material bounces off the core creating a shock wave moving outwards. This causes all of the outer layers to be ejected.

The cosmological principle and mathematical models

THE COSMOLOGICAL PRINCIPLE

The **cosmological principle** is a pair of assumptions about the structure of the Universe upon which current models are based. The two assumptions are that the Universe, providing one only considers the large scale structures in the Universe, is **isotropic** and **homogeneous**.

An isotropic universe is one that looks the same in every direction – no particular direction is different to any other. From the perspective of an observer on Earth, this appears to be a true statement about the large scale structure of the universe, but the assumption does not only apply to observers on the Earth. In an isotropic universe **all** observers, wherever they are in the universe, are expected to see the same basic random distribution of galaxies and galaxy clusters as we do on Earth and this is true in whatever direction they observe.

A homogeneous universe is one where the local distribution of galaxies and galaxy clusters that exists in one region of the universe turns out to be the same distribution in all regions of the universe. Provided one is considering a reasonably large section of space (e.g. a sphere of radius equal to several hundreds of Mpc), then the number of galaxies in that volume of space will be effectively the same wherever we choose to look in the universe. Recent discoveries of apparently very large scale structures in the Universe cause some astrophysicists to question the validity of the cosmological principle.

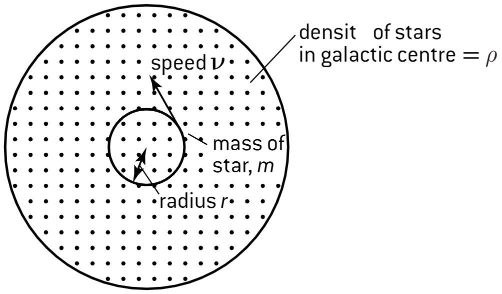
Einstein used the cosmological principle to develop a model of the Universe in which the Universe was static. He did this by proposing that the gravitational attraction between galaxies would be balanced by a yet-to-be-discovered cosmological repulsion. Subsequent analysis of the equations of general relativity showed that, if the cosmological principle is correct, the Universe must be non-static. Hubble's observational discovery of the expansion of the Universe and the existence of CMB has meant that many physicists now agree that the Universe is non-static based around the Big Bang model of an expanding universe. The cosmological principle is also linked to three possible models for the future of the Universe (see page 211).

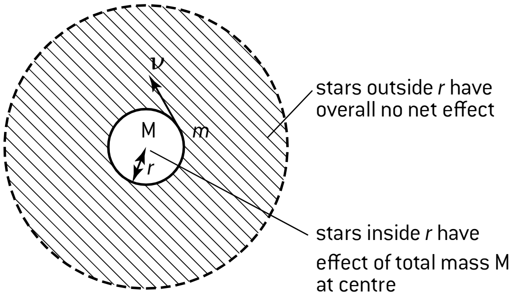
ROTATION CURVES – MATHEMATICAL MODELS

The stars in a galaxy rotate around their common centre of mass. Different models can be used to predict how the speed varies with distance from the galactic centre.

1. Near the galactic centre

A simple model to explain the different speeds of rotation of stars near the galactic centre assumes that density of the galaxy near its centre, ρ, is constant. A star of mass m feels a resultant force of gravitational attraction in towards the centre. The value of this resultant force is the same as if the total mass M of all the stars that are closer to the galactic centre were concentrated in the centre. An important point to note is that the net effect of all the stars that are orbiting at radius that is greater than r sums to zero.





The star at a given distance r from the centre will orbit in circular motion because its centripetal force is provided by the gravitational attraction:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v^2 = \frac{GM}{r}$$

The total mass of stars that orbit closer than of this star, M, is given by

$$M = \text{volume} \times \text{density} = \frac{4}{3}\pi r^3 \times \rho$$

$$v^2 = \frac{G\frac{4}{3}\pi r^3 \rho}{r} = \frac{4\pi G\rho}{3} r^2$$

$$v = \sqrt{\frac{4\pi G\rho}{3}} \cdot r$$

i.e. $v \propto r$

2. Far away from the galactic centre

Far away from the galactic centre, observations of the number of visible stars show that the effective density of the galaxy has reduced so much that individual stars at these distances can be considered to be freely orbiting the central mass and to be unaffected by their neighbouring stars. In this situation,

$v^2 = \frac{GM}{r}$ where M is the mass of the galaxy

i.e. $v \propto \sqrt{\frac{1}{r}}$

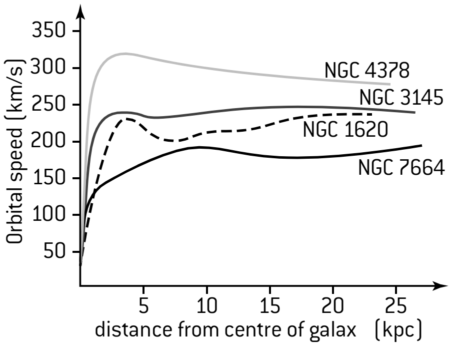
Comparisons with observations of real galaxies show good agreement with mathematical model (1) but no agreement with mathematical model (2). The proposed solution is discussed on page 209.

HL Rotation curves and dark matter

ROTATION CURVES

Galaxies rotate around their centre of mass and the speeds of this rotation can be calculated for individual stars from an analysis of the star's spectra. A **rotation curve** for a galaxy show how this orbital speed varies with distance from the galactic centre. Most galaxies show:

- an initial linear increase in orbital velocity with distance within the galactic centre
- a flat or slightly increasing curve showing a roughly constant speed of rotation away from the galactic centre.



EVIDENCE FOR DARK MATTER

As shown above, observed rotation curves for real galaxies agree with theoretical models within the galactic centre ($v \propto r$) **but** the orbital velocity of stars is not observed to decrease with distance away from the centre as would be expected. Instead, the orbital velocity is roughly constant whatever the radius. If the orbital velocity v of a star is constant at different values of radius, then

$$\text{since } v^2 = \frac{GM}{r}$$

$$\frac{M}{r} = \textit{constant} \text{ or } M \propto r$$

Thus the total mass that is keeping the star orbiting in its galaxy must be increasing with distance from the galactic centre. This is certainly not true of the visible mass (the stars emitting light) that we can see so the suggestion is that there must be **dark matter**. In this situation it would have to be concentrated outside the galactic centre forming a halo around the galaxy. Further evidence suggests that only a very small amount of this matter could be imagined to be made up of the protons and neutrons that constitute ordinary, or **baryonic**, matter.

Dark matter:

- gravitationally attracts ordinary matter
- does not emit radiation and cannot be inferred from its interactions
- is unknown in structure
- makes up the majority of the Universe with less than 5% of the Universe made up of ordinary baryonic matter.

MACHOS, WIMPS AND OTHER THEORIES

Astrophysicists are attempting to come up with theories to explain why there is so much dark matter and what it consists of. There are a number of possible theories:

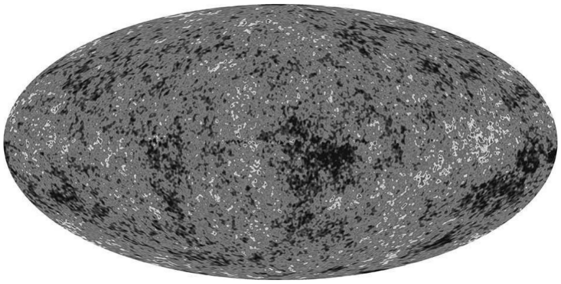
- The matter could be found in **M**assive **A**stronomical **C**ompact **H**alo **O**bjects or **MACHOs** for short. There is some evidence that lots of ordinary matter does exist in these groupings. These can be thought of as low-mass 'failed' stars or high-mass planets. They could even be black holes. These would produce little or no light. Evidence suggests that these could only account for a small proportion.
- There could be new particles that we do not know about. These are the **W**eakly **I**nteracting **M**assive **P**articles. Many experimenters around the world are searching for these so-called **WIMPs**.
- Perhaps our current theories of gravity are not completely correct. Some theories try to explain the missing matter as simply a failure of our current theories to take everything into account.

HL The history of the Universe

FLUCTUATIONS IN CMB

The cosmic microwave background radiation (CMB) is essentially **isotropic** (the same in all directions). This implies that the matter in the early Universe was uniformly distributed throughout space with no random temperature variations at all. If this was precisely the case then the development of the Universe would be expected to be absolutely identical everywhere and matter would be uniformly distributed throughout the Universe – it would be without any structure. We know, however, that matter is not uniformly distributed as it is concentrated into stars and galaxies.

Further analysis of the CMB reveals tiny fluctuations (**anisotropies**) in the temperature distribution of the early Universe in different directions. These temperature variations are typically a few μK compared with the background effective temperature of 2.73 K. The diagram right is an enhanced projection which highlights the minor observed variations in the CMB (with the effects of our own galaxy removed). Just like a map includes all the countries of the world, this projection shows the variation in received CMB from the whole Universe.



Variation in CMB as observed by the Wilkinson Microwave Anisotropy Probe (WMAP)

The minute differences in temperature imply minor differences in densities, which allow structures to be developed as the Universe expands.

THE HISTORY OF THE UNIVERSE

We can 'work backwards' and imagine the process that took place soon after the Big Bang.

- Very soon after the Big Bang, the Universe must have been very hot.
- As the Universe expanded it cooled. It had to cool to a certain temperature before atoms and molecules could be formed.
- The Universe underwent a short period of huge expansion (Inflation) that would have taken place from about 10^{-36} s after the Big Bang to 10^{-32} s.

Time	What is happening	Comments
10^{-45} s → 10^{-36} s	Unification of forces	This is the starting point.
10^{-36} s → 10^{-32} s	Inflation	A rapid period of expansion – the so-called inflationary epoch. The reasons for this rapid expansion are not fully understood.
10^{-32} s → 10^{-5} s	Quark–lepton era	Matter and antimatter (quarks and leptons) are interacting all the time. There is slightly more matter than antimatter.
10^{-5} s → 10^{-2} s	Hadron era	At the beginning of this short period it is just cool enough for hadrons (e.g. protons and neutrons) to be stable.
10^{-2} s → 10^{3} s	Nucleosynthesis	During this period some of the protons and neutrons have combined to form helium nuclei. The matter that now exists is the 'small amount' that is left over when matter and antimatter have interacted.
10^{3} s → 3×10^{5} years	Plasma era (radiation era)	The formation of light nuclei has now finished and the Universe is in the form of a plasma with electrons, protons, neutrons, helium nuclei and photons all interacting.
3×10^{5} years → 10^{9} years	Formation of atoms	At the beginning of this period, the Universe has become cool enough for the first atoms to exist. Under these conditions, the photons that exist stop having to interact with the matter. It is these photons that are now being received as part of the background microwave radiation. The Universe is essentially 75% hydrogen and 25% helium.
10^{9} years → now	Formation of stars, galaxies and galactic clusters	Some of the matter can be brought together by gravitational interactions. If this matter is dense enough and hot enough, nuclear reactions can take place and stars are formed.

COSMIC SCALE FACTOR AND TEMPERATURE

The expansion of the Universe means that the wavelength of any radiation that has been emitted in the past will be 'stretched' over time (see page 202). Thus the radiation that was emitted approximately 12 billion years ago (shortly after the Big Bang) at very short wavelengths is now being received as much longer microwaves – the CMB radiation.

The spectrum of CMB radiation received corresponds to black-body radiation at a temperature of 2.73 K. The calculation uses Wien's law to link the peak wavelength, λ_{max}, of the radiation to the temperature, T, of the black body in kelvins:

$$\lambda_{max} = \frac{2.9 \times 10^{-3}}{T}$$

$$\lambda_{max} \propto \frac{1}{T}$$

When the radiation was emitted the temperature of the universe was much hotter, the cosmic scale factor, R, was much smaller and λ_{max} was also proportionally much smaller.

Since the stretching of the Universe is the cause of the change in wavelength, then the ratio of cosmic scale factors at two different times must be the same as the ratio of peak wavelengths so

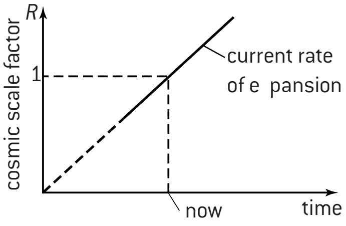
$$\lambda_{max} \propto R$$

$$\frac{1}{T} \propto R \text{ or } T \propto \frac{1}{R}$$

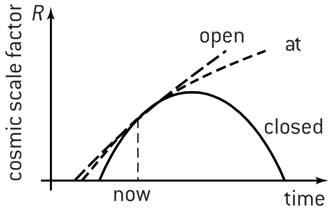
The future of the Universe

FUTURE OF THE UNIVERSE (WITHOUT DARK ENERGY)

If the Universe is expanding at the moment, what is it going to do in the future? As a result of the Big Bang, other galaxies are moving away from us. If there were no forces between the galaxies, then this expansion could be thought of as being constant.



The expansion of the Universe cannot, however, have been uniform. The force of gravity acts between all masses. This means that if two masses are moving apart from one another there is a force of attraction pulling them back together. This force must have slowed the expansion down in the past. What it is going to do in the future depends on the current rate of expansion and the density of matter in the Universe.



An **open Universe** is one that continues to expand forever. The force of gravity slows the rate of recession of the galaxies down a little bit but it is not strong enough to bring the expansion to a halt. This would happen if the density in the Universe were low.

A **closed Universe** is one that is brought to a stop and then collapses back on itself. The force of gravity is enough to bring the expansion to an end. This would happen if the density in the Universe were high.

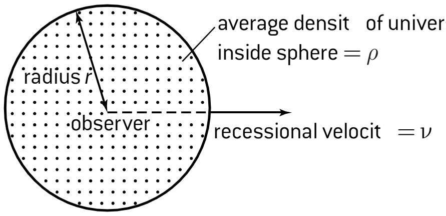
A **flat Universe** is the mathematical possibility between open and closed. The force of gravity keeps on slowing the expansion down but it takes an infinite time to get to rest. This would only happen if the Universe were exactly the right density. One electron-positron pair more, and the gravitational force would be a little bit bigger. Just enough to start the contraction and make the Universe closed.

CRITICAL DENSITY, ρ_c

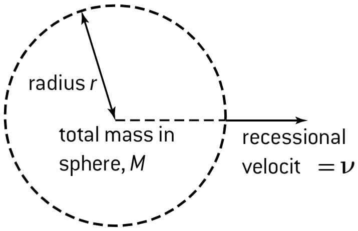
The theoretical value of density that would create a flat Universe is called the **critical density**, ρ_c. Its value is not certain because the current rate of expansion is not easy to measure. Its order of magnitude is 10^{-26} kg m^{-3} or a few proton masses every cubic metre. If this sounds very small remember that enormous amounts of space exist that contain little or no mass at all.

The density of the Universe is not an easy quantity to measure. It is reasonably easy to estimate the mass in a galaxy by estimating the number of stars and their average mass but the majority of the mass in the Universe is dark matter.

The value of ρ_c can be estimated using Newtonian gravitation. We consider a galaxy at a distance r away from an observer with a recessional velocity of v with respect to the observer.



The net effect of all the masses in the Universe outside the sphere on the galaxy is zero (see page 208 for an analogous situation). The galaxy is thus gravitationally attracted in by a total mass M which acts as though it was located at the observer as shown (above).



The total energy E_T of the galaxy is the addition of its kinetic energy E_K and gravitational potential energy, E_P given by:

$$E_T = E_K + E_P$$

$$E_K = \frac{1}{2}mv^2 \text{ but Hubble's law gives } v = H_0 r$$

$$\therefore E_K = \frac{1}{2}m(H_0 r)^2$$

$$E_P = -\frac{GMm}{r} \text{ but } M = \text{volume} \times \text{density} = \frac{4}{3}\pi r^3 \rho$$

$$E_P = -\frac{G4\pi r^3 \rho m}{3r} = -\frac{4G\pi r^2 \rho m}{3}$$

If E_T is positive, the galaxy will escape the inward attraction – the universe is open.

If E_T is negative, the galaxy will eventually fall back in – the universe is closed.

If E_T is exactly zero, the galaxy will take an infinite time to be brought to rest – the universe is flat. This will occur when the density of the universe ρ is equal to the critical density ρ_c.

$$\therefore \frac{1}{2}m(H_0 r)^2 = \frac{4G\pi r^2 \rho_c m}{3}$$

$$\therefore mH_0^2 r^2 = \frac{8G\pi r^2 \rho_c m}{3}$$

$$\therefore \rho_c = \frac{3H_0^2}{8\pi G}$$

HL Dark energy

COSMIC DENSITY PARAMETER

The cosmic density parameter, Ω_0 is the ratio of the average density of matter and energy in the Universe, ρ, to the critical density, ρ_c

$$\Omega_0 = \frac{\rho}{\rho_C}$$

If $\Omega_0 > 1$, the universe is closed.

If $\Omega_0 < 1$, the universe is open.

If $\Omega_0 = 1$, the universe is flat.

DARK ENERGY

Gravitational attraction between masses means that the rate of expansion of the Universe would be expected to decrease with time. Measurements using type Ia supernovae as standard candles have provided strong evidence that the expansion has not, in fact, been slowing down over time (see page 204). Observations currently indicate that the Universe's rate of expansion has been increasing.

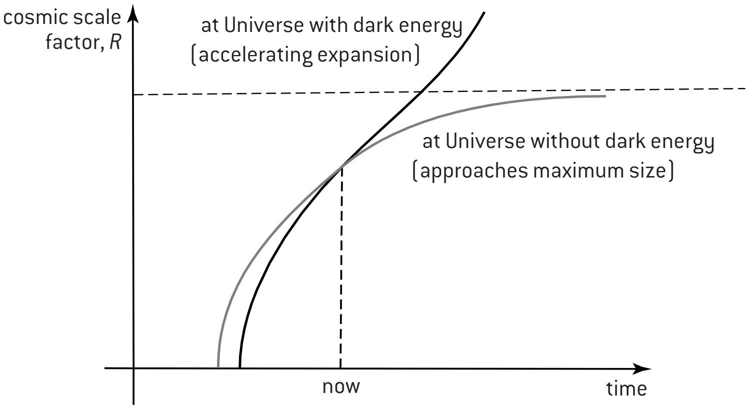
Currently there is no single accepted explanation for this observation and, of course, it is possible that our theories of gravity and general relativity need to be modified. Perhaps we are on the brink of discovery of new physics. Whatever the cause, the reason for the Universe's accelerating expansion has been given the general name '**dark energy**'.

Dark energy and dark matter are two different concepts. In both cases experimental evidence implies their existence but physicists have yet to agree a theoretical basis that explains the existence of either concept.

- Dark matter is hypothesized to explain the 'missing matter' that must exist within galaxies for the known laws of gravitational attraction to be able to explain a galaxy's rate of rotation. Dark matter **adds to the attractive force of gravity acting within galaxies** implying more unseen mass than had been previously expected, hence the name *dark mass*.
- The observation that expansion of the Universe is accelerating means that then there must be a force that is counteracting the attractive force of gravity. Dark energy **opposes the attractive force of gravity between galaxies**. The resulting increase in energy implies an unseen source of energy, hence the name *dark energy*.

EFFECT OF DARK ENERGY ON THE COSMIC SCALE FACTOR

The existence of dark energy counteracts the attractive force of gravity. This will cause the cosmic scale factor to increase over time. The graph below compares how a flat Universe is predicted to develop with and without dark energy.



Astrophysics research

ASTROPHYSICS RESEARCH

Much of the current fundamental research that is being undertaken in astrophysics involves close international collaboration and the sharing of resources. Scientists can be proud of their record of international collaboration. For example, at the time that the previous edition of this book was being published, the Cassini spacecraft had been in orbit around Saturn for several years sending information about the planet back to Earth and is currently (2014) continuing to produce data.

The Cassini–Huygens spacecraft was funded jointly by ESA (the European Space Agency), NASA (the National Aeronautics and Space Administration of the United States of America) and ASI (Agenzia Spaziale Italiana – the Italian Space Agency). As well as general information about Saturn, an important focus of the mission was a moon of Saturn called Titan. The Huygens probe was released and sent back information as it descended towards the surface. The information discovered is shared among the entire scientific community. Many current projects, for example the Dark Energy Survey (involving more than 120 scientists for 23 institutions worldwide), continue this process.

All countries have a limited budget available for the scientific research that they can undertake. There are arguments both for, and against, investing significant resources into researching the nature of the Universe.

Future research, such as the Euclid mission to map the geometry of the dark Universe continues to be planned.

Arguments for:

- Understanding the nature of the Universe sheds light on fundamental philosophical questions like:
  - Why are we here?
  - Is there (intelligent) life elsewhere in the Universe?
- It is one of the most fundamental, interesting and important areas for humankind as a whole and it therefore deserves to be properly researched.
- All fundamental research will give rise to technology that may eventually improve the quality of life for many people.
- Life on Earth will, at some time in the distant future, become an impossibility. If humankind's descendents are to exist in this future, we must be able to travel to distant stars and colonize new planets.

Arguments against:

- The money could be more usefully spent providing food, shelter and medical care to the many millions of people who are suffering from hunger, homelessness and disease around the world.
- If money is to be allocated on research, it is much more worthwhile to invest limited resources into medical research. This offers the immediate possibility of saving lives and improving the quality of life for some sufferers.
- It is better to fund a great deal of small diverse research rather than concentrating all funding into one expensive area. Sending a rocket into space is expensive, thus funding space research should not be a priority.
- Is the information gained really worth the cost?

CURRENT OBSERVATIONS

Three recent scientific experiments that have studied the CMB in detail have together added a great deal to our understanding of the Universe. Particular experiments of note include:

- NASA's **Co**smic **B**ackground **E**xplorer (COBE)
- NASA's Wilkinson Microwave Anisotropy Probe (WMAP)
- ESA's Planck space observatory.

Together these experiments have:

- mapped the anisotropies of the CMB in great detail and with precision
- discovered that the first generation of stars to shine did so 200 million years after the Big Bang, much earlier than many scientists had previously expected
- calculated the age of the Universe as 13.75 ± 0.14 billion years old
- calculated the Hubble constant to be 67.15 km s^{-1} Mpc^{-1}
- showed that their results were consistent with the Big Bang and specific inflation theories
- showed the Universe to be flat, $\Omega_0 = 1$
- calculated the Universe to be composed of 4.6% atoms, 23% dark matter and 71.4% dark energy.

In summary, current scientific evidence suggests that, when dark matter and dark energy are taken into consideration, the Universe:

- is flat
- has a density that is, within experimental error, very close to the critical density
- has an accelerating expansion
- is composed mainly of dark matter and dark energy.

IB Questions – astrophysics

1. This question is about determining some properties of the star Wolf 359.
   a) The star Wolf 359 has a parallax angle of 0.419 seconds.
      (i) Describe how this parallax angle is measured. [4]
      (ii) Calculate the distance in light-years from Earth to Wolf 359. [2]
      (iii) State why the method of parallax can only be used for stars at a distance of less than a few hundred parsecs from Earth. [1]
   b) The ratio [4]

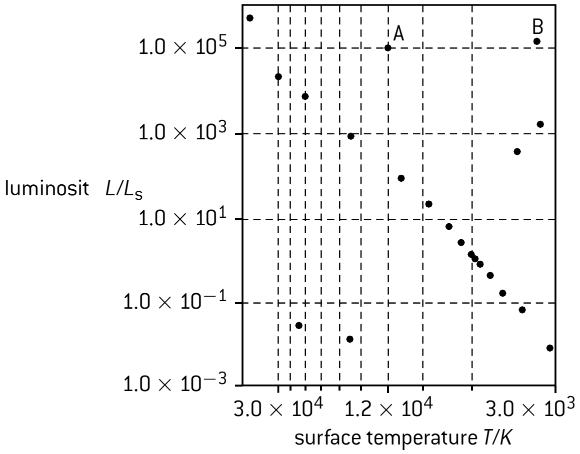
$$\frac{\text{apparent brightness of Wolf 359}}{\text{apparent brightness of the Sun}} \text{ is } 3.7 \times 10^{-15}.$$

   Show that the ratio

$$\frac{\text{luminosity of Wolf 359}}{\text{luminosity of the Sun}} \text{ is } 8.9 \times 10^{-4}. \ (1\text{ly} = 6.3 \times 10^{4} \text{ AU})$$

   c) The surface temperature of Wolf 359 is 2800 K and its luminosity is 3.5×10^{23} W. Calculate the radius of Wolf 359. [2]
   d) By reference to the data in (c), suggest why Wolf 359 is neither a white dwarf nor a red giant. [2]

2. The diagram below shows the grid of an HR diagram, on which the positions of selected stars are shown. (L_S = luminosity of the Sun.)

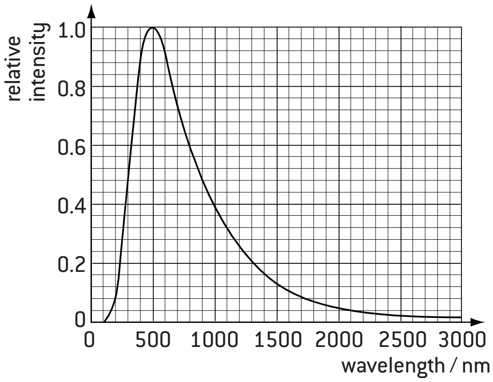


   a) (i) Draw a circle around the stars that are red giants. Label this circle R. [1]
      (ii) Draw a circle around the stars that are white dwarfs. Label this circle W. [1]
      (iii) Draw a line through the stars that are main sequence stars. [1]
   b) Explain, without doing any calculation, how astronomers can deduce that star B has a larger diameter than star A. [3]
   c) Using the following data and information from the HR diagram, show that star A is at a distance of about 800 pc from Earth.

      Apparent brightness of the Sun = 1.4×10^3 W m^{-2}
      Apparent brightness of star A = 4.9×10^{-9} W m^{-2}
      Mean distance of Sun from Earth = 1.0 AU
      1 pc = 2.1×10^5 AU [4]

   d) Explain why the distance of star A from Earth cannot be determined by the method of stellar parallax. [1]

3. a) The spectrum of light from the Sun is shown below.



   Use this spectrum to estimate the surface temperature of the Sun. [2]
   b) Outline how the following quantities can, in principle, be determined from the spectrum of a star.
      (i) The elements present in its outer layers. [2]
      (ii) Its speed relative to the Earth. [2]

4. a) Explain how Hubble's law supports the Big Bang model of the Universe. [2]
   b) Outline **one** other piece of evidence for the model, saying how it supports the Big Bang. [3]
   c) The Andromeda galaxy is a relatively close galaxy, about 700 kpc from the Milky Way, whereas the Virgo nebula is 2.3 Mpc away. If Virgo is moving away at 1200 km s^{-1}, show that Hubble's law predicts that Andromeda should be moving away at roughly 400 km s^{-1}. [1]
   d) Andromeda is in fact moving **towards** the Milky Way, with a speed of about 100 km s^{-1}. How can this discrepancy from the prediction, in both magnitude and direction, be explained? [3]
   e) If light of wavelength 500 nm is emitted from Andromeda, what would be the wavelength observed from Earth? [3]

5. A quasar has a redshift of 6.4. Calculate the ratio of the current size of the universe to its size when the quasar emitted the light that is being detected. [3]

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6. Explain the following:
   a) Why more massive stars have shorter lifetimes [2]
   b) The jeans criterion [2]
   c) How elements heavier than iron are produced by stars [2]
   d) How type 1a supernovae can be used as standard candles [2]
   e) The significance of observed anisotropies in the Cosmic Microwave background [2]
   f) The significance of the critical density of universe [2]
   g) The evidence for dark matter [2]
   h) What is meant by dark energy [2]

7. Calculate the critical density for of the universe using the Hubble constant of 71 km s^{-1} Mpc^{-1} [3]