02 Mechanics

A steam train crosses an arched bridge. Without a knowledge of mechanics trains and bridges would never have been built.

Essential ideas

2.1 Motion
Motion may be described and analysed by the use of graphs and equations.

2.2 Forces
Classical physics requires a force to change a state of motion, as suggested by Newton in his laws of motion.

2.3 Work, energy, and power
The fundamental concept of energy lays the base upon which all areas of science are built.

2.4 Momentum and impulse
Conservation of momentum is an example of a law which is never violated.

NATURE OF SCIENCE

From the definitions of velocity and acceleration we can use mathematics to derive a set of equations that predict the position and velocity of a particle at any given time. We can show by experiment that these equations give the correct result for some examples, then make the generalization that the equations apply in all cases.

2.1 Motion

2.1 Motion

Understandings, applications, and skills:

Distance and displacement
Speed and velocity
- Determining instantaneous and average values for velocity, speed, and acceleration.

Acceleration
- Determining the acceleration of free fall experimentally.

Graphs describing motion
- Sketching and interpreting motion graphs.

Equations of motion for uniform acceleration
- Solving problems using equations of motion for uniform acceleration.

Projectile motion
- Analysing projectile motion, including the resolution of vertical and horizontal components of acceleration, velocity, and displacement.

Guidance
- *Calculations will be restricted to those neglecting air resistance.*
- *Projectile motion will only involve problems using a constant value of g close to the surface of the Earth.*
- *The equation of the path of a projectile will not be required.*

Fluid resistance and terminal speed
- Qualitatively describing the effect of fluid resistance on falling objects or projectiles, including reaching terminal speed.

In Chapter 1, we observed that things move and now we are going to mathematically model that movement. Before we do that, we must define some quantities that we are going to use.

Displacement and distance

It is important to understand the difference between distance travelled and displacement. To explain this, consider the route marked out on the map shown in Figure 2.1

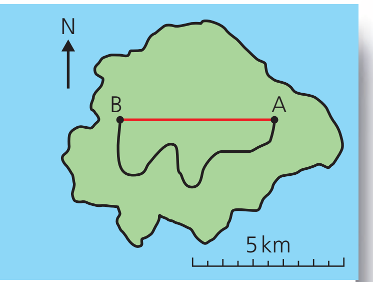


Figure 2.1

Displacement is the shortest path moved in a particular direction.

The unit of displacement is the metre (m).

Displacement is a vector quantity.

On the map, the displacement is the length of the straight line from A to B, a distance of 5 km west. (Note: since displacement is a vector you should always say what the direction is.)

Distance is how far you have travelled from A to B.

The unit of distance is also the metre.

Distance is a scalar quantity.

In this example, the distance travelled is the length of the path taken, which is about 10 km.

Sometimes this difference leads to a surprising result. For example, if you run all the way round a running track you will have travelled a distance of 400 m but your displacement will be 0 m.

In everyday life, it is often more important to know the distance travelled. For example, if you are going to travel from Paris to Lyon by road you will want to know that the distance by road is 450 km, not that your final displacement will be 336 km SE. However, in physics, we break everything down into its simplest parts, so we start by considering motion in a straight line only. In this case it is more useful to know the displacement, since that also has information about which direction you have moved.

Velocity and speed

Both speed and velocity are a measure of how fast a body is moving, but velocity is a vector quantity and speed is a scalar.

Velocity is defined as the displacement per unit time.

$$\textbf{velocity} = \frac{\textbf{displacement}}{\textbf{time}}$$

The unit of velocity is m s^{-1}.

Velocity is a vector quantity.

Speed is defined as the distance travelled per unit time.

$$\textbf{speed} = \frac{\textbf{distance}}{\textbf{time}}$$

The unit of speed is also m s^{-1}.

Speed is a scalar quantity.

Exercise

1 Convert the following speeds into m s^{-1}.

(a) A car travelling at 100 km h^{-1}.
(b) A runner running at 20 km h^{-1}.

Average velocity and instantaneous velocity

Consider travelling by car from the north of Bangkok to the south – a distance of about 16 km. If the journey takes 4 hours, you can calculate your velocity to be $\frac{16}{4} = 4\,\text{km h}^{-1}$ in a southwards direction. This doesn't tell you anything about the journey, just the difference between the beginning and the end (unless you managed to travel at a constant speed in a straight line). The value calculated is the *average velocity* and in this example it is quite useless. If we broke the trip down into lots of small pieces, each lasting only one second, then for each second the car could be considered to be travelling in a straight line at a constant speed. For these short stages we could quote the car's *instantaneous velocity* – that's how fast it's going at that moment in time and in which direction.

Figure 2.2 It's not possible to take this route across Bangkok with a constant velocity.

The bus in the photo has a constant velocity for a very short time.

Exercise

2 A runner runs around a circular track of length 400 m with a constant speed in 96 s starting at the point shown. Calculate:

(a) the average speed of the runner.
(b) the average velocity of the runner.
(c) the instantaneous velocity of the runner after 48 s.
(d) the displacement after 24 s.

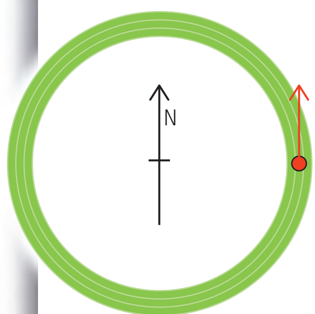


Figure 2.3.

Constant velocity

If the velocity is constant then the instantaneous velocity is the same all the time so instantaneous velocity = average velocity.

Since velocity is a vector this also implies that the direction of motion is constant.

Measuring a constant velocity

From the definition of velocity we see that velocity = $\frac{\text{displacement}}{\text{time}}$.

Rearranging this gives displacement = velocity × time.

So, if velocity is constant, displacement is proportional to time. To test this relationship and find the velocity we can measure the displacement of a body at different times. To do this you either need a lot of clocks or a stop clock that records many times. This is called a *lap timer*. In this example a bicycle was ridden at constant speed along a straight road past 6 students standing 10 m apart, each operating a stop clock as in Figure 2.4. The clocks were all started when the bike, already moving, passed the start marker and stopped as the bike passed each student.

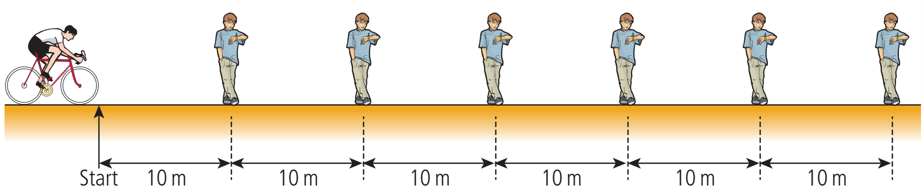


Figure 2.4 Measuring the time for a bike to pass.

Distance/m ±0.1 m	Time/s ±0.02 s
10.0	3.40
20.0	5.62
30.0	8.55
40.0	12.21
50.0	14.17
60.0	17.21

Table 2.1.

The results achieved have been entered into Table 2.1.

The uncertainty in position is quoted as 0.1 m since it is difficult to decide exactly when the bike passes the marker.

The digital stop clock had a scale with 2 decimal places so the uncertainty was 0.01 s. However, the uncertainty quoted is 0.02 s since the clocks all had to be started at the same time.

Since s is proportional to t, then a graph of s vs t should give a straight line with gradient = velocity as shown in Figure 2.5.

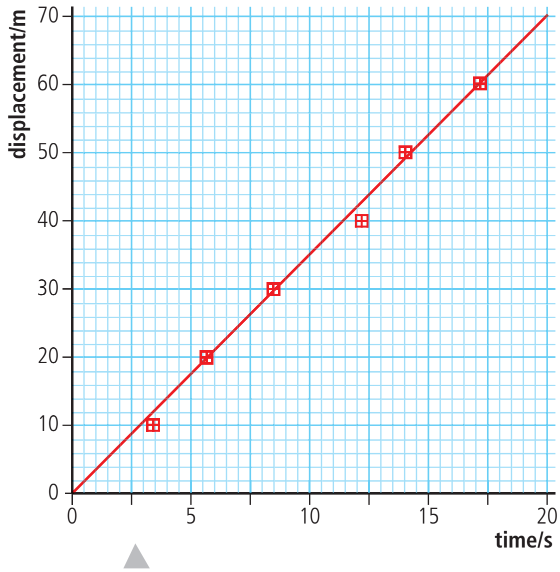


Figure 2.5 Graph of displacement vs time for a bike.

Notice in this graph the line does not quite hit all the dots. This is because the uncertainty in the measurement in time is almost certainly bigger than the uncertainty in the clock (±0.02 s) due to the reaction time of the students stopping the clock. To get a better estimate of the uncertainty we would have to have several students standing at each 10 m position. Repeating the experiment isn't possible in this example since it is very difficult to ride at the same velocity several times.

The gradient indicates that the velocity = 3.4 m s^{-1}.

Laboratory example

Most school laboratories aren't big enough to ride bikes in so when working indoors we need to use shorter distances. This means that the times are going to be shorter so hand-operated stop clocks aren't going to be accurate enough. One way of timing in the lab is by using photogates. These are connected to the computer via an interface and record the time when a body passes in or out of the gate. So to replicate the bike experiment in the lab using a ball we would need 7 photogates as in Figure 2.6, one extra to represent the start.

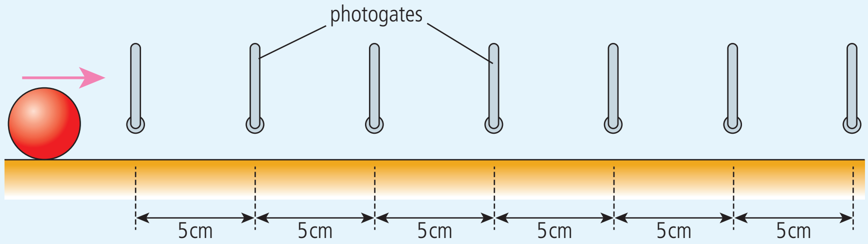
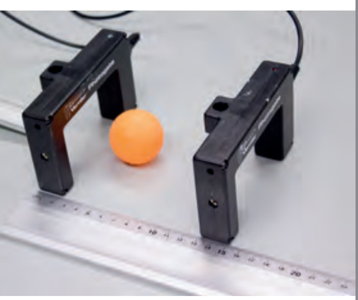


Figure 2.6 How to measure the time for a rolling ball if you've got 7 photogates.



This is would be quite expensive so we compromise by using just two photogates and a motion that can be repeated. An example could be a ball moving along a horizontal section of track after it has rolled down an inclined plane. Provided the ball starts from the same point it should have the same velocity. So instead of using seven photogates we can use two, one is set at the start of the motion and the other is moved to different positions along the track as in Figure 2.7.

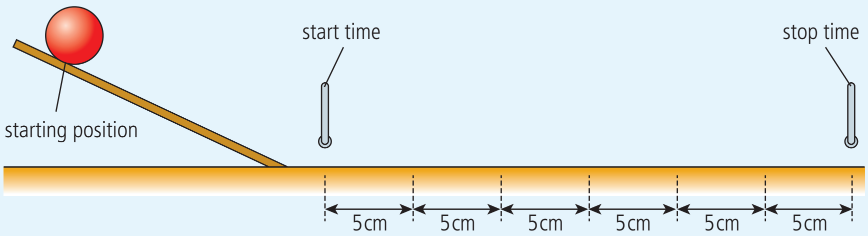


Figure 2.7 Measuring the velocity of a ball with 2 photogates.

Table 2.2 shows the results were obtained using this arrangement.

Table 2.2.

Displacement/cm ±0.1 cm	Time(t)/s ±0.0001 s					Mean t/s	Δt/s
5.0	0.0997	0.0983	0.0985	0.1035	0.1040	0.101	0.003
10.0	0.1829	0.1969	0.1770	0.1824	0.1825	0.18	0.01
15.0	0.2844	0.2800	0.2810	0.2714	0.2779	0.28	0.01
20.0	0.3681	0.3890	0.3933	0.3952	0.3854	0.39	0.01
25.0	0.4879	0.5108	0.5165	0.4994	0.5403	0.51	0.03
30.0	0.6117	0.6034	0.5978	0.6040	0.5932	0.60	0.01

Notice the uncertainty calculated from $\frac{(max - min)}{2}$ is much more than the instrument uncertainty. Graphing displacement *vs* time gives Figure 2.8.

From this graph we can see that within the limits of the experiment's uncertainties the displacement was proportional to time, so we can conclude that the velocity was constant. However, if we look closely at the data we see that there seems to be a slight curve indicating that perhaps the ball was slowing down. To verify this we would have to collect more data.

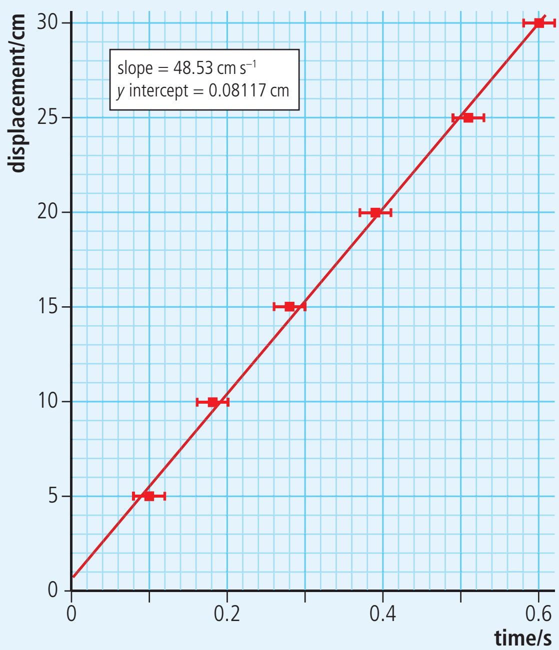


Figure 2.8 Graph of displacement *vs* time for a rolling ball.

Measuring the constant velocity of a ball rolling across a flat table

A worksheet with full details of how to carry out this experiment is available on your eBook.

Measuring instantaneous velocity

To measure instantaneous velocity, a very small displacement must be used. This could be achieved by placing two photogates close together or attaching an obstructer such as a piece of card to the moving body as shown in Figure 2.9. The time taken for the card to pass through the photogate is recorded and the instantaneous velocity calculated from the $\frac{\text{length of card}}{\text{time taken}}$ $(\frac{d}{t})$.

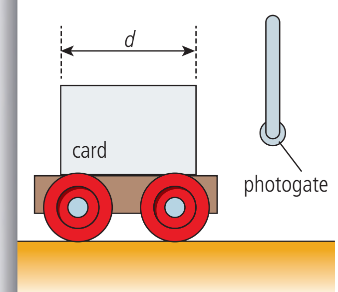


Figure 2.9 A card and photogate used to measure instantaneous velocity.

Relative velocity

Velocity is a vector so must be added vectorially. Imagine you are running north at $3\,\mathrm{m\,s^{-1}}$ on a ship that is also travelling north at $4\,\mathrm{m\,s^{-1}}$ as shown in Figure 2.10. Your velocity relative to the ship is $3\,\mathrm{m\,s^{-1}}$ but your velocity relative to the water is $7\,\mathrm{m\,s^{-1}}$. If you now turn around and run due south your velocity will still be $3\,\mathrm{m\,s^{-1}}$ relative to the ship but $1\,\mathrm{m\,s^{-1}}$ relative to the water. Finally, walking towards the east the vectors add at right angles to give a resultant velocity of magnitude $5\,\mathrm{m\,s^{-1}}$ relative to the water. You can see that the velocity vectors have been added.

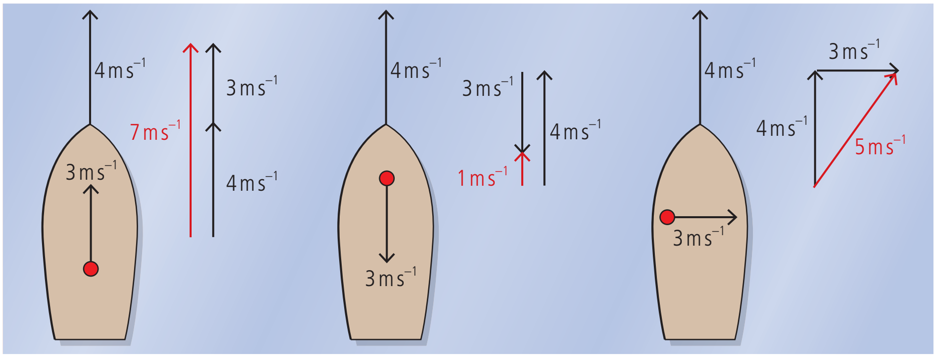


Figure 2.10 Running on board a ship.

Imagine now that you are floating in the water watching two boats travelling towards each other as in Figure 2.11.

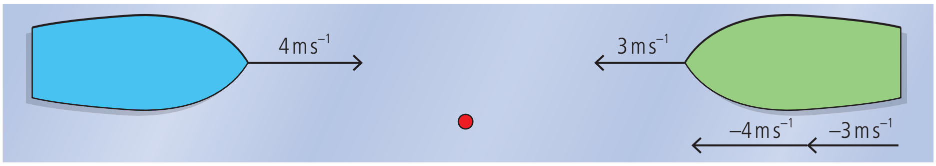


Figure 2.11 Two boats approach each other.

The blue boat is travelling east at $4\,\mathrm{m\,s^{-1}}$ and the green boat west at $-3\,\mathrm{m\,s^{-1}}$. Remember that the sign of a vector in one dimension gives the direction, so if east is positive then west is negative. If you were standing on the blue boat you would see the water going past at $-4\,\mathrm{m\,s^{-1}}$ so the green boat would approach with the velocity of the water plus its velocity in the water: $-4 + -3 = -7\,\mathrm{m\,s^{-1}}$. This can also be done in 2 dimensions as in Figure 2.12.

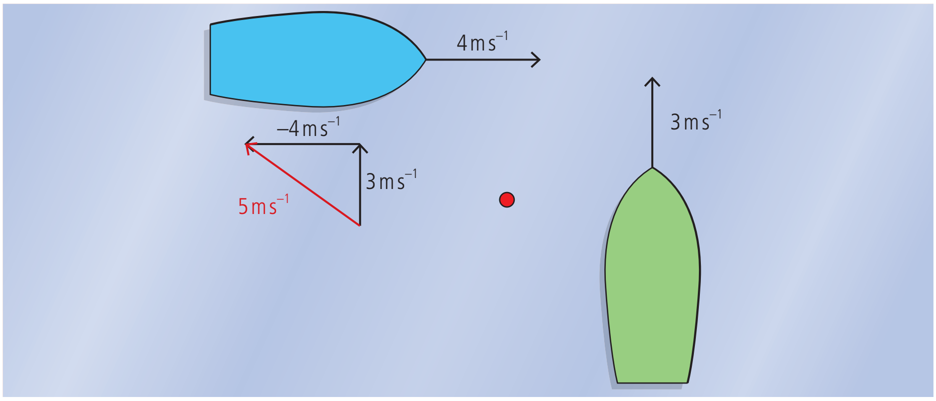


Figure 2.12 Two boats travelling perpendicular to each other.

According to the swimmer floating in the water, the green boat travels north and the blue boat travels east. but an observer on the blue boat will see the water travelling towards the west and the boat travelling due north; adding these two velocities gives a velocity of $5\,\mathrm{m\,s^{-1}}$ in an approximately northwest direction.

Exercises

3 An observer standing on a road watches a bird flying east at a velocity of $10\,m\,s^{-1}$. A second observer, driving a car along the road northwards at $20\,m\,s^{-1}$ sees the bird. What is the velocity of the bird relative to the driver?

4 A boat travels along a river heading north with a velocity $4\,m\,s^{-1}$ as a woman walks across a bridge from east to west with velocity $1\,m\,s^{-1}$. Calculate the velocity of the woman relative to the boat.

Acceleration

In everyday usage, the word *accelerate* means to go faster. However, in physics, acceleration is defined as the rate of change of velocity.

$$\text{acceleration} = \frac{\text{change of velocity}}{\text{time}}$$

The unit of acceleration is $m\,s^{-2}$.

Acceleration is a vector quantity.

This means that whenever a body changes its velocity, it accelerates. This could be because it is getting faster, slower, or just changing direction. In the example of the journey across Bangkok, the car would have been slowing down, speeding up, and going round corners almost the whole time, so it would have had many different accelerations. However, this example is far too complicated for us to consider in this course (and probably any physics course). For most of this chapter we will only consider the simplest example of accelerated motion, constant acceleration.

Constant acceleration in one dimension

In one-dimensional motion, the acceleration, velocity, and displacement are all in the same direction. This means they can simply be added without having to draw triangles. Figure 2.13 shows a body that is starting from an initial velocity u and accelerating at a constant rate a to velocity v in t seconds. The distance travelled in this time is s. Since the motion is in a straight line, this is also the displacement.

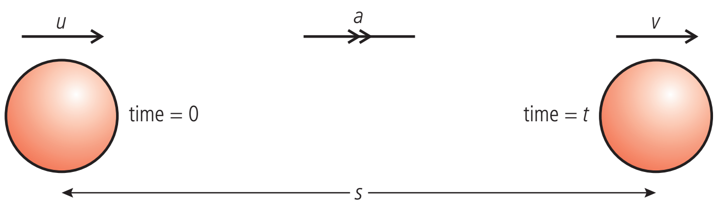


Figure 2.13 A red ball is accelerated at a constant rate.

Using the definitions already stated, we can write equations related to this example.

Average velocity

From the definition, the average velocity $= \dfrac{\text{change of velocity}}{\text{time}}$.

So $$\text{average velocity} = \frac{s}{t} \qquad (1)$$

Since the velocity changes at a constant rate from the beginning to the end, we can also calculate the average velocity by adding the velocities and dividing by two.

$$\text{Average velocity} = \frac{(u + v)}{2} \qquad (2)$$

Acceleration

Acceleration is defined as the rate of change of velocity.

So $$a = \frac{(v - u)}{t} \qquad (3)$$

We can use these equations to solve any problem involving constant acceleration. However, to make problem solving easier, we can derive two more equations by substituting from one into the other.

Equating equations (1) and(2)

$$\frac{s}{t} = \frac{(u + v)}{2}$$

so $$s = \frac{(u + v)\,t}{2} \qquad (4)$$

Rearranging (3) gives $v = u + at$.

If we substitute for v in equation (4) we get $s = ut + \frac{1}{2}at^2$ (5)

Rearranging (3) again gives $t = \frac{(v - u)}{a}$

If t is now substituted in equation (4) we get $v^2 = u^2 + 2as$ (6)

These equations are sometimes known as the *suvat* equations. If you know any 3 of s, u, v, a, and t you can find either of the other two in one step.

***suvat* equations**

$a = \frac{(v - u)}{t}$

$s = \frac{(v + u)t}{2}$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

Worked example

A car travelling at 10 m s^{-1} accelerates at 2 m s^{-2} for 5 s. What is its displacement?

Solution

The first thing to do is draw a simple diagram like Figure 2.14.

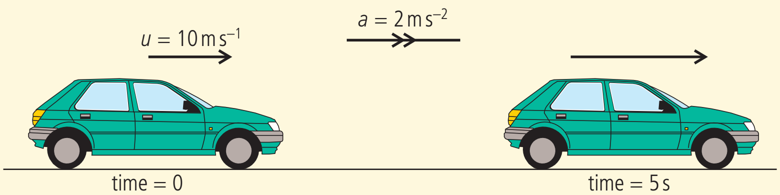


Figure 2.14 A simple diagram is always the best start.

This enables you to see what is happening at a glance rather than reading the text. The next stage is to make a list of *suvat*.

$s = ?$

$u = 10\text{ m s}^{-1}$

$v = ?$

$a = 2\text{ m s}^{-2}$

$t = 5\text{ s}$

To find s you need an equation that contains *suat*. The only equation with all 4 of these quantities is $s = ut + \frac{1}{2}at^2$

Using this equation gives: $s = 10 \times 5 + \frac{1}{2} \times 2 \times 5^2$

$s = 75\text{ m}$

You don't need to include units in all stages of a calculation, just the answer.

The signs of displacement, velocity, and acceleration

We must not forget that displacement, velocity, and acceleration are vectors. This means that they have direction. However, since this is a one-dimensional example, there are only two possible directions, forward and backward. We know which direction the vector is in from its sign.

A positive displacement means that the body has moved to the right.

A positive velocity means the body is moving to the right.

A positive acceleration means that the body is either moving to the right and getting faster or moving to the left and getting slower. This can be confusing, so consider the following example.

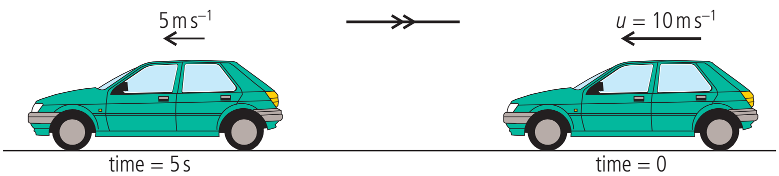


Figure 2.15

The car is travelling in a negative direction so the velocities are negative.

$$u = -10\,\mathrm{m\,s^{-1}}$$
$$v = -5\,\mathrm{m\,s^{-1}}$$
$$t = 5\,\mathrm{s}$$

The acceleration is therefore given by

$$a = \frac{(v-u)}{t} = \frac{-5 - -10}{5} = 1\,\mathrm{m\,s^{-2}}$$

The positive sign tells us that the acceleration is in a positive direction (right) even though the car is travelling in a negative direction (left).

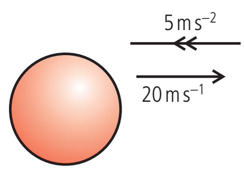


Figure 2.16 The acceleration is negative so pointing to the left.

Example

A body with a constant acceleration of $-5\,\mathrm{m\,s^{-2}}$ is travelling to the right with a velocity of $20\,\mathrm{m\,s^{-1}}$. What will its displacement be after 20 s?

$$s = ?$$
$$u = 20\,\mathrm{m\,s^{-1}}$$
$$v = ?$$
$$a = -5\,\mathrm{m\,s^{-2}}$$
$$t = 20\,\mathrm{s}$$

To calculate s we can use the equation $s = ut + \frac{1}{2}at^2$

$$s = 20 \times 20 + \tfrac{1}{2}(-5) \times 20^2 = 400 - 1000 = -600\,\mathrm{m}$$

This means that the final displacement of the body is to the left of the starting point. It has gone forward, stopped, and then gone backwards.

g

The acceleration due to gravity is not constant all over the Earth. $9.81\,\mathrm{m\,s^{-2}}$ is the average value. The acceleration also gets smaller the higher you go. However we ignore this change when conducting experiments in the lab since labs aren't that high.

To make the examples easier to follow, $g = 10\,\mathrm{m\,s^{-2}}$ is used throughout; you should only use this approximate value in exam questions if told to do so.

Exercises

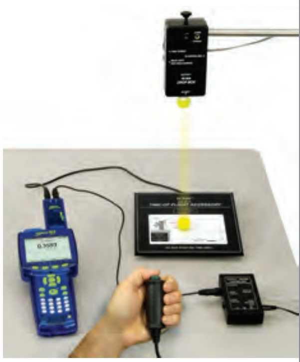
5 Calculate the final velocity of a body that starts from rest and accelerates at $5\,\mathrm{m\,s^{-2}}$ for a distance of 100 m.

6 A body starts with a velocity of $20\,\mathrm{m\,s^{-1}}$ and accelerates for 200 m with an acceleration of $5\,\mathrm{m\,s^{-2}}$. What is the final velocity of the body?

7 A body accelerates at $10\,\mathrm{m\,s^{-2}}$ reaching a final velocity of $20\,\mathrm{m\,s^{-1}}$ in 5 s. What was the initial velocity of the body?

The effect of air resistance

If you jump out of a plane (with a parachute on) you will feel the push of the air as it rushes past you. As you fall faster and faster, the air will push upwards more and more until you can't go any faster. At this point you have reached terminal velocity. We will come back to this after introducing forces.



Apparatus for measuring g.

If a parachutist kept accelerating at a constant rate they would break the sound barrier after a little more than 30 s of flight. By understanding the forces involved it has been possible to design wing suits so that base jumpers can achieve forward velocities greater than their rate of falling.

Free fall motion

Although a car was used in one of the previous illustrations, the acceleration of a car is not usually constant, so we shouldn't use the *suvat* equations. The only example of constant acceleration that we see in everyday life is when a body is dropped. Even then the acceleration is only constant for a short distance.

Acceleration of free fall

When a body is allowed to fall freely we say it is in free fall. Bodies falling freely on the Earth fall with an acceleration of about 9.81 m s^{-2}. (It depends where you are.) The body falls because of gravity. For that reason we use the letter g to denote this acceleration. Since the acceleration is constant, we can use the *suvat* equations to solve problems.

Exercises

In these calculations use g = 10 m s^{-2}.

8 A ball is thrown upwards with a velocity of 30 m s^{-1}. What is the displacement of the ball after 2 s?

9 A ball is dropped. What will its velocity be after falling 65 cm?

10 A ball is thrown upwards with a velocity of 20 m s^{-1}. After how many seconds will the ball return to its starting point?

Measuring the acceleration due to gravity

When a body falls freely under the influence of gravity it accelerates at a constant rate. This means that time to fall, t, and distance, s, are related by the equation $s = ut + \frac{1}{2}at^2$. If the body starts from rest then $u = 0$ so the equation becomes $s = \frac{1}{2}at^2$. Since s is directly proportional to t^2, a graph of s vs t^2 would therefore be a straight line with gradient $\frac{1}{2}g$. It is difficult to measure the time for a ball to pass different markers but if we assume the ball falls with the same acceleration when repeatedly dropped we can measure the time taken for the ball to fall from different heights. There are many ways of doing this. All involve some way of starting a clock when the ball is released, and stopping it when it hits the ground. Table 2.3 shows a set of results from a 'ball drop' experiment.

Table 2.3 Measuring g.

Height(h)/m ±0.001	Time(t)/s ±0.001					Mean t/s	t^2/s^2	Δt^2/s^2
0.118	0.155	0.153	0.156	0.156	0.152	0.154	0.024	0.001
0.168	0.183	0.182	0.183	0.182	0.184	0.183	0.033	0.004
0.218	0.208	0.205	0.210	0.211	0.210	0.209	0.044	0.001
0.268	0.236	0.235	0.237	0.239	0.231	0.236	0.056	0.002
0.318	0.250	0.254	0.255	0.250	0.256	0.253	0.064	0.002
0.368	0.276	0.277	0.276	0.278	0.276	0.277	0.077	0.001
0.418	0.292	0.293	0.294	0.291	0.292	0.292	0.085	0.001
0.468	0.310	0.310	0.303	0.300	0.311	0.307	0.094	0.003
0.518	0.322	0.328	0.330	0.328	0.324	0.326	0.107	0.003
0.568	0.342	0.341	0.343	0.343	0.352	0.344	0.118	0.004

Notice the uncertainty in t^2 is calculated from $\frac{(t_{max}^2 - t_{min}^2)}{2}$.

Notice how the line in Figure 2.17 is very close to the points, and that the uncertainties reflect the actual random variation in the data. The gradient of the line is equal to $\frac{1}{2}g$ so $g = 2 \times$ gradient.

$$g = 2 \times 4.814 = 9.624 \text{ m s}^{-2}$$

The uncertainty in this value can be estimated from the steepest and least steep lines.

$$g_{max} = 2 \times 5.112 = 10.224 \text{ m s}^{-2}$$

$$g_{min} = 2 \times 4.571 = 9.142 \text{ m s}^{-2}.$$

$$\Delta g = \frac{(g_{max} - g_{min})}{2} = \frac{(10.224 - 9.142)}{2} = 0.541 \text{ m s}^{-2}$$

So the final value including uncertainty is $9.6 \pm 0.5 \text{ m s}^{-2}$.

This is in good agreement with the accepted average value which is 9.81 m s^{-2}.

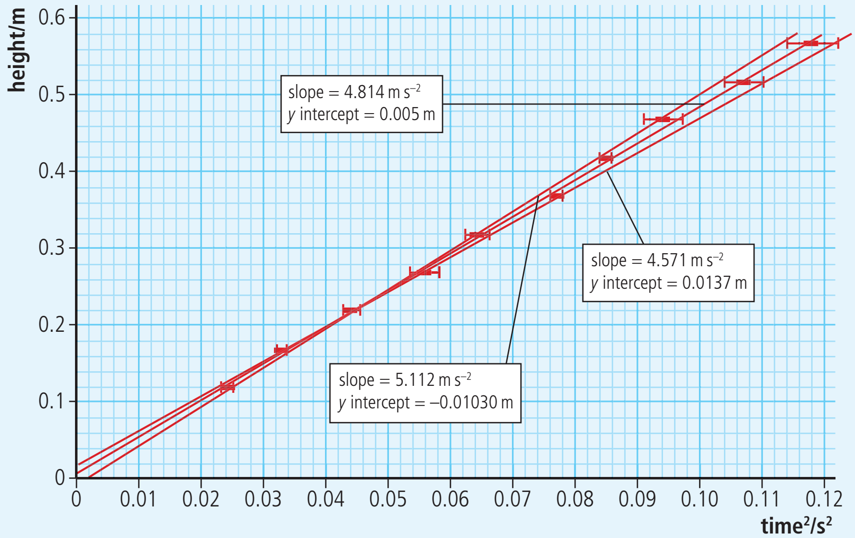


Figure 2.17 Height *vs* time^2 for a falling object.

Measuring the acceleration due to gravity by timing a freefalling object

A worksheet with full details of how to carry out this experiment is available on your eBook.

Graphical representation of motion

Graphs are used in physics to give a visual representation of relationships. In kinematics they can be used to show how displacement, velocity, and acceleration change with time. Figure 2.18 shows the graphs for four different examples of motion. They are placed vertically since they all have the same time axis.

The best way to go about sketching graphs is to split the motion into sections then plot where the body is at different times; joining these points will give the displacement–time graph. Once you have done that you can work out the v–t and a–t graphs by looking at the s–t graph rather than the motion.

You need to be able to

- figure out what kind of motion a body has by looking at the graphs
- sketch graphs for a a given motion.

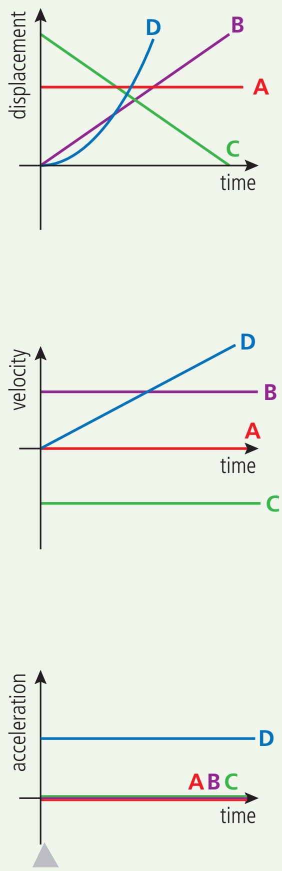
Gradient of displacement–time

The gradient of a graph is $\frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$

In the case of the displacement–time graph this will give

$$\text{gradient} = \frac{\Delta s}{\Delta t}$$

This is the same as velocity.



Line A

A body that is not moving.
Displacement is always the same.
Velocity is zero.
Acceleration is zero.

Line B

A body that is travelling with a constant positive velocity.
Displacement increases linearly with time.
Velocity is a constant positive value.
Acceleration is zero.

Line C

A body that has a constant negative velocity.
Displacement is decreasing linearly with time.
Velocity is a constant negative value.
Acceleration is zero.

Line D

A body that is accelerating with constant acceleration.
Displacement is increasing at a non-linear rate. The shape of this line is a parabola since displacement is proportional to t^2 ($s = ut + \frac{1}{2}at^2$).
Velocity is increasing linearly with time.
Acceleration is a constant positive value.

Figure 2.18 Graphical representation of motion.

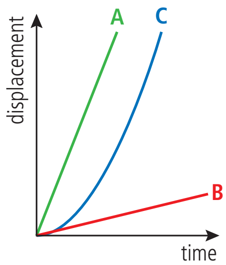


Figure 2.19.

So the gradient of the displacement–time graph equals the velocity. Using this information, we can see that line A in Figure 2.19 represents a body with greater velocity than line B and that since the gradient of line C is increasing, this must be the graph for an accelerating body.

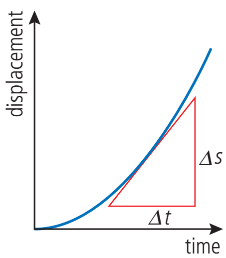


Figure 2.20.

Instantaneous velocity

When a body accelerates its velocity is constantly changing. The displacement–time graph for this motion is therefore a curve. To find the instantaneous velocity from the graph we can draw a tangent to the curve and find the gradient of the tangent as shown in Figure 2.20.

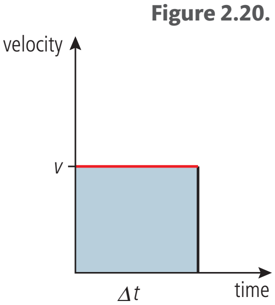


Figure 2.21.

Area under velocity–time graph

The area under the velocity–time graph for the body travelling at constant velocity v shown in Figure 2.21 is given by

$$\text{area} = v\Delta t$$

But we know from the definition of velocity that $v = \dfrac{\Delta s}{\Delta t}$

Rearranging gives $\Delta s = v\Delta t$ so the area under a velocity–time graph gives the displacement.

This is true not only for simple cases such as this but for all examples.

Gradient of velocity–time graph

The gradient of the velocity–time graph is given by $\frac{\Delta v}{\Delta t}$. This is the same as acceleration.

Area under acceleration–time graph

The area under an acceleration–time graph in Figure 2.22 is given by $a\Delta t$. But we know from the definition of acceleration that $a = \frac{(v-u)}{t}$

Rearranging this gives $v - u = a\Delta t$ so the area under the graph gives the change in velocity.

If you have covered calculus in your maths course you may recognize these equations:

$$v = \frac{ds}{dt},\ a = \frac{dv}{dt} = \frac{d^2s}{d^2t},\ \text{and}\ s = \int v dt,\ v = \int a dt$$

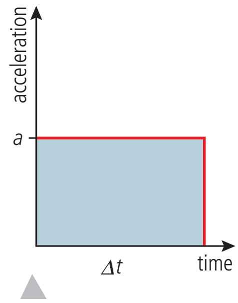


Figure 2.22.

Exercises

11 Sketch a velocity–time graph for a body starting from rest and accelerating at a constant rate to a final velocity of 25 m s^{-1} in 10 seconds. Use the graph to find the distance travelled and the acceleration of the body.

12 Describe the motion of the body whose velocity–time graph is shown in Figure 2.23. What is the final displacement of the body?

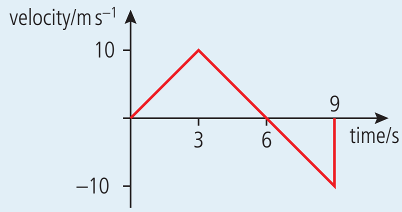


Figure 2.23

13 A ball is released from rest on the hill in Figure 2.24. Sketch the *s*–*t*, *v*–*t*, and *a*–*t* graphs for its horizontal motion.

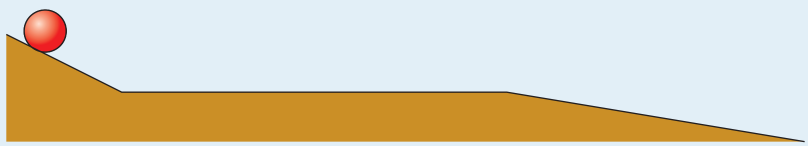


Figure 2.24.

14 A ball rolls along a table then falls off the edge landing on soft sand. Sketch the *s*–*t*, *v*–*t*, and *a*–*t* graphs for its vertical motion.

Example 1: the *suvat* example

As an example let us consider the motion we looked at when deriving the *suvat* equations.

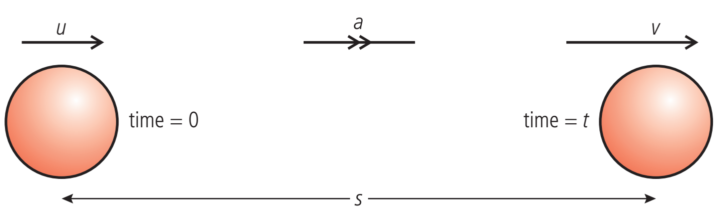


Figure 2.25 A body with constant acceleration.

Negative time

Negative time doesn't mean going back in time – it means the time before you started the clock.

Displacement–time

The body starts with velocity *u* and travels to the right with constant acceleration, *a* for a time *t*. If we take the starting point to be zero displacement, then the displacement–time graph starts from zero and rises to *s* in *t* seconds. We can therefore plot the two

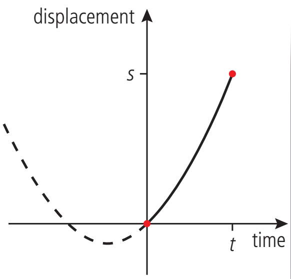


Figure 2.26.

points shown in Figure 2.26. The body is accelerating so the line joining these points is a parabola. The whole parabola has been drawn to show what it would look like – the reason it is offset is because the body is not starting from rest. The part of the curve to the left of the origin tells us what the particle was doing before we started the clock.

Velocity–time

Figure 2.27 is a straight line with a positive gradient showing that the acceleration is constant. The line doesn't start from the origin since the initial velocity is u.

The gradient of this line is $\frac{(v-u)}{t}$ which we know from the *suvat* equations is acceleration.

The area under the line makes the shape of a trapezium. The area of this trapezium is $\frac{1}{2}(v+u)t$. This is the *suvat* equation for s.

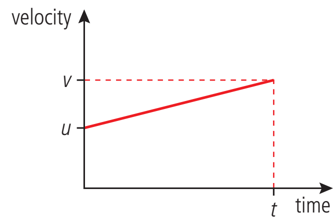


Figure 2.27.

Acceleration–time

The acceleration is constant so the acceleration–time graph is simply a horizontal line as shown in Figure 2.28. The area under this line is $a \times t$ which we know from the *suvat* equations equals $(v - u)$.

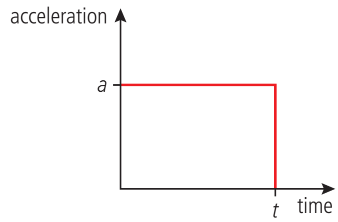
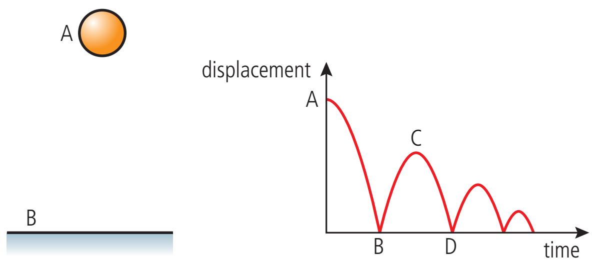


Figure 2.28.

Example 2: The bouncing ball

Consider a rubber ball dropped from some position above the ground A onto a hard surface B. The ball bounces up and down several times. Figure 2.29 shows the displacement–time graph for 4 bounces. From the graph we see that the ball starts above the ground then falls with increasing velocity (as deduced by the increasing negative gradient). When the ball bounces at B the velocity suddenly changes from negative to positive as the ball begins to travel back up. As the ball goes up, its velocity gets less until it stops at C and begins to fall again.

Figure 2.29.



Exercise

15 By considering the gradient of the displacement–time graph in Figure 2.29, plot the velocity–time graph for the motion of the bouncing ball.

Example 3: A ball falling with air resistance

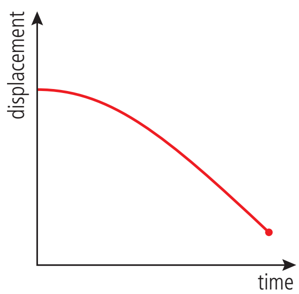


Figure 2.30.

Figure 2.30 represents the motion of a ball that is dropped several hundred metres through the air. It starts from rest and accelerates for some time. As the ball accelerates, the air resistance gets bigger, which prevents the ball from getting any faster. At this point the ball continues with constant velocity.

Exercise

16 By considering the gradient of the displacement–time graph plot the velocity–time graph for the motion of the falling ball in Figure 2.30.

Projectile motion

A stroboscopic photograph of a projected ball.

We all know what happens when a ball is thrown; it follows a curved path like the one in the photo. We can see from this photo that the path is parabolic, and later we will show why that is the case.

Modelling projectile motion

All examples of motion up to this point have been in one dimension but projectile motion is two-dimensional. However, if we take components of all the vectors vertically and horizontally, we can simplify this into two simultaneous one-dimensional problems. The important thing to realize is that the vertical and horizontal components are independent of each other; you can test this by dropping a stone off a cliff and throwing one forward at the same time: they both hit the bottom together. The downward motion is not altered by the fact that one stone is also moving forward.

Consider a ball that is projected at an angle θ to the horizontal, as shown in Figure 2.31. We can split the motion into three parts, beginning, middle, and end, and analyse the vectors representing displacement, velocity, and time at each stage. Note that the path is symmetrical so the motion on the way down is the same as on the way up.

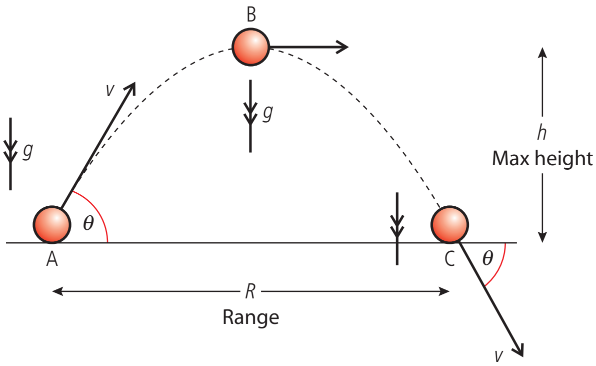


Figure 2.31 A projectile launched at an angle θ.

Horizontal components

Table 2.4 Horizontal components.

At A (time = 0)	At B (time = $\frac{t}{2}$)	At C (time = t)
Displacement = zero	Displacement = $\frac{R}{2}$	Displacement = R
Velocity = $v\cos\theta$	Velocity = $v\cos\theta$	Velocity = $v\cos\theta$
Acceleration = 0	Acceleration = 0	Acceleration = 0

Vertical components

Table 2.5 Vertical components.

At A	At B	At C
Displacement = zero Velocity = $v\sin\theta$ Acceleration = $-g$	Displacement = h Velocity = zero Acceleration = $-g$	Displacement = zero Velocity = $-v\sin\theta$ Acceleration = $-g$

We can see that the vertical motion is constant acceleration and the horizontal motion is constant velocity. We can therefore use the *suvat* equations.

suvat for horizontal motion

Since acceleration is zero there is only one equation needed to define the motion.

Table 2.6 *suvat* for horizontal motion.

suvat	A to C
Velocity = $v = \frac{s}{t}$	$R = v\cos\theta t$

suvat for vertical motion

When considering the vertical motion it is worth splitting the motion into two parts.

Table 2.7 *suvat* for vertical motion.

suvat	At B	At C
$s = \frac{1}{2}(u + v)t$	$h = \frac{1}{2}(v\sin\theta)\frac{t}{2}$	$0 = \frac{1}{2}(v\sin\theta - v\sin\theta)t$
$v^2 = u^2 + 2as$	$0 = v^2\sin^2\theta - 2gh$	$(-v\sin\theta)^2 = (v\sin\theta)^2 - 0$
$s = ut + \frac{1}{2}at^2$	$h = v\sin\theta t - \frac{1}{2}g(\frac{t}{2})^2$	$0 = v\sin\theta t - \frac{1}{2}gt^2$
$a = \frac{v-u}{t}$	$g = \frac{v\sin\theta - 0}{\frac{t}{2}}$	$g = \frac{v\sin\theta - -v\sin\theta}{t}$

Some of these equations are not very useful since they simply state that 0 = 0. However we do end up with three useful ones (highlighted):

$$R = v\cos\theta t \qquad (1)$$

$$0 = v^2\sin^2\theta - 2gh \quad \text{or} \quad h = \frac{v^2\sin^2\theta}{2g} \qquad (2)$$

$$0 = v\sin\theta t - \frac{1}{2}gt^2 \quad \text{or} \quad t = \frac{2v\sin\theta}{g} \qquad (3)$$

Solving problems

In a typical problem you will be given the magnitude and direction of the initial velocity and asked to find either the maximum height or range. To calculate h you can use equation (2) but to calculate R you need to find the time of flight so must use

Parabolic path

Since the horizontal displacement is proportional to t the path has the same shape as a graph of vertical displacement plotted against time. This is parabolic since the vertical displacement is proportional to t^2.

(3) first (you could also substitute for t into equation (1) to give a fourth equation but maybe we have enough equations already).

You do not have to remember a lot of equations to solve a projectile problem. If you understand how to apply the *suvat* equations to the two components of the projectile motion, you only have to remember the *suvat* equations (and they are in the databook).

Worked example

A ball is thrown at an angle of 30° to the horizontal at a speed of $20\,\text{m s}^{-1}$. Calculate its range and the maximum height reached.

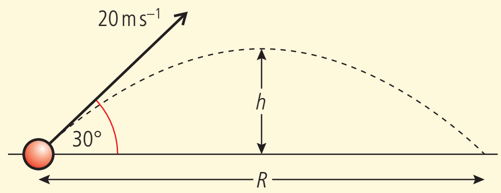


Figure 2.32.

Solution

First, as always, draw a diagram, including labels defining all the quantities known and unknown.

Now we need to find the time of flight. If we apply $s = ut + \frac{1}{2}at^2$ to the whole flight we get

$$t = \frac{2v\sin\theta}{g} = \frac{(2 \times 20 \times \sin 30°)}{10} = 2\,\text{s}$$

We can now apply $s = vt$ to the whole flight to find the range:

$$R = v\cos\theta t = 20 \times \cos 30° \times 2 = 34.6\,\text{m}$$

Finally to find the height, we use $s = ut + \frac{1}{2}at^2$ to the vertical motion, but remember, this is only half the complete flight so the time is 1 s.

$$h = v\sin\theta t - \tfrac{1}{2}gt^2 = 20 \times \sin 30° \times 1 - \tfrac{1}{2} \times 10 \times 1^2 = 10 - 5 = 5\,\text{m}$$

Worked example

A ball is thrown horizontally from a cliff top with a horizontal speed of $10\,\text{m s}^{-1}$. If the cliff is 20 m high what is the range of the ball?

Solution

This is an easy one since there aren't any angles to deal with. The initial vertical component of the velocity is zero and the horizontal component is $10\,\text{m s}^{-1}$. To calculate the time of flight we apply $s = ut + \frac{1}{2}at^2$ to the vertical component. Knowing that the final displacement is −20 m this gives

$$-20\,\text{m} = 0 - \tfrac{1}{2}gt^2 \text{ so } t = \sqrt{\frac{(2 \times 20)}{10}} = 2\,\text{s}$$

We can now use this value to find the range by applying the formula $s = vt$ to the horizontal component: $R = 10 \times 2 = 20\,\text{m}$

Maximum range

For a given value of v the maximum range is when $v\cos\theta t$ is a maximum value.

Now $t = \frac{2v\sin\theta}{g}$.

If we substitute this for t we get

$R = \frac{2v^2\cos\theta\sin\theta}{g}$

This is a maximum when $\cos\theta\sin\theta$ is maximum, which is when $\theta = 45°$.

When a bullet is fired at a distant target it will travel in a curved path due to the action of gravity and air resistance. Precision marksmen adjust their sights to compensate for this. The angle of this adjustment could be based on calculation or experiment (trial and error).

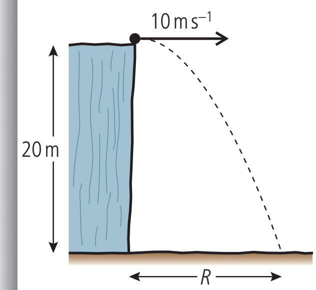


Figure 2.33.

If you have ever played golf you will know it is not true that the maximum range is achieved with an angle of 45°; it is actually much less. This is because the ball is held up by the air like an airplane is. In this photo Alan Shepard is playing golf on the Moon. Here the maximum range will be at 45°.

CHALLENGE YOURSELF

1 A projectile is launched perpendicular to a 30° slope at 20 m s^{-1}. Calculate the distance between the launching position and landing position.

Exercises

17 Calculate the range of a projectile thrown at an angle of 60° to the horizontal with velocity 30 m s^{-1}.

18 You throw a ball at a speed of 20 m s^{-1}.

(a) At what angle must you throw it so that it will just get over a wall that is 5 m high?
(b) How far away from the wall must you be standing?

19 A gun is aimed so that it points directly at the centre of a target 200 m away. If the bullet travels at 200 m s^{-1} how far below the centre of the target will the bullet hit?

20 If you can throw a ball at 20 m s^{-1} what is the maximum distance you can throw it?

Projectile motion with air resistance

In all the examples above we have ignored the fact that the air will resist the motion of the ball. The actual path of a ball including air resistance is likely to be as shown in Figure 2.34.

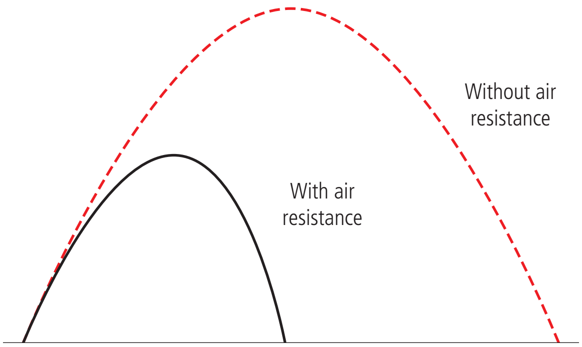


Figure 2.34.

Notice both the height and range are less. It is also no longer a parabola – the way down is steeper than the way up.

To learn more about motion, go to the hotlinks site, search for the title or ISBN and click on Chapter 2.

2.2 Forces

2.2 Forces

Understandings, applications, and skills:

Objects as point particles
Free body diagrams
- Representing forces as vectors.
- Sketching and interpreting free body diagrams.

Guidance
- *Students should label forces using commonly accepted names or symbols. e.g. Weight.*
- *Free body diagrams should show scaled vector lengths acting from the point of application.*

Translational equilibrium
Newton's laws of motion
- Describing the consequences of Newton's first law for translational equilibrium.
- Using Newton's second law quantitatively and qualitatively. Identifying force pairs in the context of Newton's third law. Solving problems involving forces and determining resultant force.

Guidance
- *Examples and questions will be limited to constant mass.*
- mg *should be identified as weight.*
- *Calculations relating to the determination of resultant forces will be restricted to one and two dimensional situations.*

Solid friction
- Describing solid friction (static and dynamic) by coefficients of friction.

NATURE OF SCIENCE

Newton's three laws of motion are a set of statements, based on observation and experiment, that can be used to predict the motion of a point object from the forces acting on it. Einstein showed that the laws do not apply when speeds approach the speed of light. However, we still use them to predict outcomes at the lower velocities achieved by objects travelling in the lab.

Force (F)

We can now model the motion of a constantly accelerating body but what makes it accelerate? From experience we know that to make something move we must push or pull it. We call this *applying a force*. One simple way of applying a force to a body is to attach a string and pull it. Imagine a sphere floating in space with two strings attached; the sphere will not start to move unless one of the astronauts pulls the string as in Figure 2.35.

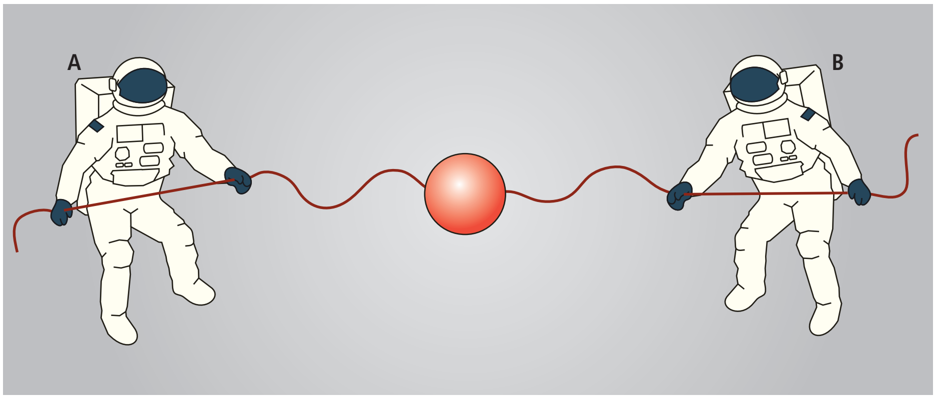


Figure 2.35 Two astronauts and a red ball.

A force is a push or a pull.

The unit of force is the newton.

The size of one newton

If you hold an object of mass 100 g in your hand then you will be exerting an upward force of about one newton (1 N).

If A pulls the string then the body will move to the left, and if B pulls it will move to the right. We can see that force is a *vector* quantity since it has *direction*.

Addition of forces

Since force is a vector we must add forces vectorially, so if A applies a force of 50 N and B applies a force of 60 N the resultant force will be 10 N towards B, as can be seen in Figure 2.36.

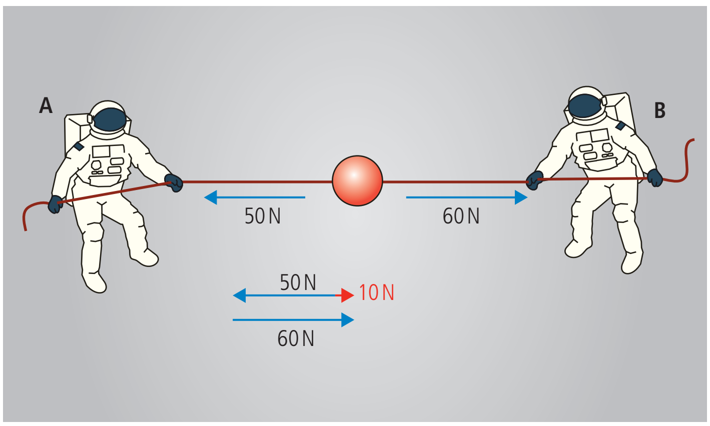


Figure 2.36 Astronaut A pulls harder than B.

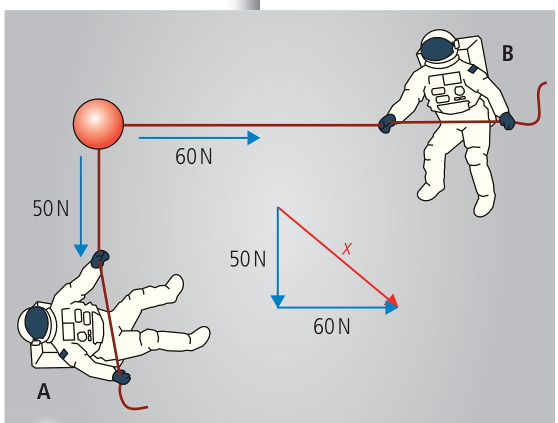


Figure 2.37 Astronauts pulling at right angles.

Or, in two dimensions, we can use trigonometry as in Figure 2.37.

In this case we can use Pythagoras to find x

$$x = \sqrt{50^2 + 60^2} = 78\,\text{N}$$

Astronauts in space are considered here so that no other forces are present. This makes things simpler.

Taking components

As with other vector quantities we can calculate components of forces, for example we might want to know the resultant force in a particular direction.

In Figure 2.38 the component of the force in the x direction is $F_x = 60 \times \cos 30° = 52\,\text{N}$.

This is particularly useful when we have several forces.

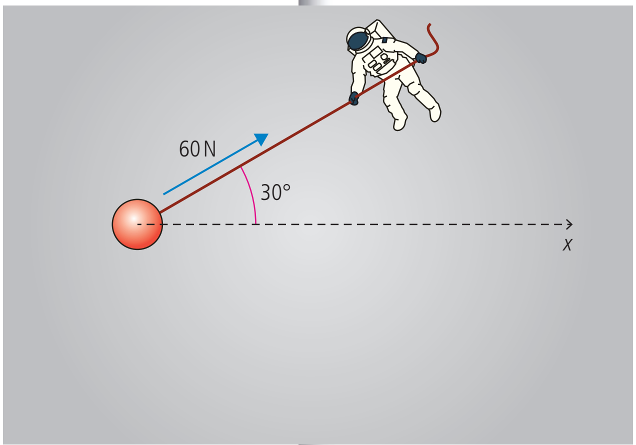


Figure 2.38 Pulling at an angle.

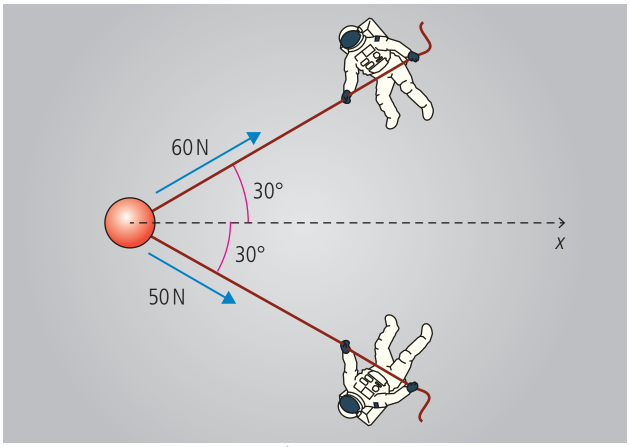


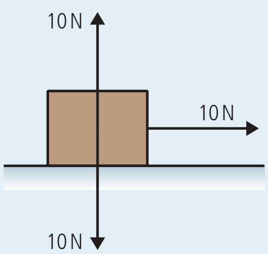
Figure 2.39 Astronauts not pulling in line.

In the example shown in Figure 2.39 we can use components to calculate the resultant force in the x direction $= 60 \times \cos 30° + 50 \times \cos 30° = 52 + 43 = 95\,\text{N}$.

Exercise

21 Find the resultant force in the following examples:

(a)



(b)

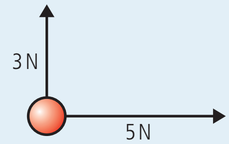


Figure 2.40.

Equilibrium

If the resultant force on a body is zero, as in Figure 2.41, then we say the forces are *balanced* or the body is in *equilibrium*.

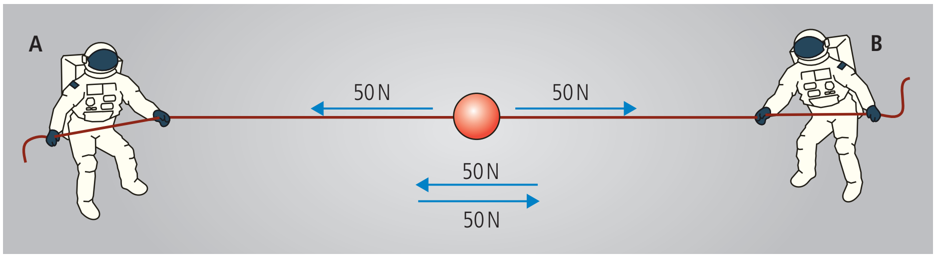


Figure 2.41 Balanced forces.

Or with three forces as in Figure 2.42.

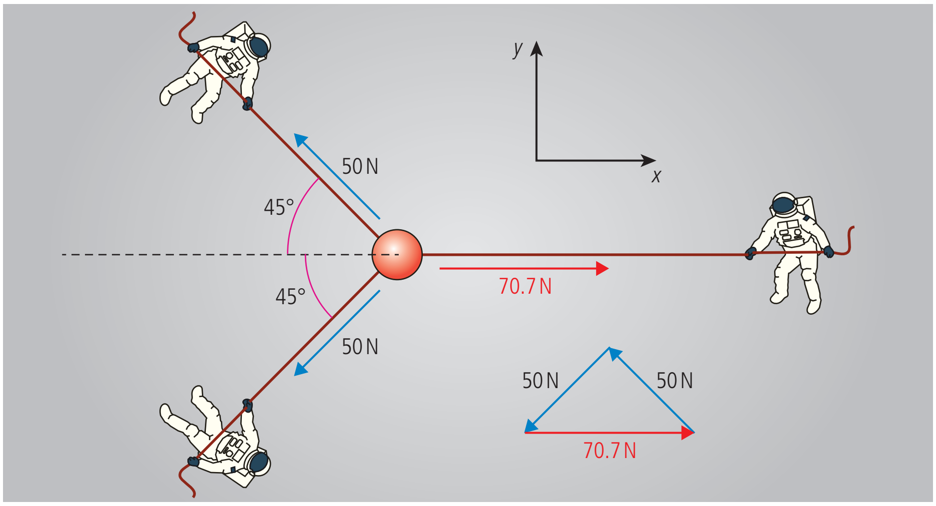


Figure 2.42 Three balanced forces.

In this example the two blue forces are perpendicular making the trigonometry easy. Adding all three forces gives a right-angled triangle. We can also see that if we take components in any direction then the forces must be balanced.

Taking components in the *x* direction:

$-50 \times \cos 45° - 50 \times \cos 45° + 70.7 = -35.35 - 35.35 + 70.7 = 0$

Taking components in the *y* direction:

$50 \times \sin 45° - 50 \times \sin 45° = 0$

Free body diagrams

Problems often involve more than one body; for example, the previous problem involved four bodies, three astronauts, and one red ball. All of these bodies will experience forces but if we drew them all on the diagram it would be very confusing, For that reason we only draw forces on the body we are interested in, in this case the red ball. This is called a free body diagram as shown in Figure 2.43. Note that we treat the red ball as a point object by drawing the forces acting on the centre. Not all forces actually act on the centre but when adding forces it can be convenient to draw them as if they do.

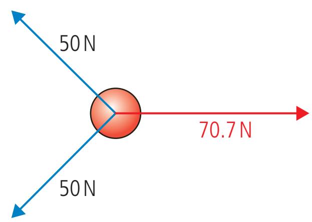
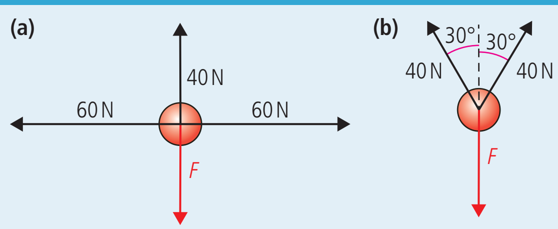


Figure 2.43 A free body diagram of the forces in Figure. 2.42.

Exercises

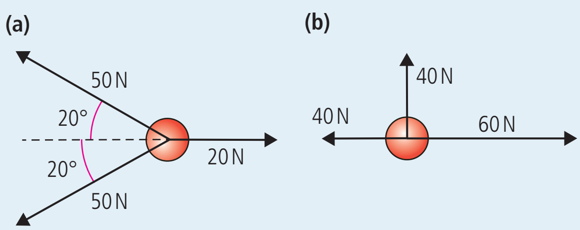
22 In the following examples calculate the force *F* required to balance the forces.

Figure 2.44.



23 Calculate the resultant force for the following.

Figure 2.45.



24 By resolving the vectors into components, calculate if the following bodies are in translational equilibrium or not. If not, calculate the resultant force.

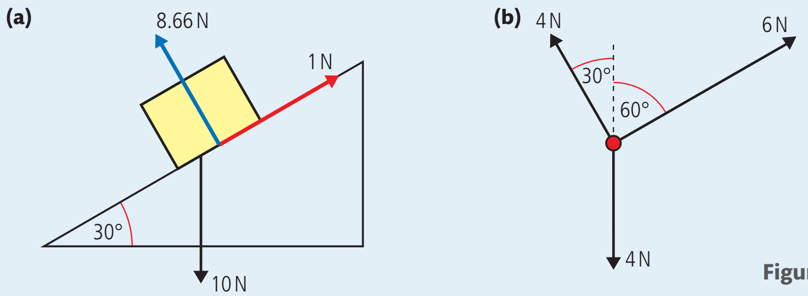


Figure 2.46.

25 If the following two examples are in equilibrium, calculate the unknown forces F_1, F_2, and F_3.

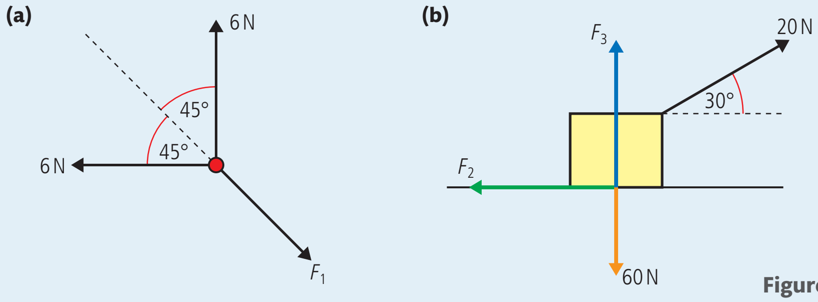


Figure 2.47.

Using laws in physics

TOK

A law in physics is a very useful tool. If applied properly, it enables us to make a very strong argument that what we say is true. If asked 'will a box move?' you can say that you think it will and someone else could say it won't. You both have your opinions and you would then argue as to who is right. However, if you say that Newton's law says it will move, then you have a much stronger argument (assuming you have applied the law correctly).

Newton's first law of motion

From observation we can conclude that to make a body move we need to apply an unbalanced force to it. What isn't so obvious is that once moving it will continue to move with a constant velocity unless acted upon by another unbalanced force. Newton's first law of motion is a formal statement of this:

A body will remain at rest or moving with constant velocity unless acted upon by an unbalanced force.

The reason that this is not obvious to us on Earth is that we don't tend to observe bodies travelling with constant velocity with no forces acting on them; in space it would be more obvious. Newton's first law can be used in two ways. If the forces on a body are balanced then we can use Newton's first law to predict that it will be at rest or moving with constant velocity. If the forces are unbalanced then the body will not be at rest or moving with constant velocity. This means its velocity changes – in other words, it accelerates. Using the law the other way round, if a body accelerates then Newton's first law predicts that the forces acting on the body are unbalanced. To apply this law in real situations we need to know a bit more about the different types of force.

Types of force

Tension

Tension is the name of the force exerted by the astronauts on the red ball. If you attach a string to a body and pull it then you are exerting tension, as in Figure 2.48.

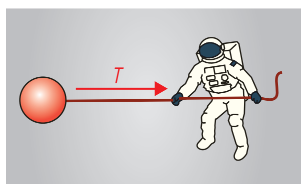


Figure 2.48 Exerting tension with a string.

Normal reaction

Whenever two surfaces are in contact with (touching) each other there will be a force between them. This force is perpendicular to the surface so it is called the *normal reaction force*. If the astronaut pushes the ball with his hand as in Figure 2.49, then there will be a normal reaction between the hand and the ball.

Note that the force acts on both surfaces so the astronaut will also experience a normal force. However, since we are interested in the ball, not the astronaut, we take the ball as our 'free body' so only draw the forces acting on it.

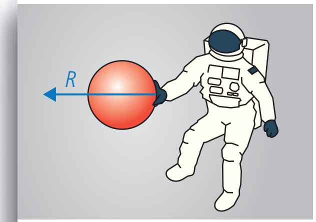


Figure 2.49 A normal reaction force is exerted when a hand is in contact with a ball.

Gravitational force (weight)

Back on Earth, if a body is released above the ground as in Figure 2.50, it accelerates downwards. According to Newton's first law there must be an unbalanced force causing this motion; this force is called the *weight*. The weight of a body is directly proportional to its mass: $W = mg$ where $g = 9.81\,\mathrm{N\,kg^{-1}}$. Note that this is the same as the acceleration of free fall. You will find out why later on.

Note that the weight acts at the *centre* of the body.

If a block is at rest on the floor then Newton's first law implies that the forces are balanced. The forces involved are weight (because the block has mass and is on the Earth) and normal force (because the block is in contact with the ground). Figure 2.51 shows the forces.

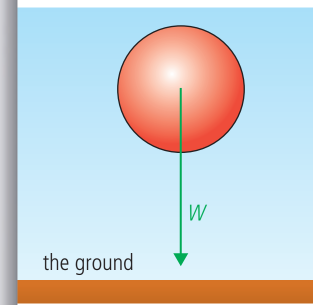


Figure 2.50 A ball in free fall.

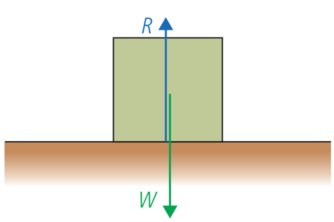


Figure 2.51 A free body diagram of a box resting on the ground.

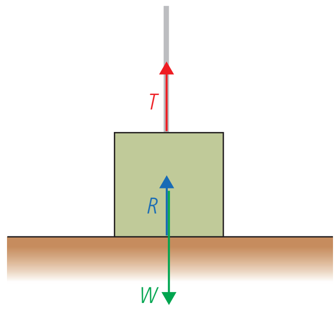


Figure 2.52 A string applies an upward force on the box.

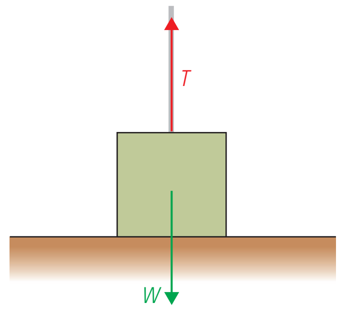


Figure 2.53 The block is lifted as the tension is bigger than its weight.

These forces are balanced so $-W + R = 0$ or $W = R$.

If the mass of the block is increased then the normal reaction will also increase.

If a string is added to the block then we can exert tension on the block as in Figure 2.52.

The forces are still balanced since $T + R = W$. Notice how W has remained the same but R has got smaller. If we pull with more force we can lift the block as in Figure 2.53. At this point the normal reaction R will be zero. The block is no longer in contact with the ground; now $T = W$.

The block in Figure 2.54 is on an inclined plane (slope) so the weight still acts downwards. In this case it might be convenient to split the weight into components, one acting down the slope and one into the slope.

The component of weight perpendicular to the slope is $W \cos \theta$. Since there is no movement in this direction the force is balanced by R. The component of weight

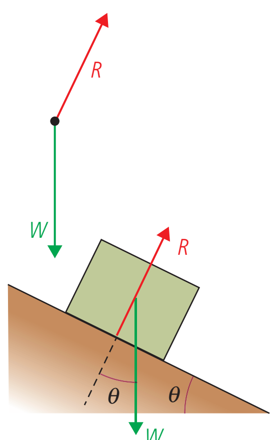


Figure 2.54 Free body diagram for a block on a slope.

parallel to the slope is $W \sin \theta$. This force is unbalanced, causing the block to accelerate down the slope. If the angle of the slope is increased then $\sin \theta$ will also increase, resulting in a greater force down the slope.

Friction

There are two types of friction: *static friction*, which is the force that stops the relative motion between two touching surfaces and *dynamic friction*, which opposes the relative motion between two touching surfaces. In both cases the force is related to both the normal force and the nature of the surfaces, so pushing two surfaces together increases the friction between them.

$F = \mu R$ where μ is the coefficient of friction (static or dynamic).

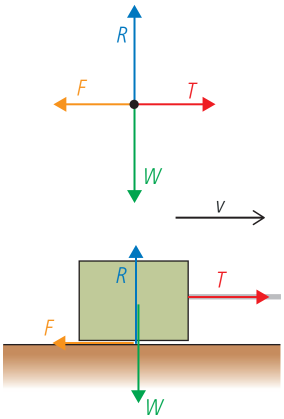


Figure 2.55 The force experienced by a block pulled along a table.

Dynamic friction

In Figure 2.55 a block is being pulled along a table at a constant velocity.

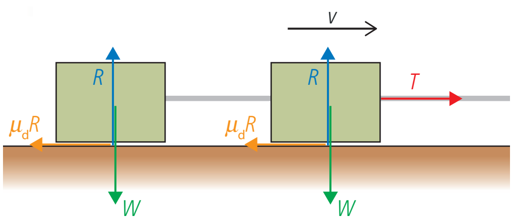


Figure 2.56 Two blocks joined with a rope.

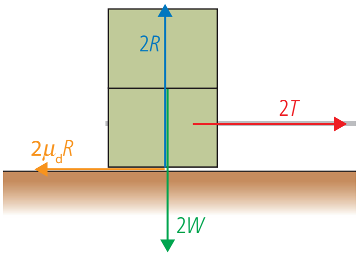


Figure 2.57 Two blocks on top of each other.

Since the velocity is constant, Newton's first law implies that the forces are balanced so $T = F$ and $W = R$. Notice that friction doesn't depend on the area of contact. We can show this by considering two identical blocks sliding at constant velocity across a table top joined together by a rope as in Figure 2.56. The friction under each cube is $\mu_d R$ so the total friction would be $2\mu_d R$.

If the one cube is now placed on top of the other as in Figure 2.57, the normal force under the bottom cube will be twice as much so the friction is now $2\mu_d R$. It doesn't matter if the blocks are side by side (large area of contact) or on top of each other (small area of contact); the friction is the same.

If this is the case, then why do racing cars have wide tyres with no tread pattern (slicks)? There are several reasons for this but one is to increase the friction between the tyres and the road. This is strange because friction is not supposed to depend on area of contact. In practice friction isn't so simple. When one of the surfaces is sticky like the tyres of a racing car the force *does* depend upon the surface area. The type of surfaces we are concerned with here are quite smooth, non-sticky surfaces like wood and metal.

Static friction

If a very small force is applied to a block at rest on the ground it won't move. This means that the forces on the block are *balanced* (Newton's first law): the applied force is balanced by the static friction.

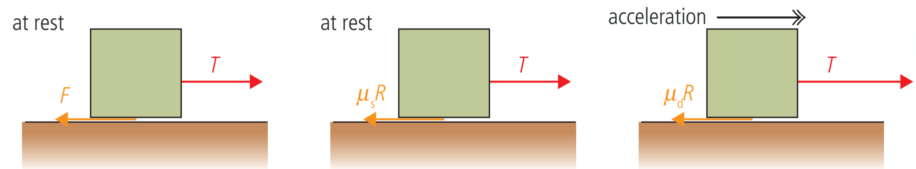


Figure 2.58 μR is the maximum size of friction.

In this case the friction simply equals the applied force: $F = T$. As the applied force is increased the friction will also increase. However, there will be a point when the friction cannot be any bigger. If the applied force is increased past that point the block will start to move; the forces have become *unbalanced* as illustrated in Figure 2.58. The maximum value that friction can have is $\mu_s R$ where μ_s is the coefficient of static friction. The value of static friction is always *greater* than dynamic friction. This can easily be demonstrated with a block on an inclined plane as shown in Figure 2.59.

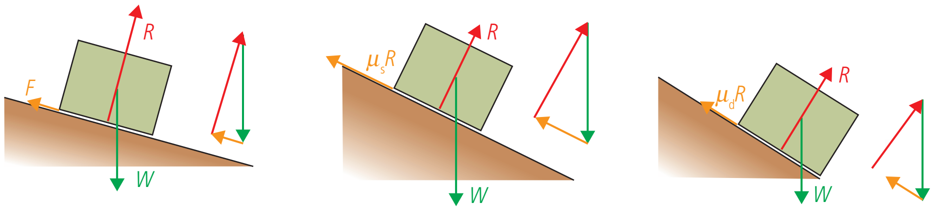


Figure 2.59 A block rests on a slope until the forces become unbalanced.

In the first example the friction is balancing the component of weight down the plane which equals $W \sin \theta$, where θ is the angle of the slope. As the angle of the slope is increased, the point is reached where the static friction = $\mu_s R$. The forces are still balanced but the friction cannot get any bigger so if the angle is increased further the forces become unbalanced and the block will start to move. Once the block moves the friction becomes dynamic friction. Dynamic friction is less than static friction, so this results in a bigger resultant force down the slope, causing the block to accelerate.

Friction doesn't just slow things down, it is also the force that makes things move. Consider the tyre of a car as it starts to drive away from the traffic lights. The rubber of the tyre is trying to move relative to the road. In fact, if there wasn't any friction the wheel would spin as the tyre slipped backwards on the road. The force of friction that opposes the motion of the tyre slipping backwards on the road is therefore in the *forwards* direction.

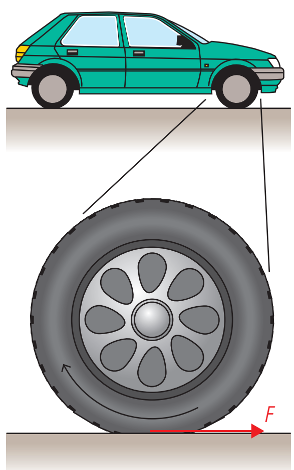


Figure 2.60 Friction pushes the car forwards.

If the static friction between the tyre and the road is not big enough the tyre will slip. Once this happens the friction becomes dynamic friction which is less than static friction, so once tyres start to slip they tend to continue slipping.

Buoyancy

Buoyancy is the name of the force experienced by a body totally or partially immersed in a fluid (a fluid is a liquid or gas). The size of this force is equal to the weight of fluid displaced. It is this force that enables a boat to float and a helium balloon to rise in the air. Let us consider a football and a bucket full of water.

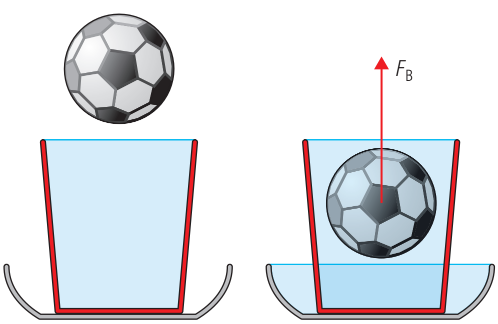


Figure 2.61 A football immersed in a bucket of water.

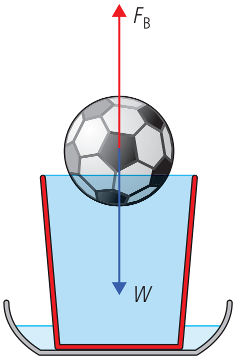


Figure 2.62 A football floats in a bucket of water.

If you take the football and push it under the water then water will flow out of the bucket, (luckily a big bowl was placed there to catch it). The weight of this displaced water is equal to the upward force on the ball. To keep the ball under water you would therefore have to balance that force by pushing the ball down.

The forces on a floating object are balanced so the weight must equal the buoyant force. This means that the ball must have displaced its own weight of water as in Figure 2.62.

Air resistance

Air resistance, or *drag force*, is the force that opposes the motion of a body through the air. The size of this force depends on the speed, size and shape of the body. At slow speeds the drag force experienced by a sphere is given by Stoke's law:

$$F = 6\pi\eta vr$$

where η = viscosity (a constant)
v = velocity
r = radius

When a balloon is dropped it accelerates downwards due to the force of gravity. As it falls through the air it experiences a drag force opposing its motion. As the balloon's velocity gets bigger so does the drag force, until the drag force balances its weight at which point its velocity will remain constant (Figure 2.63). This maximum velocity is called its *terminal velocity*.

The same thing happens when a parachutist jumps out of a plane. The terminal velocity in this case is around 54 m s^{-1} (195 km h^{-1}). Opening the parachute increases the drag force which slows the parachutist down to a safer 10 m s^{-1} for landing.

As it is mainly the air resistance that limits the top speed of a car, a lot of time and money is spent by car designers to try to reduce this force. This is particularly important at high speeds when the drag force is related to the square of the speed.

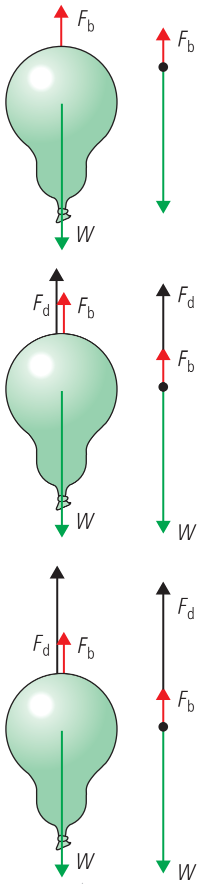


Figure 2.63 A balloon reaches terminal velocity as the forces become balanced. Notice the buoyant force is also present.

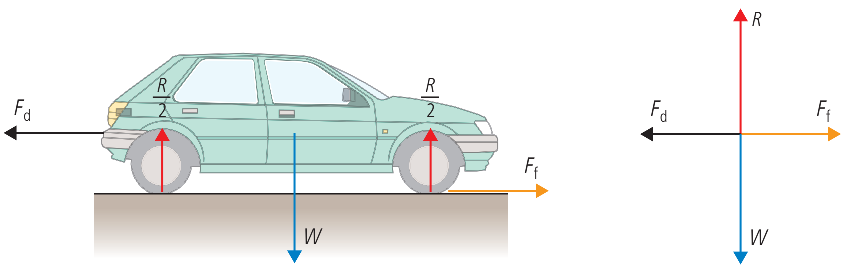


Figure 2.64 The forces acting on a car travelling at constant velocity.

Speed skiers wear special clothes and squat down like this to reduce air resistance.

Exercises

26 A ball of weight 10 N is suspended on a string and pulled to one side by another horizontal string as shown in Figure 2.65. If the forces are balanced:

(a) write an equation for the horizontal components of the forces acting on the ball
(b) write an equation for the vertical components of the forces acting on the ball
(c) use the second equation to calculate the tension in the upper string, *T*
(d) use your answer to (c) plus the first equation to find the horizontal force *F*.

27 The condition for the forces to be balanced is that the sum of components of the forces in any two perpendicular components is zero. In the 'box on a ramp' example the vertical and horizontal components were taken. However, it is sometimes more convenient to consider components parallel and perpendicular to the ramp.

Consider the situation in Figure 2.66. If the forces on this box are balanced:

(a) write an equation for the components of the forces parallel to the ramp
(b) write an equation for the forces perpendicular to the ramp
(c) use your answers to find the friction (*F*) and normal force (*N*).

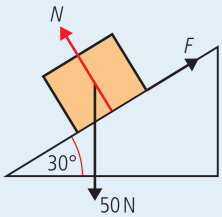


Figure 2.66.

28 A rock climber is hanging from a rope attached to the cliff by two bolts as shown in Figure 2.67. If the forces are balanced

(a) write an equation for the vertical component of the forces on the knot
(b) write an equation for the horizontal forces exerted on the knot
(c) calculate the tension *T* in the ropes joined to the bolts.

The result of this calculation shows why ropes should not be connected in this way.

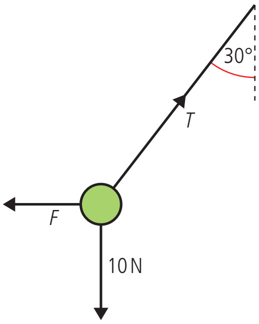


Figure 2.65.

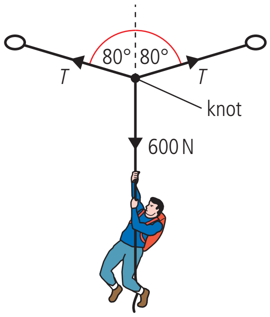


Figure 2.67.

2.3 Momentum and impulse

2.4 Momentum and impulse

Understandings, applications, and skills:

Newton's second law expressed as a rate of change of momentum

- Applying conservation of momentum in isolated systems including (but not limited to) the motion of rockets, collisions, explosions, or water jets.
- Using Newton's second law quantitatively and qualitatively in cases where mass is not constant.

Guidance

- *Students should be aware that* F = ma *is equivalent to* F = Δp/Δt *only when mass is constant.*

Impulse and force–time graphs

- Sketching and interpreting force–time graphs.
- Determining impulse in various contexts including (but not limited to) car safety and sports.

Conservation of linear momentum

Elastic collisions, inelastic collisions, and explosions

- Qualitatively and quantitatively comparing situations involving elastic collisions, inelastic collisions, and explosions.

Guidance

- *Solving simultaneous equations involving conservation of momentum and energy in collisions will not be required.*
- *Calculations relating to collisions and explosions will be restricted to one-dimensional situations.*
- *A comparison between inelastic collisions (in which kinetic energy is not conserved) and the conservation of (total) energy should be made.*

The relationship between force and acceleration

Newton's first law states that a body will accelerate if an unbalanced force is applied to it. Newton's second law tells us how big the acceleration will be and in which direction.

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The principle of conservation of momentum is a consequence of Newton's laws of motion applied to the collision between two bodies. If this applies to two isolated bodies we can generalize that it applies to any number of isolated bodies. Here we will consider colliding balls but it also applies to collisions between microscopic particles such as atoms.

Before we look in detail at Newton's second law we should look at the factors that affect the acceleration of a body when an unbalanced force is applied. Let us consider the example of catching a ball. When we catch the ball we change its velocity, Newton's first law tells us that we must therefore apply an unbalanced force to the ball. The size of that force depends upon two things, the mass and the velocity. A heavy ball is more difficult to stop than a light one travelling at the same speed, and a fast one is harder to stop than a slow one. Rather than having to concern ourselves with two quantities we will introduce a new quantity that incorporates both mass and velocity: *momentum*.

Momentum (p)

Momentum is defined as the product of mass and velocity: $\mathbf{p} = m\mathbf{v}$

The unit of momentum is kg m s^{-1}.

Momentum is a vector quantity.

Impulse

When you get hit by a ball the effect it has on you is greater if the ball bounces off you than if you catch it. This is because the change of momentum is greater when the ball bounces, as shown in Figure 2.68.

The unit of impulse is kg m s^{-1}.

Impulse is a vector.

Red ball

Momentum before = mv
Momentum after = $-mv$ (remember momentum is a vector)
Change in momentum = $-mv - mv = -2mv$

Blue ball

Momentum before = mv
Momentum after = 0
Change in momentum = $0 - mv = -mv$

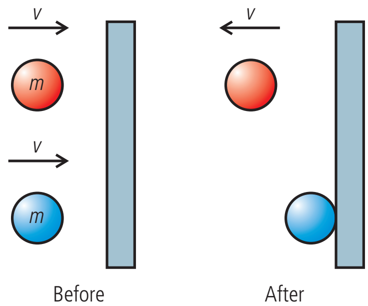


Figure 2.68 The change of momentum of the red ball is greater.

The impulse is defined as the change of momentum.

Exercises

29 A ball of mass 200 g travelling at 10 m s^{-1} bounces off a wall as in Figure 2.68. If after hitting the wall it travels at 5 m s^{-1}, what is the impulse?

30 Calculate the impulse on a tennis racket that hits a ball of mass 67 g travelling at 10 m s^{-1} so that it comes off the racket at a velocity of 50 m s^{-1}.

Newton's second law of motion

The rate of change of momentum of a body is directly proportional to the unbalanced force acting on that body and takes place in same direction.

Let us once again consider a ball with a constant force acting on it as in Figure 2.69.

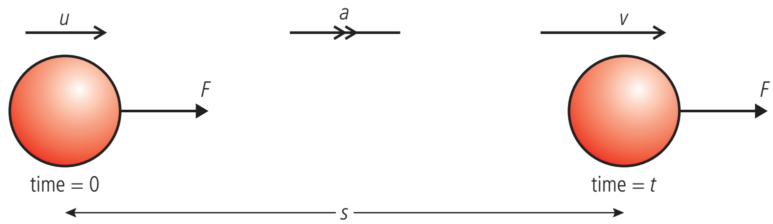


Figure 2.69.

Newton's first law tells us that there must be an unbalanced force acting on the ball since it is accelerating.

Newton's second law tells us that the size of the unbalanced force is directly proportional to the rate of change of momentum. We know that the force is constant so the rate of change of momentum is also constant, which, since the mass is also constant, implies that the acceleration is uniform so the *suvat* equations apply.

Unit of momentum

If F = change in momentum/time then momentum = force × time.
So the unit of momentum is N s. This is the same as kg m s^{-1}.

If the ball has mass m we can calculate the change of momentum of the ball.

Initial momentum = mu

Final momentum = mv

Change in momentum = $mv - mu$

The time taken is t so the rate of change of momentum = $\dfrac{mv - mu}{t}$

This is the same as $\dfrac{m(v-u)}{t} = ma$

Newton's second law states that the rate of change of momentum is proportional to the force, so $F \propto ma$.

To make things simple the newton is defined so that the constant of proportionality is equal to 1 so:

$$F = ma$$

So when a force is applied to a body in this way, Newton's second law can be simplified to:

The acceleration of a body is proportional to the force applied and inversely proportional to its mass.

Not all examples are so simple. Consider a jet of water hitting a wall as in Figure 2.70. The water hits the wall and loses its momentum, ending up in a puddle on the floor.

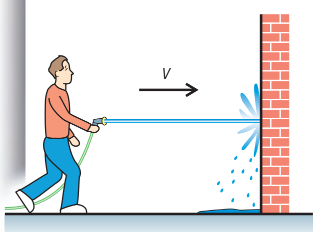


Figure 2.70.

Newton's first law tells us that since the velocity of the water is changing, there must be a force on the water,

Newton's second law tells us that the size of the force is equal to the rate of change of momentum. The rate of change of momentum in this case is equal to the amount of water hitting the wall per second multiplied by the change in velocity; this is not the same as ma. For this reason it is best to use the first, more general statement of Newton's second law, since this can always be applied.

However, in this course most of the examples will be of the $F = ma$ type.

Example 1: Elevator accelerating upwards

An elevator has an upward acceleration of 1 m s^{-2}. If the mass of the elevator is 500 kg, what is the tension in the cables pulling it up?

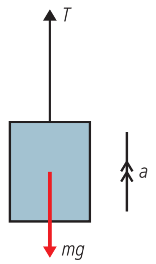


Figure 2.71 An elevator accelerating upwards. This could either be going up getting *faster* or going down getting *slower*.

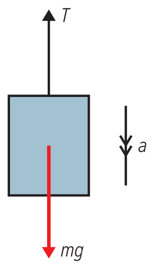


Figure 2.72 The elevator with downward acceleration.

First draw a free body diagram as in Figure 2.71. Now we can see what forces are acting. Newton's first law tells us that the forces must be unbalanced. Newton's second law tells us that the unbalanced force must be in the direction of the acceleration (upwards). This means that T is bigger than mg.

Newton's second law also tells us that the size of the unbalanced force equals ma so we get the equation

$$T - mg = ma$$

Rearranging gives

$$\begin{aligned} T &= mg + ma \\ &= 500 \times 10 + 500 \times 1 \\ &= 5500\,\text{N} \end{aligned}$$

Example 2: Elevator accelerating downwards

The same elevator as in example 1 now has a downward acceleration of $1\,\text{m s}^{-2}$ as in Figure 2.72.

This time Newton's laws tell us that the weight is bigger than the tension so $mg - T = ma$

Rearranging gives

$$\begin{aligned} T &= mg - ma \\ &= 500 \times 10 - 500 \times 1 \\ &= 4500\,\text{N} \end{aligned}$$

Example 3: Joined masses

Two masses are joined by a rope. One of the masses sits on a frictionless table, the other hangs off the edge as in Figure 2.73.

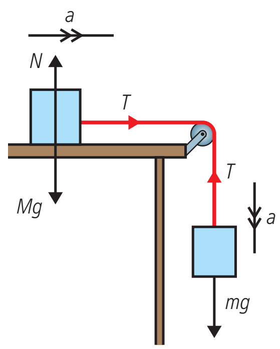


Figure 2.73.

M is being dragged to the edge of the table by m.

Both are connected to the same rope so T is the same for both masses. This also means that the acceleration a is the same.

We do not need to consider N and Mg for the mass on the table because these forces are balanced. However the horizontally unbalanced force is T.

Applying Newton's laws to the mass on the table gives

$$T = Ma$$

The hanging mass is accelerating down so mg is bigger than T. Newton's second law implies that $mg - T = ma$

Substituting for T gives $mg - Ma = ma$ so $a = \dfrac{mg}{M + m}$

Example 4: The free fall parachutist

After falling freely for some time, a free fall parachutist whose weight is 60 kg opens his parachute. Suddenly the force due to air resistance increases to 1200 N. What happens?

Looking at the free body diagram in Figure 2.74 we can see that the forces are unbalanced and that according to Newton's second law the acceleration, *a*, will be upwards.

The size of the acceleration is given by

$$ma = 1200 - 600 = 60 \times a$$

so
$$a = 10\,\mathrm{m\,s^{-2}}$$

The acceleration is in the opposite direction to the motion. This will cause the parachutist to slow down. As he slows down, the air resistance gets less until the forces are balanced. He will then continue down with a constant velocity.

Figure 2.74 The parachutist just after opening the parachute.

Even without a parachute base jumpers reach terminal velocity.

Exercises

31 The helium in a balloon causes an upthrust of 0.1 N. If the mass of the balloon and helium is 6 g, calculate the acceleration of the balloon.

32 A rope is used to pull a felled tree (mass 50 kg) along the ground. A tension of 1000 N causes the tree to move from rest to a velocity of 0.1 m s^{-1} in 2 s. Calculate the force due to friction acting on the tree.

33 Two masses are arranged on a frictionless table as shown in Figure 2.75. Calculate:

(a) the acceleration of the masses

(b) the tension in the string.

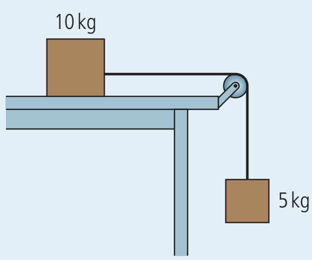


Figure 2.75.

34 A helicopter is lifting a load of mass 1000 kg with a rope. The rope is strong enough to hold a force of 12 kN. What is the maximum upward acceleration of the helicopter?

35 A person of mass 65 kg is standing in an elevator that is accelerating upwards at 0.5 m s^{-2}. What is the normal force between the floor and the person?

36 A plastic ball is held under the water by a child in a swimming pool. The volume of the ball is 4000 cm^3.

(a) If the density of water is 1000 kg m^{-3}, calculate the buoyant force on the ball (remember buoyant force = weight of fluid displaced).

(b) If the mass of the ball is 250 g, calculate the theoretical acceleration of the ball when it is released. Why won't the ball accelerate this quickly in a real situation?

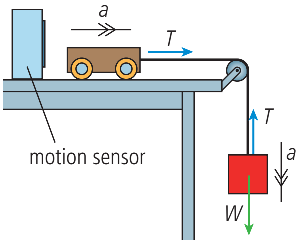


Figure 2.76 Apparatus for finding the relationship between force and acceleration.

The relationship between force and acceleration

Full details of how to carry out this experiment with a worksheet are available online.

Experiment to test the relationship between acceleration and force

It isn't easy to apply a constant, known force to a moving body: just try pulling a cart along the table with a force meter and you will see. One way this is often done in the laboratory is by hanging a mass over the edge of the table as shown in Figure 2.76.

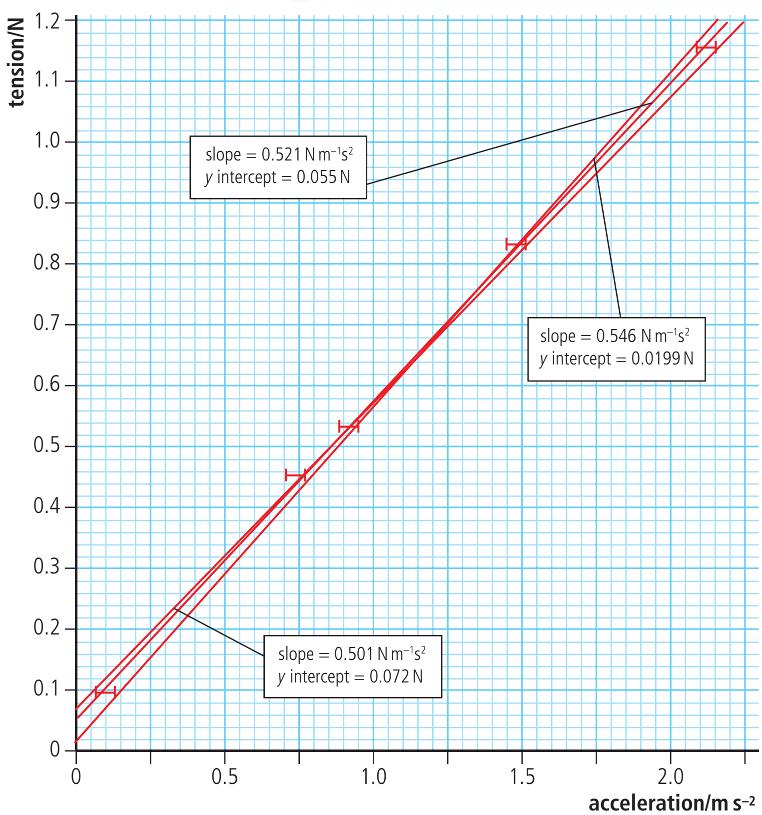
If we ignore any friction in the pulley or in the wheels of the trolley then the unbalanced force on the trolley = T. Since the mass is accelerating down then the weight is bigger than T so $W - T = ma$ where m is the mass hanging on the string. The tension is therefore given by $T = mg - ma = m(g - a)$.

There are several ways to measure the acceleration of the trolley; one is to use a motion sensor. This senses the position of the trolley by reflecting an ultrasonic pulse off it. Knowing the speed of the pulse, the software can calculate the distance between the trolley and sensor. As the trolley moves away from the sensor the time taken for the pulse to return increases; the software calculates the velocity from these changing times. Using this apparatus, the acceleration of the trolley for different masses was measured, and the results are given in the Table 2.8.

Table 2.8 Results from the force and acceleration experiment.

Mass/kg ±0.0001	Acceleration/m s^{-2} ±0.03	Tension ($T = mg - ma$)/N	Max T/N	Min T/N	ΔT/N
0.0100	0.10	0.097	0.098	0.096	0.001
0.0500	0.74	0.454	0.453	0.451	0.001
0.0600	0.92	0.533	0.532	0.531	0.001
0.1000	1.49	0.832	0.830	0.828	0.001
0.1500	2.12	1.154	1.150	1.148	0.001

Figure 2.77 Graph of tension against acceleration.



The uncertainty in mass is given by the last decimal place in the scale, and the uncertainty in acceleration by repeating one run several times. To calculate the uncertainty in tension the maximum and minimum values have been calculated by adding and subtracting the uncertainties.

These results are shown in Figure 2.77.

Applying Newton's second law to the trolley the relationship between T and a should be $T = Ma$ where M is the mass of the trolley. This implies that the gradient of the line should be M. From the graph we can see that the gradient is 0.52 ± 0.02 kg which is quite close to the 0.5 kg mass of the trolley.

According to theory the intercept should be (0, 0) but we can see that there is a positive intercept of 0.05 N. It appears that each value is 0.05 N too big. The reason for this could be friction. If there was friction then the actual unbalanced force acting on the trolley would be tension – friction. If this is the case then the results would imply that friction is about 0.05 N.

Newton’s third law of motion

When dealing with Newton’s first and second laws, we are careful to consider only the body that is *experiencing* the forces, not the body that is *exerting* the forces. Newton’s third law relates these forces.

If body A exerts a force on body B then body B will exert an equal and opposite force on body A.

So if someone is pushing a car with a force *F* as shown in Figure 2.78 the car will push back on the person with a force –*F*. In this case both of these forces are the normal force.

Incorrect statements

It is very important to realise that Newton’s third law is about two bodies. Avoid statements of this law that do not mention anything about there being two bodies.

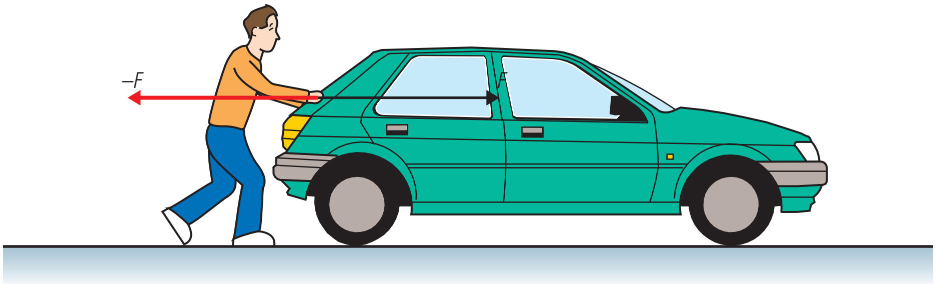


Figure 2.78 The man pushes the car and the car pushes the man.

You might think that since these forces are equal and opposite, they will be balanced, and in that case how does the person get the car moving? This is wrong; the forces act on different bodies so can’t balance each other.

Example 1: A falling body

A body falls freely towards the ground as in Figure 2.79. If we ignore air resistance, there is only one force acting on the body – the force due to the gravitational attraction of the Earth, that we call weight.

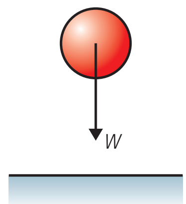


Figure 2.79 A falling body pulled down by gravity.

Applying Newton’s third law:

If the Earth pulls the body down, then the body must pull the Earth up with an equal and opposite force. We have seen that the gravitational force always acts on the centre of the body, so Newton’s third law implies that there must be a force equal to *W* acting upwards on the centre of the Earth as in Figure 2.80.

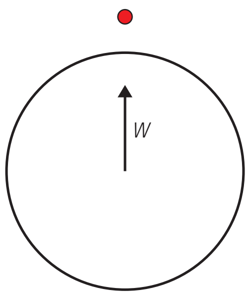


Figure 2.80 The Earth pulled up by gravity.

Example 2: A box rests on the floor

A box sits on the floor as shown in Figure 2.81. Let us apply Newton’s third law to this situation.
There are two forces acting on the box.

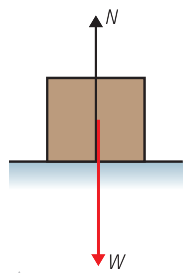


Figure 2.81 Forces acting on a box resting on the floor.

Normal force: The floor is pushing up on the box with a force *N*. According to Newton’s third law the box must therefore push down on the floor with a force of magnitude *N*.

Weight: The Earth is pulling the box down with a force *W*. According to Newton’s third law, the box must be pulling the Earth up with a force of magnitude *W* as shown in Figure 2.82.

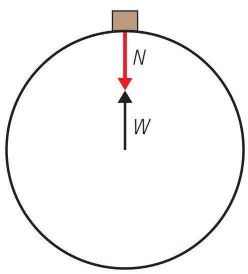


Figure 2.82 Forces acting on the Earth according to Newton’s third law.

Students often think that Newton’s third law implies that the normal force = –weight, but *both* of these forces act on the box. If the box is at *rest* these forces are indeed equal and opposite but this is due to Newton’s *first* law.

Example 3: Recoil of a gun

When a gun is fired the velocity of the bullet changes. Newton's first law implies that there must be an unbalanced force on the bullet; this force must come from the gun. Newton's third law says that if the gun exerts a force on the bullet the bullet must exert an equal and opposite force on the gun. This is the force that makes the gun recoil or 'kick back'.

Example 4: The water cannon

When water is sprayed at a wall from a hosepipe it hits the wall and stops. Newton's first law says that if the velocity of the water changes, there must be an unbalanced force on the water. This force comes from the wall. Newton's third law says that if the wall exerts a force on the water then the water will exert a force on the wall. This is the force that makes a water cannon so effective at dispersing demonstrators.

A boat tests its water cannons.

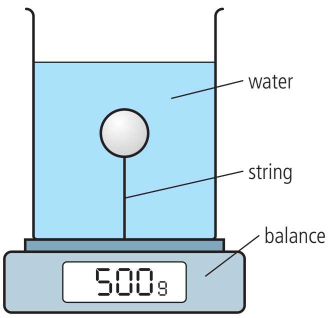


Figure 2.83.

Exercise

37 Use Newton's first and third laws to explain the following:

(a) When burning gas is forced downwards out of a rocket motor, the rocket accelerates up.
(b) When the water cannons on the boat in the photo are operating, the boat accelerates forwards.
(c) When you step forwards off a skateboard, the skateboard accelerates backwards.
(d) A table tennis ball is immersed in a fluid and held down by a string as shown in Figure 2.83. The container is placed on a balance. What will happen to the reading of the balance if the string breaks?

Collisions

In this section we have been dealing with the interaction between two bodies (gun–bullet, skater–skateboard, hose–water). To develop our understanding of the interaction between bodies, let us consider a simple collision between two balls as illustrated in Figure 2.84.

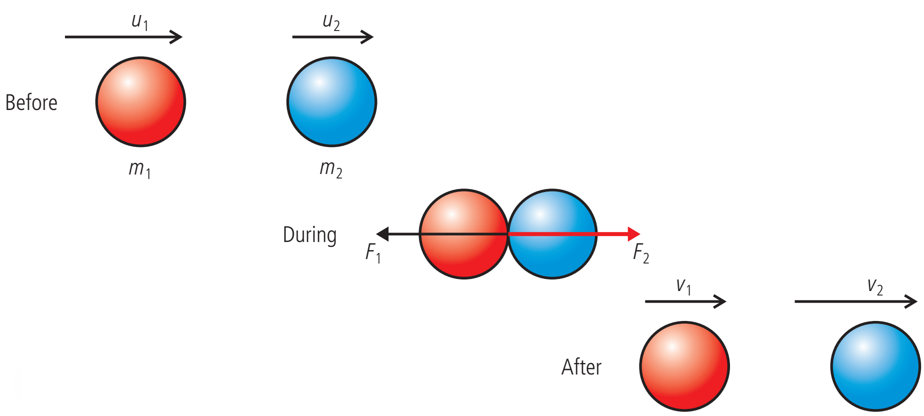


Figure 2.84 Collision between two balls.

Let us apply Newton's three laws to this problem.

Newton's first law

In the collision the red ball slows down and the blue ball speeds up. Newton's first law tells us that that this means there is a force acting to the left on the red ball (F_1) and to the right on the blue ball (F_2).

Newton's second law

This law tells us that the force will be equal to the rate of change of momentum of the balls so if the balls are touching each other for a time Δt:

$$F_1 = \frac{m_1v_1 - m_1u_1}{\Delta t}$$

$$F_2 = \frac{m_2v_2 - m_2u_2}{\Delta t}$$

Newton's third law

According to the third law, if the red ball exerts a force on the blue ball, then the blue ball will exert an equal and opposite force on the red ball.

$$F_1 = -F_2$$

$$\frac{m_1v_1 - m_1u_1}{\Delta t} = \frac{-(m_2v_2 - m_2u_2)}{\Delta t}$$

Rearranging gives $\quad m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

In other words the momentum at the start equals the momentum at the end. We find that this applies not only to this example but to all interactions.

The law of the conservation of momentum

For a system of isolated bodies the total momentum is always the same.

This is not a new law since it is really just a combination of Newton's laws. However it provides a useful short cut when solving problems.

Isolated system

An isolated system is one in which no external forces are acting. When a ball hits a wall the momentum of the ball is not conserved because the ball and wall is not an isolated system, since the wall is attached to the ground. If the ball and wall were floating in space then momentum would be conserved.

Examples

In these examples we will have to pretend everything is in space isolated from the rest of the Universe, otherwise they are not isolated and the law of conservation of momentum won't apply.

1. A collision where the bodies join together

If two balls of modelling clay collide with each other they stick together as shown in Figure 2.85. We want to find the velocity, v, of the combined lump after the collision.

Simplified models

Pieces of clay floating in space are not exactly everyday examples, but most everyday examples (like balls on a pool table) are not isolated systems, so we can't solve them in this simple way.

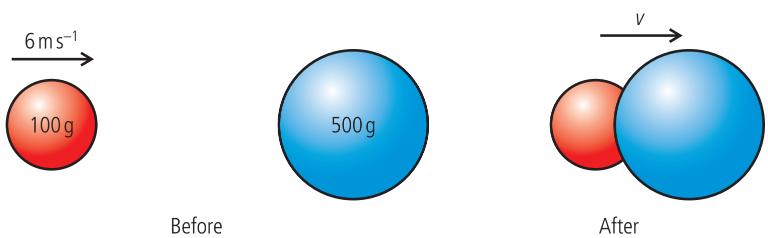


Figure 2.85 Two bodies stick together after colliding.

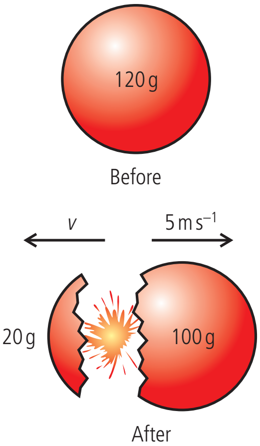


Figure 2.86 A piece of modelling clay suddenly explodes.

If bodies are isolated then momentum is conserved so:

$$\text{momentum before} = \text{momentum after}$$

$$0.1 \times 6 + 0.5 \times 0.0 = 0.6 \times v$$

$$v = \frac{0.6}{0.6} = 1\,\text{m}\,\text{s}^{-1}$$

2. An explosion

A ball of clay floating around in space suddenly explodes into a big piece and a small piece, as shown in Figure 2.86. If the big bit has a velocity of $5\,\text{m}\,\text{s}^{-1}$, what is the velocity of the small bit?

Since this is an isolated system, momentum is conserved so:

$$\text{momentum before} = \text{momentum after}$$

$$0 \times 0.12 = 0.02 \times (-v) + 0.1 \times 5$$

$$0.02 \times v = 0.5$$

$$v = 25\,\text{m}\,\text{s}^{-1}$$

Rocket engine

The momentum of a rocket plus fuel floating in space is zero but when the engines are fired gas is expelled at high speed. This gas has momentum towards the left so in order to conserve momentum the rocket will move towards the right.

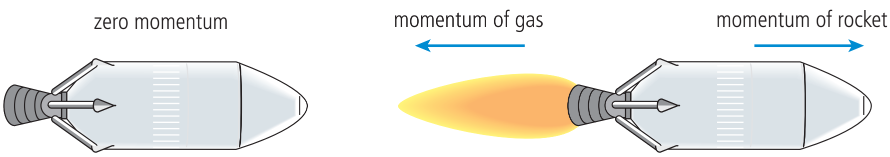


Figure 2.87 Rocket engine expels gas to give thrust.

The momentum of the rocket will equal the momentum of the expelled gases, so increasing the rate at which the gases are expelled will increase the acceleration of the rocket. Note that the situation for a rocket about to blast off on the Earth is rather more complex since the rocket + fuel can no longer be considered isolated.

Jet engine

A passenger jet aeroplane powered by four jet engines.

A jet engine produces thrust (the force that pushes the plane forwards) by increasing the speed of air taken in at the front by passing it through a series of turbines. The fast-moving air expelled from the back of the engine has an increased momentum so if momentum is to be conserved the plane must have increased momentum in the forwards direction.

Exercise

38 Draw diagrams to represent the following collisions then use the law of conservation of momentum to find the unknown velocity. Assume all collisions are head-on, in other words they take place in one dimension.

(a) Two identical isolated balls collide with each other. Before the collision, one ball was travelling at $10\,\mathrm{m\,s^{-1}}$ and the other was at rest. After the collision the first ball continues in the same direction with a velocity of $1\,\mathrm{m\,s^{-1}}$. Find the velocity of the other ball.

(b) Two identical balls are travelling towards each other; each is travelling at a speed of $5\,\mathrm{m\,s^{-1}}$. After they hit, one ball bounces off with a speed of $1\,\mathrm{m\,s^{-1}}$. What is the speed of the other?

(c) A spaceman of mass 100 kg is stranded 2 m from his spaceship as shown in Figure 2.88. He happens to be holding a hammer of mass 2 kg what must he do?

If he only has enough air to survive for 2 minutes, how fast must he throw the hammer if he is to get back in time? Is it possible?

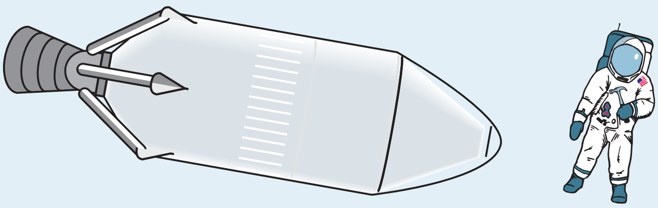


Figure 2.88 If you are ever in this position this course could save your life.

Momentum and force

If a constant unbalanced force acts on an isolated body the graph of the force against time would be as shown in Figure 2.89, force remaining constant all the time.

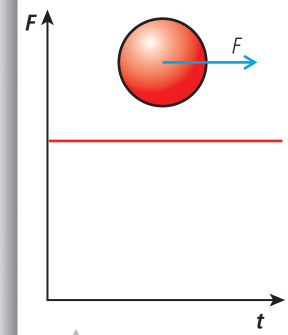


Figure 2.89 Graph of force against time for constant force.

According to Newton's second law the bodies rate of change of momentum will be equal to force applied so $F = \frac{\Delta mv}{\Delta t}$.

Rearranging this gives $\Delta mv = F\Delta t$ where $F\Delta t$ is the area under the graph. This is the case whenever a force is applied over a time.

Now let us consider a more difficult example of a 2 kg steel ball travelling at $10\,\mathrm{m\,s^{-1}}$ bouncing off a concrete wall as illustrated by the graph in Figure 2.90.

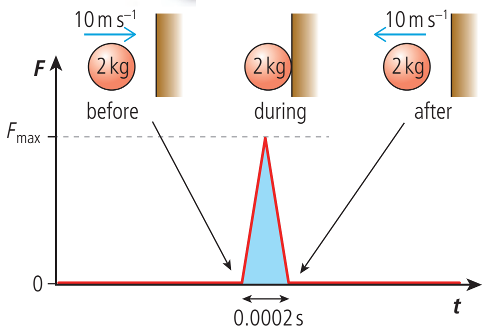


Figure 2.90 Graph of force against time for a hard collision.

From the graph you can see that the ball is only in contact with the wall for 0.0002 s, the force is not constant but has its maximum value, F_{max} in the middle of the bounce.

The change in momentum (impulse) = momentum after − momentum before $= -2 \times 10 - 2 \times 10 = -40\,\mathrm{N\,s}$. This is the same as the area under the graph $= \frac{1}{2} \times F_{max} \times 0.0002$

so $\frac{1}{2} \times F_{max} \times 0.0002 = -40\,\mathrm{N\,s}$

which makes $F_{max} = -400\,\mathrm{kN}$

Note that the force on the ball is *negative* because it is to the *left*.

If the same ball is thrown against a soft wall, the soft wall bends inwards as the ball hits it, resulting in a bounce lasting a much longer time as represented by the graph in Figure 2.91, let's say 0.2 s.

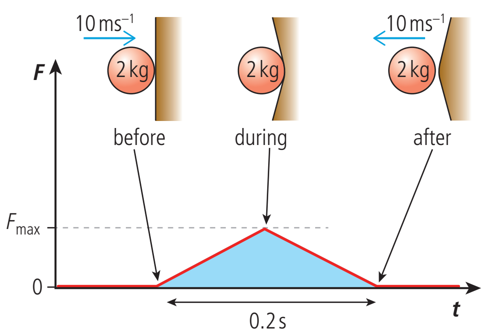


Figure 2.91 Force against time for a soft collision.

The mass and change in velocity are the same as last time so impulse Δmv = 40 N s. This means that the area under the graph is the same as last time so:

$$\frac{1}{2} \times F_{max} \times 0.2 = -40\,\text{N s}$$
$$F_{max} = -400\,\text{N}.$$

The author testing a climbing rope.

From this example we can see that the amount of force required to slow down a body is related to the time of application of the force. This is very important when designing climbing ropes. If a climber fell a distance of 10 m they would be travelling at about 14 m s^{-1}. If the rope didn't stretch they would be brought to rest very quickly resulting in a large force applied to the climber. This sudden large force would certainly injure the climber and probably even break the rope. It is for this very reason that stretchy ropes are used, making the result of being stopped by the rope pleasant rather than life threatening. Similar considerations also come to play when designing cars. If cars were made out of perfectly rigid materials, when involved in a crash they would stop very rapidly. The driver would also subjected to large forces resulting in injury. To make the time of collision longer and hence the force smaller, cars are designed with crumple zones that collapse when the car hits something. This doesn't make crashing a car pleasant but can reduce injury.

The amount that a climbing rope stretches is related to the length of the rope. If the length is short then the stretch may not be enough to reduce the force of the fall sufficiently. In this case the person holding the rope can allow the rope to run by moving forwards. This is a little disconcerting for the climber who falls further than expected.

Figure 2.92 shows some real data obtained by dropping a ball of modelling clay onto a force sensor. The negative peak is probably due to the ball sticking to the force sensor on the way back up. This graph is part of the data collected by IB student Gustav Gordon researching the relationship between temperature and maximum force experienced when a plasticine ball hits the floor for his Extended Essay.

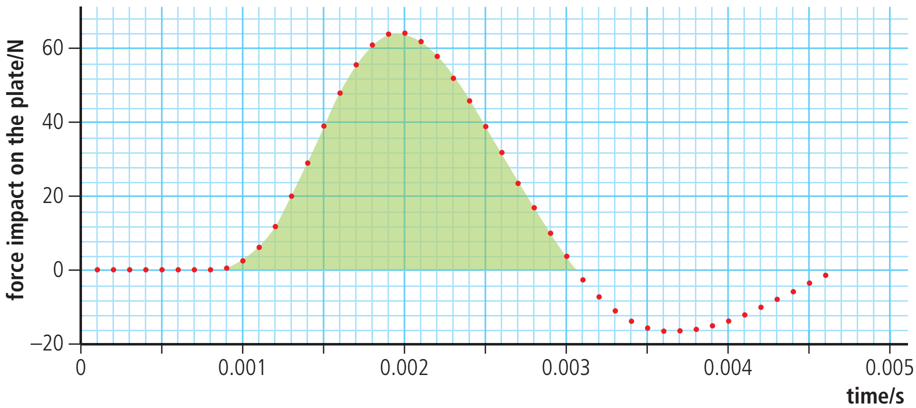


Figure 2.92 Force against time for a plasticine ball landing on a force sensor.

Exercises

39 **(a)** Calculate the impulse of the body for the motion represented in Figure 2.93.

(b) If the mass of the object is 20 g, what is the change of velocity?

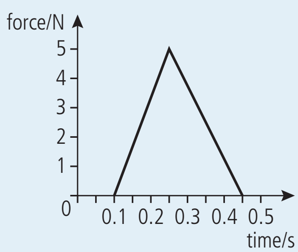


Figure 2.93.

40 Use the data in Figure 2.93 to estimate the height that the 20 g plasticine ball was dropped from.

Pressure (P)

If we take the example of a block resting on the ground, the bottom surface of the block is in contact with the ground so will exert a normal reaction force on the ground. The normal reaction on the ground is equal to the normal reaction on the block (Newton's third law) which, since the block is in equilibrium, is equal to the weight of the object (Newton's first law). The force on the ground will therefore be the same no matter what the area of contact is, so in Figure 2.94 the normal reaction is the same in both cases. Even though the force is the same, the *effect* of the force might be different. Imagine the block was placed on something soft like snow, mud, or sand. The block with the smallest area would then push into the soft material the most. This is because the force per unit area (*pressure*) is greater.

$$\textbf{pressure} = \frac{\textbf{force}}{\textbf{area}}$$

The unit pressure is the $\mathrm{N\,m^{-2}}$. This is called a pascal (Pa).

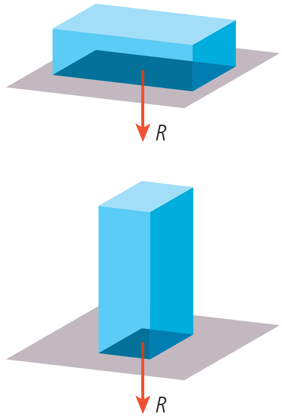


Figure 2.94 Comparing the force under two different areas.

2.4 Work, energy, and power

2.3 Work, energy, and power

Understandings, applications, and skills:

Principle of conservation of energy
- Discussing the conservation of total energy within energy transformations.

Kinetic energy
Gravitational potential energy
Elastic potential energy
- Sketching and interpreting force–distance graphs.

Work done as energy transfer
- Determining work done including cases where a resistive force acts.

Guidance
- *Cases where the line of action of the force and the displacement are not parallel should be considered.*
- *Examples should include force–distance graphs for variable forces.*

Power as rate of energy transfer
- Solving problems involving power.

Efficiency
- Quantitatively describing efficiency in energy transfers.

NATURE OF SCIENCE

Scientists should remain sceptical but that doesn't mean you have to doubt everything you read. The law of conservation of energy is supported by many experiments, and is the basis of countless predictions that turn out to be true. If someone now found that energy was not conserved then there would be a lot of explaining to do. Once a law is accepted it gives us an easy way to make predictions. For example, if you are shown a device that produces energy from nowhere you know it must be a fake without even finding out how it works because it violates the law of conservation of energy.

We have so far dealt with the motion of a small red ball and understand what causes it to accelerate. We have also investigated the interaction between a red ball and a blue one and have seen that the red one can cause the blue one to move when they collide. But what enables the red one to push the blue one? To answer this question we need to define some more quantities.

Work

In the introduction to this book it was stated that by developing models, our aim is to understand the physical world so that we can make predictions. At this point you should understand certain concepts related to the collision between two balls, but we still can't predict the outcome. To illustrate this point let us again consider the red and blue balls. Figure 2.95 shows three possible outcomes of the collision.

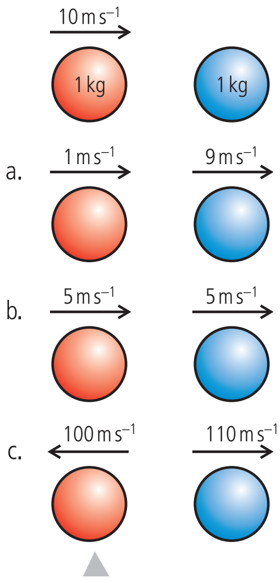


Figure 2.95 The red ball hits the blue ball but what happens?

If we apply the law of conservation of momentum, we realize that all three outcomes are possible. The original momentum is 10 N s and the final momentum is 10 N s in all three cases. But which one actually happens? This we cannot say (yet). All we know is that from experience the last option is not possible – but why?

When body A hits body B, body A exerts a force on body B. This force causes B to have an increase in velocity. The amount that the velocity increases depends upon how big the force is and over what distance the collision takes place. To make this simpler, consider a constant force acting on two blocks as in Figure 2.96.

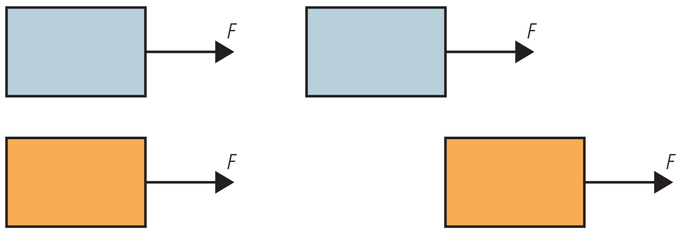


Figure 2.96 The force acts on the orange block for a greater distance

Both blocks start at rest and are pulled by the same force, but the orange block will gain more velocity because the force acts over a longer distance. To quantify this difference, we say that in the case of the orange block the force has done more work. Work is done when the point of application of a force moves in the direction of the force.

Work is defined in the following way:

Work done = force × distance moved in the direction of the force.

The unit of work is the newton metre (N m) which is the same as the joule (J).

Work is a scalar quantity.

Worked example

A tractor pulls a felled tree along the ground for a distance of 200 m. If the tractor exerts a force of 5000 N, how much work will be done?

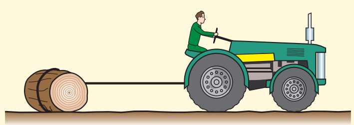


Figure 2.97.

Solution

Work done = force × distance moved in direction of force

Work done = 5000 × 200 = 1 MJ

Worked example

A force of 10 N is applied to a block, pulling it 50 m along the ground as shown in Figure 2.98. How much work is done by the force?

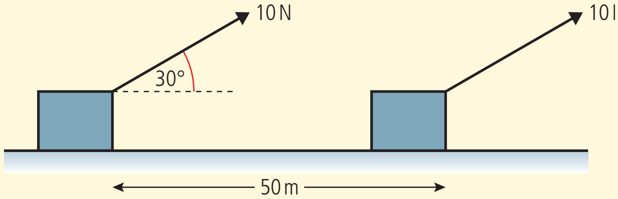


Figure 2.98.

Solution

In this example the force is not in the same direction as the movement. However, the horizontal component of the force is.

$$\text{Work done} = 10 \times \cos 30° \times 50 = 433\ \text{N}$$

Worked example

When a car brakes it slows down due to the friction force between the tyres and the road. This force opposes the motion as shown in Figure 2.99. If the friction force is a constant 500 N and the car comes to rest in 25 m, how much work is done by the friction force?

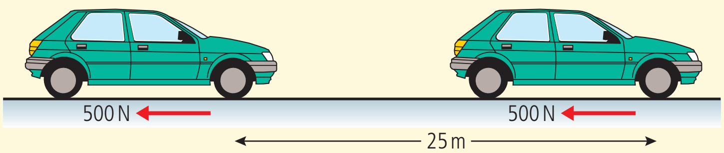


Figure 2.99 Work done against friction.

Solution

This time the force is in the opposite direction to the motion.

$$\text{Work done} = -500 \times 25 = -12\,500\ \text{J}$$

The negative sign tells us that the friction isn't doing the work but the work is being done against the friction.

Worked example

The woman in Figure 2.100 walks along with a constant velocity holding a suitcase.
How much work is done by the force holding the case?

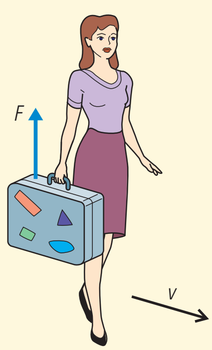


Figure 2.100.

Solution

In this example the force is acting perpendicular to the direction of motion, so there is no movement in the direction of the force.

$$\text{Work done} = \text{zero}$$

Working or not?

It may seem strange that when you carry a heavy bag you are not doing any work – that's not what it feels like. In reality, lots of work is being done, since to hold the bag you use your muscles. Muscles are made of microscopic fibres, which are continuously contracting and relaxing, so are doing work.

General formula

In general

Work = $F \cos \theta \times \Delta s$

where θ is the angle between the displacement, Δs, and force, F (see Figure 2.101).

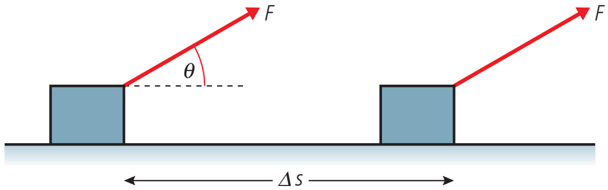


Figure 2.101.

All the previous examples can be solved using this formula.

If $\theta < 90°$, $\cos \theta$ is positive so the work is positive.

$\theta = 90°$, $\cos \theta = 0$ so the work is zero.

$\theta > 90°$, $\cos \theta$ is negative so the work is negative.

Exercises

41 Figure 2.102 shows a boy taking a dog for a walk.

(a) Calculate the work done by the force shown when the dog moves 10 m forward.

(b) Who is doing the work?

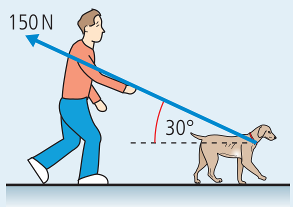


Figure 2.102.

42 A bird weighing 200 g sits on a tree branch. How much work does the bird do on the tree?

43 As a box slides along the floor it is slowed down by a constant force due to friction. If this force is 150 N and the box slides for 2 m, how much work is done against the frictional force?

Graphical method for determining work done

Let us consider a constant force acting in the direction of movement pulling a body a distance Δx. The graph of force against distance for this example is simply as shown in Figure 2.103. From the definition of work we know that work done = $F\Delta x$ which in this case is the area under the graph. From this we can deduce that:

work done = area under force *vs* distance graph

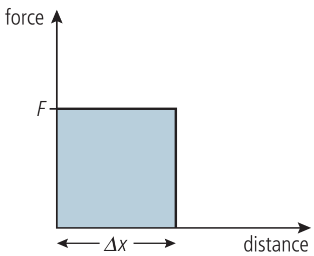


Figure 2.103 Force *vs* distance for a constant force.

Work done by a varying force

Stretching a spring is a common example of a varying force. When you stretch a spring it gets more and more difficult the longer it gets. Within certain limits the force needed to stretch the spring is directly proportional to the extension of the spring. This was first recognised by Robert Hooke in 1676, so is named 'Hooke's Law'. Figure 2.104 shows what happens if we add different weights to a spring: the more weight we add the longer it gets. If we draw a graph of force against distance as we stretch a spring, it will look like the graph in Figure 2.105. The gradient of this line, $\frac{F}{\Delta x}$ is called the spring constant, k.

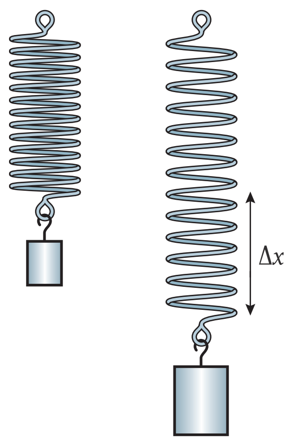


Figure 2.104 Stretching a spring.

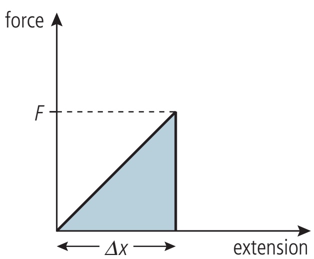


Figure 2.105 Force *vs* extension for a spring.

The work done as the spring is stretched is found by calculating the area under the graph.

$$\text{area} = \tfrac{1}{2}\text{base} \times \text{height} = \tfrac{1}{2}F\Delta x$$

So $\quad \text{work done} = \tfrac{1}{2}F\Delta x$

But if $\quad \dfrac{F}{\Delta x} = k$ then $F = k\Delta x$

Substituting for F gives

$$\text{work done} = \tfrac{1}{2}k\Delta x^2$$

Exercises

44 A spring of spring constant 2 N cm^{-1} and length 6 cm is stretched to a new length of 8 cm.

(a) How far has the spring been stretched?
(b) What force will be needed to hold the spring at this length?
(c) Sketch a graph of force against extension for this spring.
(d) Calculate the work done in stretching the spring.
(e) The spring is now stretched a further 2 cm. Draw a line on your graph to represent this and calculate how much additional work has been done.

45 Calculate the work done by the force represented by Figure 2.106.

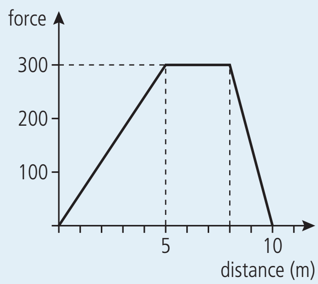


Figure 2.106.

Energy

We have seen that it is sometimes possible for body A to do work on body B but what does A have that enables it to do work on B? To answer this question we must define a new quantity, energy.

Energy is the quantity that enables body A to do work on body B.

If body A collides with body B as shown in Figure 2.107, body A has done work on body B. This means that body B can now do work on body C. Energy has been transferred from A to B.

When body A does work on body B, energy is transferred from body A to body B.

The unit of energy is the joule (J).

Energy is a scalar quantity.

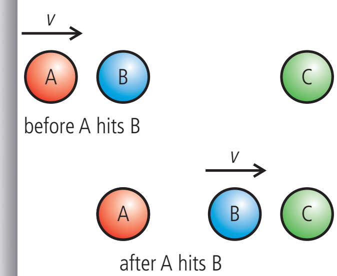


Figure 2.107 The red ball gives energy to the blue ball.

Different types of energy

If a body can do work then it has energy. There are two ways that a simple body such as a red ball can do work. In the example above, body A could do work because it was moving – this is called *kinetic energy*. Figure 2.108 shows an example where A can do work even though it isn't moving. In this example, body A is able to do work on body B because of its position above the Earth. If the hand is removed, body A will be pulled downwards by the force of gravity, and the string attached to it will then drag B along the table. If a body is able to do work because of its position, we say it has *potential energy*.

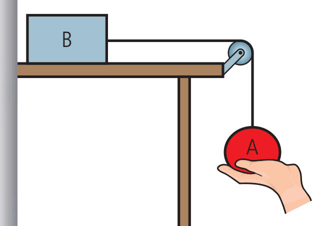


Figure 2.108.

Kinetic energy (KE)

This is the energy a body has due to its movement. To give a body KE, work must be done on the body. The amount of work done will be equal to the increase in KE. If a constant force acts on a red ball of mass m as shown in Figure 2.109, then the work done is Fs.

TOK

Use of words

If we say a body has potential energy it sounds as though it has the potential to do work. This is true, but a body that is moving has the potential to do work too. This can lead to misunderstanding. It would have been better to call it *positional energy*.

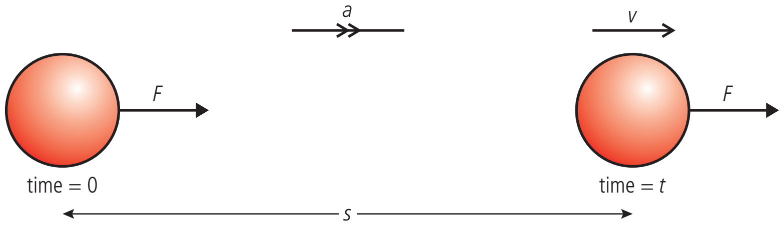


Figure 2.109.

From Newton's second law we know that $F = ma$ which we can substitute in work = Fs to give work = mas.

We also know that since acceleration is constant we can use the *suvat* equation $v^2 = u^2 + 2as$ which since $u = 0$ simplifies to $v^2 = 2as$.

Rearranging this gives $as = \frac{v^2}{2}$ so work = $\frac{1}{2}mv^2$.

This work has increased the KE of the body so we can deduce that:

$$KE = \frac{1}{2}mv^2$$

Gravitational potential energy (PE)

This is the energy a body has due to its position above the Earth.

Other types of PE

In this section we only deal with examples of PE due to a body's position close to the Earth. However there are other positions that will enable a body to do work (for example, in an electric field). These will be introduced after the concept of fields has been introduced in Chapter 6.

For a body to have PE it must have at some time been lifted to that position. The amount of work done in lifting it equals the PE. Taking the example shown in Figure 2.110, the work done in lifting the mass, m, to a height h is mgh (this assumes that the body is moving at a constant velocity so the lifting force and weight are balanced).

If work is done on the body then energy is transferred so:

$$\text{gain in PE} = mgh$$

The law of conservation of energy

We could not have derived the equations for KE or PE without assuming that the work done is the same as the gain in energy. The law of conservation of energy is a formal statement of this fact.

Energy can neither be created nor destroyed – it can only be changed from one form to another.

This law is one of the most important laws that we use in physics. If it were not true you could suddenly find yourself at the top of the stairs without having done any work in climbing them, or a car suddenly has a speed of 200 km h^{-1} without anyone touching the accelerator pedal. These things just don't happen, so the laws we use to describe the physical world should reflect that.

Worked example

A ball of mass 200 g is thrown vertically upwards with a velocity of $2\,m\,s^{-1}$ as shown in Figure 2.110. Use the law of conservation of energy to calculate its maximum height.

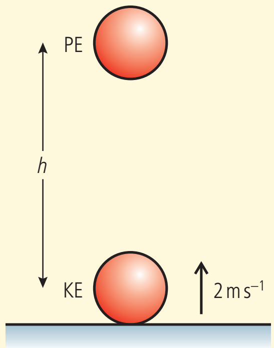


Figure 2.110 Work is done lifting the ball so it gains PE.

Solution

At the start of its motion the body has KE. This enables the body to do work against gravity as the ball travels upwards. When the ball reaches the top, all the KE has been converted into PE. So applying the law of conservation of energy:

$$\text{loss of KE} = \text{gain in PE}$$

$$\frac{1}{2}mv^2 = mgh$$

so $$h = \frac{v^2}{2g} = \frac{2^2}{2 \times 10} = 0.2\,\text{m}$$

This is exactly the same answer you would get by calculating the acceleration from $F = ma$ and using the *suvat* equations.

Worked example

A block slides down the frictionless ramp shown in Figure 2.111. Use the law of conservation of energy to find its speed when it gets to the bottom.

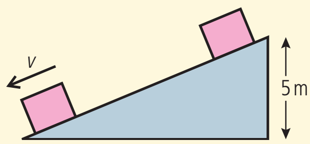


Figure 2.111 As the block slides down the slope it gains KE.

Solution

This time the body loses PE and gains KE so applying the law of conservation of energy:

$$\text{loss of PE} = \text{gain of KE}$$

$$mgh = \frac{1}{2}mv^2$$

So $$v = \sqrt{2gh} = \sqrt{(2 \times 10 \times 5)} = 10\,\text{m}\,\text{s}^{-1}$$

Again, this is a much simpler way of getting the answer than using components of the forces.

Exercises

Use the law of conservation of energy to solve the following:

46 A stone of mass 500 g is thrown off the top of a cliff with a speed of $5\,m\,s^{-1}$. If the cliff is 50 m high, what is its speed just before it hits the ground?

47 A ball of mass 250 g is dropped 5 m onto a spring as shown in Figure 2.112.

(a) How much KE will the ball have when it hits the spring?
(b) How much work will be done as the spring is compressed?
(c) If the spring constant is $250\,kN\,m^{-1}$, calculate how far the spring will be compressed.

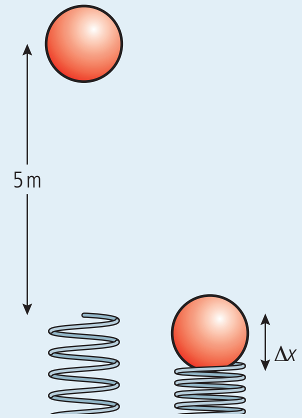


Figure 2.112 In this example the spring is compressed, not stretched, but Hooke's law still applies.

48 A ball of mass 100 g is hit vertically upwards with a bat. The bat exerts a constant force of 15 N on the ball and is in contact with it for a distance of 5 cm.

(a) How much work does the bat do on the ball?
(b) How high will the ball go?

49 A child pushes a toy car of mass 200 g up a slope. The car has a speed of $2\,m\,s^{-1}$ at the bottom of the slope.

(a) How high up the slope will the car go?
(b) If the speed of the car were doubled how high would it go now?

Forms of energy

When we are describing the motion of simple red balls there are only two forms of energy, KE and PE. However, when we start to look at more complicated systems, we discover that we can do work using a variety of different machines, such as petrol engine, electric engine, etc. To do work, these machines must be given energy and this can come in many forms, for example:

- Petrol
- Solar
- Gas
- Nuclear
- Electricity

As you learn more about the nature of matter in Chapter 3, you will discover that all of these (except solar) are related to either KE or PE of particles.

Energy conversion

Taking the example of a petrol engine, the energy stored in the petrol is converted to mechanical energy of the car by the engine. 1 litre of petrol contains 36 MJ of energy. Let's calculate how far a car could travel at a constant $36\,km\,h^{-1}$ on 1 litre of fuel; that's pretty slow but $36\,km\,h^{-1}$ is $10\,m\,s^{-1}$ so it will make the calculation easier.

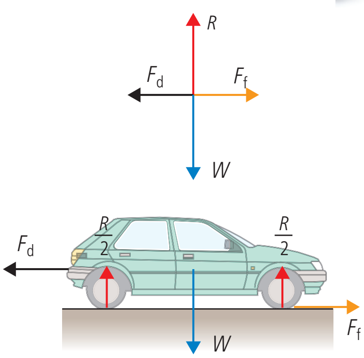


Figure 2.113 The forces acting on a car travelling at constant velocity.

The reason a car needs to use energy when travelling at a constant speed is because of air resistance. If we look at the forces acting on the car we see that there must be a constant forwards force (provided by the friction between tyres and road) to balance the air resistance or drag force.

So work is done against the drag force and the energy to do this work comes from the petrol. The amount of work done = force × distance travelled. So to calculate the work done we need to know the drag force on a car travelling at $36\,km\,h^{-1}$. One way to do this would be to drive along a flat road at a constant $36\,km\,h^{-1}$ and then take your foot off the accelerator pedal. The car would then slow down because of the unbalanced drag force.

This force will get less as the car slows down but here we will assume it is constant. From Newton's second law we know that $F = ma$ so if we can measure how fast the car slows down we can calculate the force. This will depend on the make of car but to reduce the speed by $1\,\mathrm{m\,s^{-1}}$ (about $4\,\mathrm{km\,h^{-1}}$) would take about 2 s. Now we can do the calculation:

$$\text{acceleration of car} = \frac{(v-u)}{t} = \frac{(9-10)}{2} = -0.5\,\mathrm{m\,s^{-2}}$$

$$\text{drag force} = ma = 1000 \times -0.5 = -500\,\mathrm{N}.$$

So to keep the car moving at a constant velocity this force would need to be balanced by an equal and opposite force $F = 500$ N.

Work done = force × distance so the distance travelled by the car = $\dfrac{\text{work done}}{\text{force}}$.

So if all of the energy in 1 litre of fuel is converted to work the car will move a distance = $\dfrac{36 \times 10^6}{500}$ = 72 km. Note that if you reduce the drag force on the car you increase the distance it can travel on 1 litre of fuel.

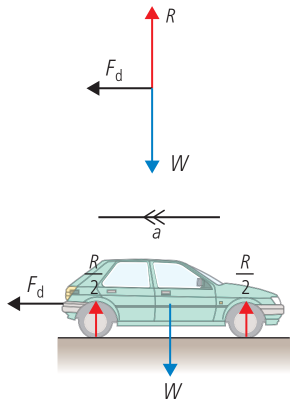


Figure 2.114 The forces on a car travelling at high speed with the foot off the accelerator pedal.

Efficiency

A very efficient road car driven carefully wouldn't be able to drive much further than 20 km on 1 litre of fuel so energy must be lost somewhere. One place where the energy is lost is in doing work against the friction that exists between the moving parts of the engine. Using oil and grease will reduce this but it can never be eliminated. The efficiency of an engine is defined by the equation

$$\textbf{efficiency} = \frac{\textbf{useful work out}}{\textbf{total energy in}}$$

so if a car travels 20 km at a speed of $10\,\mathrm{m\,s^{-1}}$ the useful work done by the engine

$$= \text{force} \times \text{distance} = 500 \times 20000 = 10\,\mathrm{MJ}.$$

The total energy put in = 36 MJ so the efficiency = $\dfrac{10}{36}$ = 0.28.

Efficiency is often expressed as a percentage so this would be 28%.

There has been a lot of research into making cars more efficient so that they use less fuel. Is this to save energy or money?

Where does all the energy go?

In this example we calculated the energy required to move a car along a flat road. The car was travelling at a constant speed so there was no increase in KE and the road was flat so there was no increase in PE. We know that energy cannot be created or destroyed so where has the energy gone? The answer is that it has been given to the particles that make up the air and car. More about that in Chapter 3.

Exercises

50 A 45% efficient machine lifts 100 kg 2 m.

(a) How much work is done by the machine?
(b) How much energy is used by the machine?

51 A 1000 kg car accelerates uniformly from rest to $100\,\mathrm{km\,h^{-1}}$.

(a) Ignoring air resistance and friction, calculate how much work was done by the car's engine.
(b) If the car is 60% efficient how much energy in the form of fuel was given to the engine?
(c) If the fuel contains 36 MJ per litre, how many litres of fuel were used?

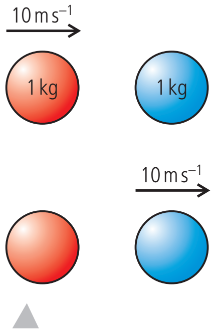


Figure 2.115 The red ball gives energy to the blue ball.

Energy and collisions

One of the reasons that we brought up the concept of energy was related to the collision between two balls as shown in Figure 2.115. We now know that if no energy is lost when the balls collide, then the KE before the collision = KE after. This enables us to calculate the velocity afterwards and the only solution in this example is quite a simple one. The red ball gives all its KE to the blue one, so the red one stops and the blue one continues, with velocity = $10\,\mathrm{m\,s^{-1}}$. If the balls become squashed, then some work needs to be done to squash them. In this case not all the KE is transferred, and we can only calculate the outcome if we know how much energy is used in squashing the balls.

Elastic collisions

An elastic collision is a collision in which both momentum and KE are conserved.

Example: two balls with equal mass

Two balls with equal mass *m* collide as shown in Figure. 2.116. As you can see, the red ball is travelling faster than the blue one before and slower after. If the collision is perfectly elastic then we can show that the velocities of the balls simply swap so $u_1 = v_2$ and $u_2 = v_1$.

Figure 2.116 Collision between 2 identical balls..

If the collision is elastic then momentum and kinetic energy are both conserved. If we consider these one at a time we get:

Conservation of momentum:

$$\text{momentum before} = \text{momentum after}$$
$$mu_1 + mu_2 = mv_1 + mv_2$$
$$u_1 + u_2 = v_1 + v_2$$

Conservation of KE:

$$\text{KE before} = \text{KE after}$$
$$\tfrac{1}{2}mu_1{}^2 + \tfrac{1}{2}mu_2{}^2 = \tfrac{1}{2}mv_1{}^2 + \tfrac{1}{2}mv_2{}^2$$
$$u_1{}^2 + u_2{}^2 = v_1{}^2 + v_2{}^2$$

So we can see that the velocities are such that both their sums are equal and the squares of their sums are equal. This is only true if the velocities swap, as in Figure 2.117.

Figure 2.117 A possible elastic collision.

Collision in 2D between 2 identical balls

Anyone who has ever played pool or snooker will know that balls don't always collide in line, they travel at *angles* to each other. Figure 2.118 shows a possible collision.

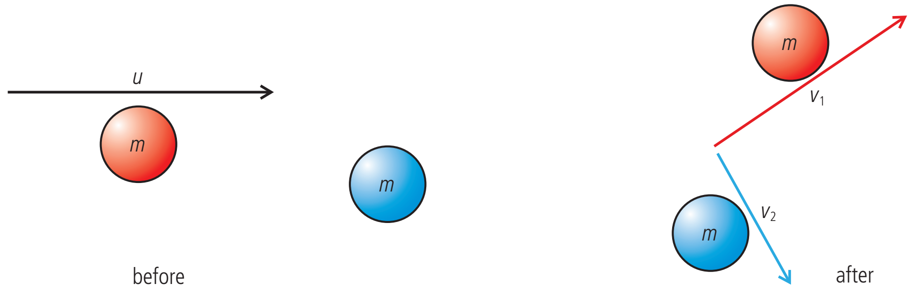


Figure 2.118 A 2D collision.

In this case the blue ball is stationary so applying the conservation laws we get slightly simpler equations:

$$\vec{u} = \vec{v}_1 + \vec{v}_2$$

$$\vec{u}^2 = \vec{v}_1{}^2 + \vec{v}_2{}^2$$

Note the vector notation to remind us that we are dealing with vectors. The first equation means that the sum of the velocity vectors after the collision gives the velocity before. This can be represented by the triangle of vectors in Figure 2.119.

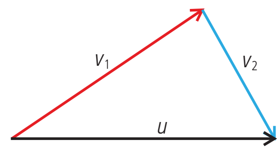


Figure 2.119 Adding the velocity vectors.

The second equation tells us that the sum of the squares of two sides of this triangle = the square of the other side. This is Pythagoras' theorem which is only true for right-angled triangles. So, after an elastic collision between two identical balls the two balls will always travel away at right angles (unless the collision is perfectly head on). This of course doesn't apply to balls rolling on a pool table since they are not isolated.

Pool balls may not collide like perfectly elastic isolated spheres but if the table is included their motion can be accurately modelled enabling scientists to calculate the correct direction and speed for the perfect shot. Taking that shot is another matter entirely.

Inelastic collisions

There are many outcomes of an inelastic collision but here we will only consider the case when the two bodies stick together. We call this *totally inelastic collision*.

Example

When considering the conservation of momentum in collisions, we used the example shown in Figure 2.120. How much work was done to squash the balls in this example?

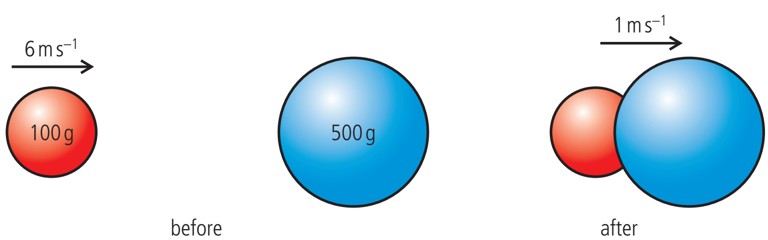


Figure 2.120.

According to the law of conservation of energy, the work done squashing the balls is equal to the loss in KE.

$$\text{KE loss} = \text{KE before} - \text{KE after} = \tfrac{1}{2} \times 0.1 \times 6^2 - \tfrac{1}{2} \times 0.6 \times 1^2$$

$$\text{KE loss} = 1.8 - 0.3 = 1.5\,\text{J}$$

So $\quad$ work done = 1.5 J

Explosions

Explosions can never be elastic since, without doing work, the parts that fly off after the explosion would not have any KE and would therefore not be moving. The energy to initiate an explosion often comes from the chemical energy contained in the explosive.

Example

Again consider a previous example where a ball exploded (shown again in Figure 2.121). How much energy was supplied to the balls by the explosive?

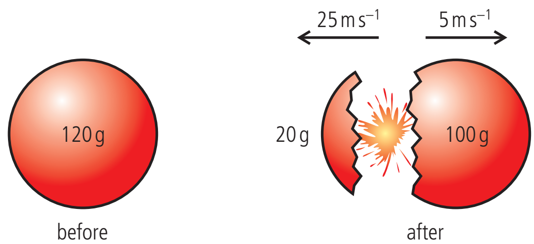


Figure 2.121.

According to the law of conservation of energy, the energy from the explosive equals the gain in KE of the balls.

$$\text{KE gain} = \text{KE after} - \text{KE before}$$

$$\text{KE gain} = (\tfrac{1}{2} \times 0.02 \times 25^2 + \tfrac{1}{2} \times 0.1 \times 5^2) - 0 = 6.25 + 1.25 = 7.5\,\text{J}$$

Sharing of energy

The result of this example is very important; we will use it when dealing with nuclear decay later on. So remember, when a body explodes into two unequal bits, the small bit gets most energy.

Exercises

52 Two balls are held together by a spring as shown in Figure 2.122. The spring has a spring constant of 10 N cm^{-1} and has been compressed a distance 5 cm.

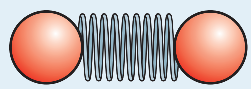


Figure 2.122.

(a) How much work was done to compress the spring?
(b) How much KE will each gain?
(c) If each ball has a mass of 10 g calculate the velocity of each ball.

53 Two pieces of modelling clay as shown in Figure 2.123 collide and stick together.

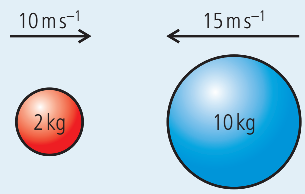


Figure 2.123.

(a) Calculate the velocity of the lump after the collision.
(b) How much KE is lost during the collision?

54 A red ball travelling at 10 m s^{-1} to the right collides with a blue ball with the same mass travelling at 15 m s^{-1} to the left. If the collision is elastic, what are the velocities of the balls after the collision?

CHALLENGE YOURSELF

2 A 200 g red ball travelling at 6 m s^{-1} collides with a 500 g blue ball at rest, such that after the collision the red ball travels at 4 m s^{-1} at an angle of 45° to its original direction. Calculate the speed of the blue ball.

Power

We know that to do work requires energy, but work can be done quickly or it can be done slowly. This does not alter the energy exchanged but the situations are certainly different. For example we know that to lift one thousand 1 kg bags of sugar from the floor to the table is not an impossible task – we can simply lift them one by one. It will take a long time but we would manage it in the end. However, if we were asked to do the same task in 5 seconds, we would either have to lift all 1000 kg at the same time or move each bag in 0.005 s, both of which are impossible. Power is the quantity that distinguishes between these two tasks.

Power and velocity

If power = $\frac{\text{work done}}{\text{time}}$

then we can also write

$$P = \frac{F\Delta s}{t}$$

so $$P = F\frac{\Delta s}{t}$$

which is the same as

$$P = Fv$$

where v is the velocity.

Power is defined as:

power = work done per unit time.

Unit of power is the $\mathrm{J\,s^{-1}}$ which is the same as the watt (W).

Power is a scalar quantity.

Example 1: The powerful car

We often use the term power to describe cars. A powerful car is one that can accelerate from 0 to $100\,\mathrm{km\,h^{-1}}$ in a very short time. When a car accelerates, energy is being converted from the chemical energy in the fuel to KE. To have a big acceleration the car must gain KE in a short time, hence be powerful.

Example 2: Power lifter

A power lifter is someone who can lift heavy weights, so shouldn't we say they are strong people rather than powerful? A power lifter certainly is a strong person (if they are good at it) but they are also powerful. This is because they can lift a big weight in a short time.

Worked example

A car of mass 1000 kg accelerates from rest to $100\,\mathrm{km\,h^{-1}}$ in 5 seconds. What is the average power of the car?

Solution

$$100\,\mathrm{km\,h^{-1}} = 28\,\mathrm{m\,s^{-1}}.$$

$$\text{The gain in KE of the car} = \tfrac{1}{2}mv^2 = \tfrac{1}{2} \times 1000 \times 28^2 = 392\,\mathrm{kJ}$$

If the car does this in 5 s then

$$\text{power} = \frac{\text{work done}}{\text{time}} = \frac{392}{5} = 78.4\,\mathrm{kW}$$

Horse power

Horse power is often used as the unit for power when talking about cars and boats.

746 W = 1 hp

So in Worked example 1, the power of the car is 105 hp.

Exercises

55 A weightlifter lifts 200 kg 2 m above the ground in 5 s. Calculate the power of the weightlifter in watts.

56 In 25 s a trolley of mass 50 kg runs down a hill. If the difference in height between the top and the bottom of the hill is 50 m, how much power will have been dissipated?

57 A car moves along a road at a constant velocity of $20\,\mathrm{m\,s^{-1}}$. If the resistance force acting against the car is 1000 N, what is the power developed by the engine?

Efficiency and power

We have define efficiency by the equation:

$$\text{efficiency} = \frac{\text{useful work out}}{\text{work in}}$$

If the work out is done at the same time as the work in then we can also write:

$$\text{efficiency} = \frac{\text{useful power out}}{\text{total power in}}$$

Exercises

58 A motor is used to lift a 10 kg mass 2 m above the ground in 4 s. If the power input to the motor is 100 W, what is the efficiency of the motor?

59 A motor is 70% efficient. If 60 kJ of energy is put into the engine, how much work is got out?

60 The drag force that resists the motion of a car travelling at $80\,\mathrm{km\,h^{-1}}$ is 300 N.

(a) What power is required to keep the car travelling at that speed?
(b) If the efficiency of the engine is 60%, what is the power of the engine?

Practice questions

1. This question is about linear motion.

A police car P is stationary by the side of a road. A car S, exceeding the speed limit, passes the police car P at a constant speed of $18\,\mathrm{m\,s^{-1}}$. The police car P sets off to catch car S just as car S passes the police car P. Car P accelerates at $4.5\,\mathrm{m\,s^{-2}}$ for a time of 6.0 s and then continues at constant speed. Car P takes a time *t* seconds to draw level with car S.

(a) (i) State an expression, in terms of *t*, for the distance car S travels in *t* seconds. (1)

(ii) Calculate the distance travelled by the police car P during the first 6.0 seconds of its motion. (1)

(iii) Calculate the speed of the police car P after it has completed its acceleration. (1)

(iv) State an expression, in terms of *t*, for the distance travelled by the police car P during the time that it is travelling at constant speed. (1)

(b) Using your answers to (a), determine the total time *t* taken for the police car P to draw level with car S. (2)

(*Total 6 marks*)

2. This question is about the kinematics of an elevator (lift).

(a) Explain the difference between the gravitational mass and the inertial mass of an object. (3)

An elevator (lift) starts from rest on the ground floor and comes to rest at a higher floor. Its motion is controlled by an electric motor. Figure 2.124 is a simplified graph of the variation of the elevator's velocity with time.

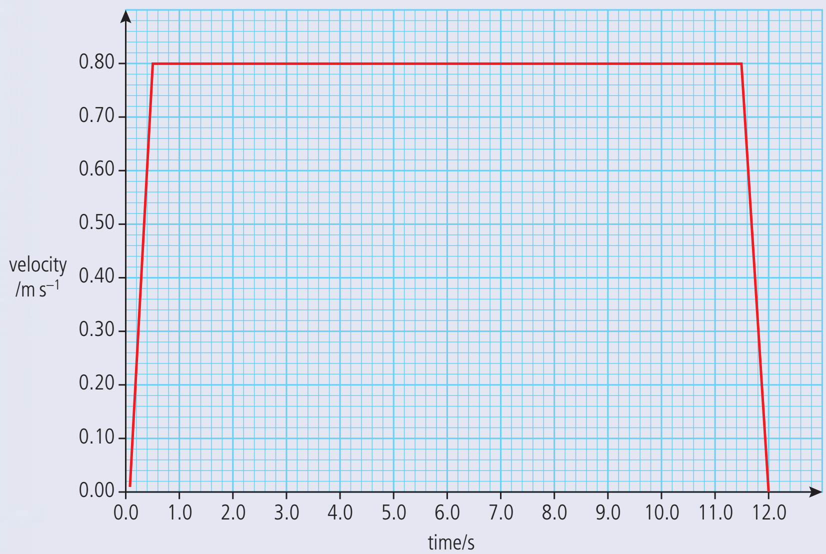


Figure 2.124.

(b) The mass of the elevator is 250 kg. Use this information to calculate:

(i) the acceleration of the elevator during the first 0.50 s. (2)

(ii) the total distance travelled by the elevator. (2)

(iii) the minimum work required to raise the elevator to the higher floor. (2)

(iv) the minimum average power required to raise the elevator to the higher floor. (2)

(v) the efficiency of the electric motor that lifts the elevator, given that the input power to the motor is 5.0 kW. (2)

(c) Copy Figure 2.125 and, on the graph axes, sketch a realistic variation of velocity for the elevator. Explain your reasoning. *(The simplified version is shown as a dotted line.)*

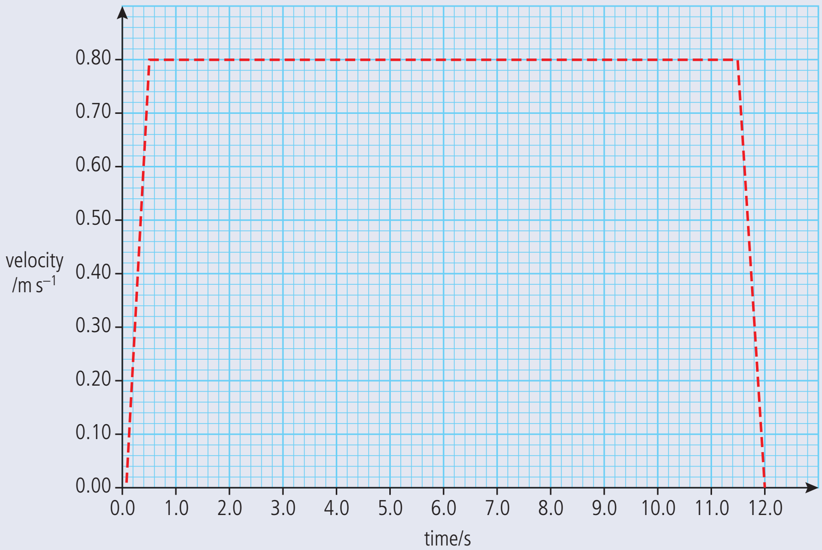


Figure 2.125.

(2)

The elevator is supported by a cable. Figure 2.126 is a free body force diagram for when the elevator is moving upwards during the first 0.50 s.

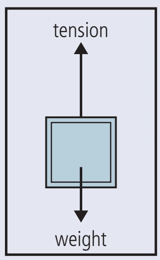


Figure 2.126.

(d) Copy Figures 2.127 and 2.128 and in the space below each box, draw free body force diagrams for the elevator during the following time intervals.

(i) 0.5 to 11.50 s

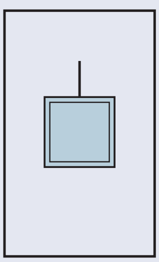


Figure 2.127.

(ii) 11.50 to 12.00 s

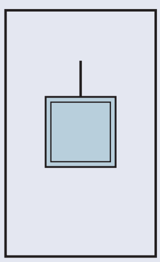


Figure 2.128.

(3)

A person is standing on weighing scales in the elevator. Before the elevator rises, the reading on the scales is *W*.

(e) Copy Figure 2.129 and on the axes, sketch a graph to show how the reading on the scales varies during the whole 12.00 s upward journey of the elevator. *(Note that this is a sketch graph – you do not need to add any values.)*

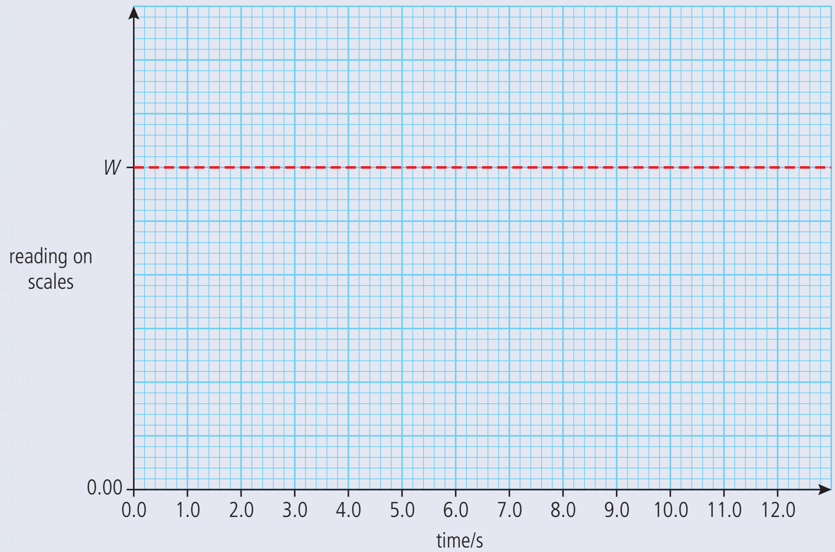


Figure 2.129.

(3)

(f) The elevator now returns to the ground floor where it comes to rest. Describe and explain the energy changes that take place during the whole up-and-down journey. (4)

(*Total 25 marks*)

3. This question is about throwing a stone from a cliff. Antonia stands at the edge of a vertical cliff and throws a stone vertically upwards.

The stone leaves Antonia's hand with a speed $v = 8.0\,\mathrm{m\,s^{-1}}$.

The acceleration of free fall g is $10\,\mathrm{m\,s^{-2}}$ and all distance measurements are taken from the point where the stone leaves Antonia's hand.

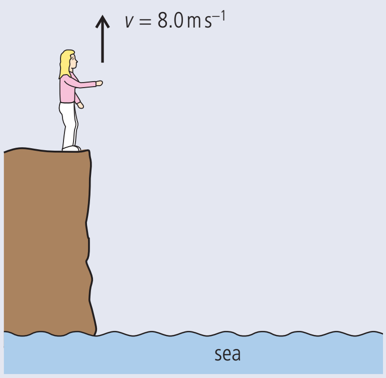


Figure 2.130.

(a) Ignoring air resistance calculate:

(i) the maximum height reached by the stone. (2)

(ii) the time taken by the stone to reach its maximum height. (1)

The time between the stone leaving Antonia's hand and hitting the sea is 3.0 s.

(b) Determine the height of the cliff. (3)

(*Total 6 marks*)

4. This question is about conservation of momentum and conservation of energy.

(a) State Newton's third law. (1)

(b) State the law of conservation of momentum. (2)

Figure 2.131 shows two identical balls A and B on a horizontal surface. Ball B is at rest and ball A is moving with speed V along a line joining the centres of the balls. The mass of each ball is M.

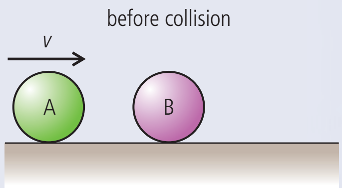


Figure 2.131.

During the collision of the balls, the magnitude of the force that ball A exerts on ball B is F_{AB} and the magnitude of the force that ball B exerts on ball A is F_{BA}.

(c) Copy Figure 2.132 and on the diagram, add labelled arrows to represent the magnitude and direction of the forces F_{AB} and F_{BA}.

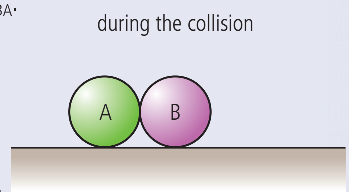


Figure 2.132.

(3)

The balls are in contact for a time Δt. After the collision, the speed of ball A is $+v_A$ and the speed of ball B is $+v_B$ in the directions shown in Figure 2.133.

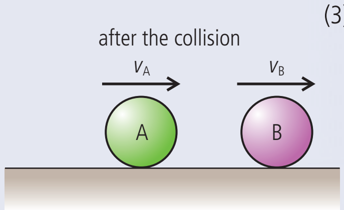


Figure 2.133.

As a result of the collision, there is a change in momentum of ball A and of ball B.

(d) Use Newton's second law of motion to deduce an expression relating the forces acting during the collision to the change in momentum of

(i) ball B. (2)

(ii) ball A. (2)

(e) Apply Newton's third law and your answers to (d), to deduce that the change in momentum of the system (ball A and ball B) as a result of this collision, is zero. (4)

(f) Deduce, that if kinetic energy is conserved in the collision, then after the collision ball A will come to rest and ball B will move with speed V. (3)

5. This question is about the collision between two railway trucks (carts).

(a) Define *linear momentum*. (1)

In Figure 2.134, railway truck A is moving along a horizontal track. It collides with a stationary truck B and on collision, the two join together. Immediately before the collision, truck A is moving with speed $5.0\,\mathrm{m\,s^{-1}}$. Immediately after collision, the speed of the trucks is v.

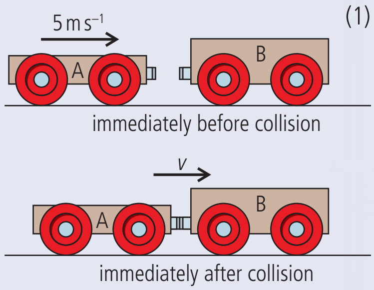


Figure 2.134.

The mass of truck A is 800 kg and the mass of truck B is 1200 kg.

(b) (i) Calculate the speed v immediately after the collision. (3)

(ii) Calculate the total kinetic energy lost during the collision. (2)

(c) Suggest what has happened to the lost kinetic energy. (2)

(*Total 8 marks*)

6. This question is about estimating the energy changes for an escalator (moving staircase).

Figure 2.135 represents an escalator. People step on to it at point A and step off at point B.

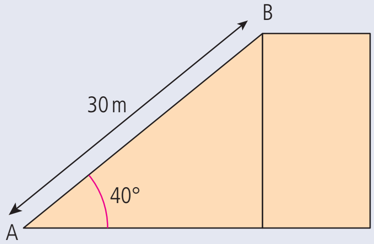


Figure 2.135.

(a) The escalator is 30 m long and makes an angle of 40° with the horizontal. At full capacity, 48 people step on at point A and step off at point B every minute.

(i) Calculate the potential energy gained by a person of weight 700 N in moving from A to B. (2)

(ii) Estimate the energy supplied by the escalator motor to the people every minute when the escalator is working at full capacity. (1)

(iii) State **one** assumption that you have made to obtain your answer to (ii). (1)

The escalator is driven by an electric motor that has an efficiency of 70%.

(b) **(i)** Using your answer to (a)(ii), calculate the minimum input power required by the motor to drive the escalator. (3)

(ii) Explain why it is not necessary to take into account the weight of the escalator when calculating the input power. (1)

(c) Explain why in practice, the power of the motor will need to be greater than that calculated in (b)(i). (1)

(*Total 9 marks*)

7. This question is about collisions.

(a) State the principle of conservation of momentum. (2)

(b) In an experiment, an air-rifle pellet is fired into a block of modelling clay that rests on a table.

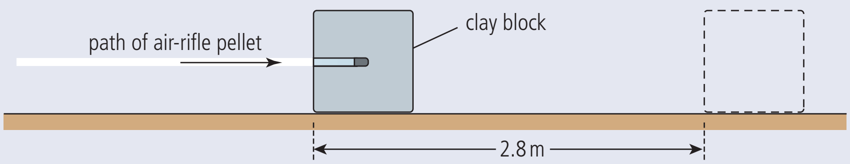


Figure 2.136. (not to scale)

The air-rifle pellet remains inside the clay block after the impact.

As a result of the collision, the clay block slides along the table in a straight line and comes to rest. Further data relating to the experiment are given below.

Mass of air-rifle pellet = 2.0 g
Mass of clay block = 56 g
Velocity of impact of air-rifle pellet = 140 m s^{-1}
Stopping distance of clay block = 2.8 m

(i) Show that the initial speed of the clay block after the air-rifle pellet strikes it is 4.8 m s^{-1}. (2)

(ii) Calculate the average frictional force that the surface of the table exerts on the clay block whilst the clay block is moving. (4)

(c) The experiment is repeated with the clay block placed at the edge of the table so that it is fired away from the table. The initial speed of the clay block is 4.3 m s^{-1} horizontally. The table surface is 0.85 m above the ground.

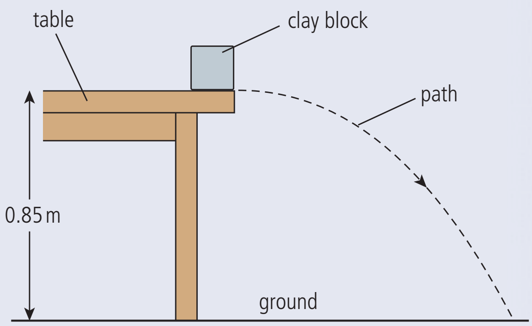


Figure 2.137.
(not to scale)

(i) Ignoring air resistance, calculate the horizontal distance travelled by the clay block before it strikes the ground. (4)

(ii) Figure 2.137 shows the path of the clay block neglecting air resistance. Copy the diagram, and on it, draw the approximate shape of the path that the clay block will take assuming that air resistance acts on the clay block. (3)

(*Total 15 marks***)**