Chapter 1: Measurement and Uncertainty

Section 1: Observation, Describing quantities, and Estimation

Section 2: Measurement uncertainty, Error propagation, and Analytical methods

Section 1 Objectives

Core Principles

- Physics and Observation
- Fundamental and derived SI units
- Scientific notation and metric multipliers
- Significant figures
- Orders of magnitude
- Estimation

Applications and skills:

- Using SI units in the correct format for all required measurements, final answers to calculations and presentation of raw and processed data
- Using scientific notation and metric multipliers
- Quoting and comparing ratios, values and approximations to the nearest order of magnitude
- Estimating quantities to an appropriate number of significant figures

ToK and Aims

Theory of knowledge:

What has influenced the common language used in science? To what extent does having a common standard approach to measurement facilitate the sharing of knowledge in physics?

Utilization:

This topic is able to be integrated into any topic taught at the start of the course and is important to all topics

Students studying more than one group 4 subject will be able to use these skills across all subjects

Aims:

Aim 2 and 3: this is an essential area of knowledge that allows scientists to collaborate across the globe

Aim 4 and 5: a common approach to expressing results of analysis, evaluation and synthesis of scientific information enables greater sharing and collaboration

What is Physics?

Science

- Science is the attempt to understand and describe the universe. (Physics ≈ Grammar, Math ≈ Language).
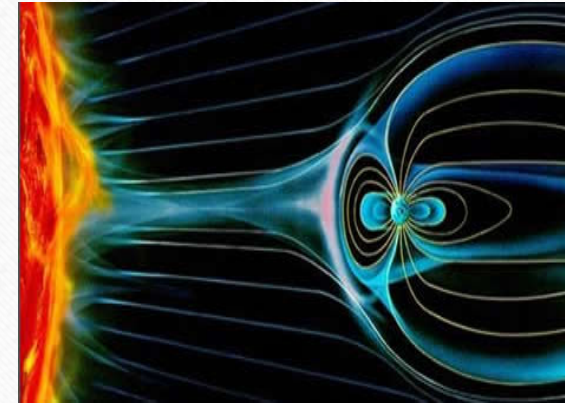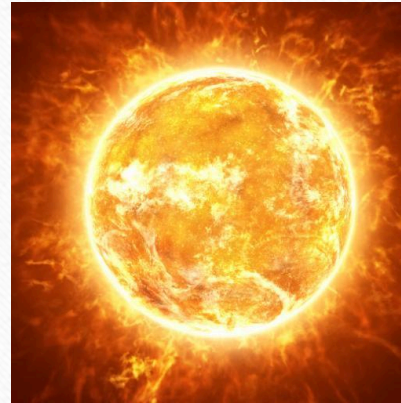
Physics

- Physics may be generally described as the study of forces and energy, and matter's reaction to these in space and time.

Fundamental Forces in Physics

4 Forces Strongest to Weakest

- Strong Nuclear Force
- Electromagnetic
- Weak Nuclear Force
- Gravity

		Strength	Range (m)	Particle
Strong	Force which holds nucleus togeher	1	10^{-15} (diameter of a medium sized nucleus)	gluons, π(nucleons)
Electro-magnetic		$\frac{1}{137}$	Infinite	photon mass = 0 spin = 1
Weak	neutrino interaction induces beta decay	10^{-6}	10^{-18} (0.1% of the diameter of a proton)	Intermediate vector bosons W^+, W^-, Z_0, mass > 80 GeV spin =1
Gravity		6×10^{-39}	Infinite	graviton ? mass = 0 spin = 2
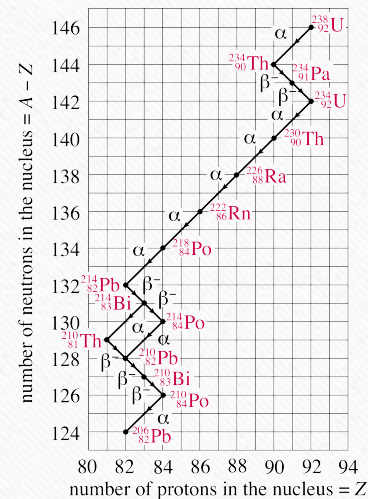
Observation and Measurement

(1) <u>Common terminology</u>: Since the 18th century, scientists have sought to establish common systems of measurements to facilitate international collaboration across science disciplines and ensure replication and comparability of experiments.

(2) <u>Definition of physical quantities</u>. Physics must carefully define the various aspects of the universe we wish to quantify. Then we need units or standards by which to measure these quantities. There are seven fundamental quantities for which we have standard units:

- Mass measured in kilograms (kg)—defines the amount of matter or inertial quantity of matter.

- Length measured in meters (m)—defines the space in which some event occurs or the distance/size of some object/space

- Time measured in seconds (s)—defines the duration over which an event takes place

- Temperature measured in Kelvin degrees (K)—defines the average kinetic energy of the molecules of a substance

- Electric Charge measured in Coulombs (C)—defines the amount of electric charge a particle or object possesses

- Luminosity measured in Candelas (cd)—defines the relative brightness of an object

- Amount of substance measured in moles (mol)—defines how many atoms of a substance exist for a given material

(3) <u>Derived</u> Units: Any unit created by algebraic combinations of these fundamental units creates a derived unit. Ex velocity = distance/time (m/s)

(4) <u>Certainty</u>: Although scientists are perceived as working towards finding "exact" answers, there is unavoidable uncertainty in **any** measurement. No measurement is infinitely precise therefore all quantities have error associated with them.

Essential idea:

- Since 1948, the *Système International d'Unités* (SI) has been used as the preferred language of science and technology across the globe and reflects current best measurement practice.

Dimensional Analysis

- By analyzing the algebra performed to generate a quantity and comparing the known unit for the quantity to the units obtained by that algebra we can confirm the correctness of our equations.

- Note: IB standard for metric units is to place all units in the numerator and to use negative exponents, rather than the fractional notation. Examples of both are provided below.

Ex: force has units of kgms^{-2} (or $kgm/s\uparrow 2$) then if we have an equation $F=mass\times length\uparrow 2 /time\uparrow 2$ is this the correct equation for calculating force?

Ex: Suppose the rate of a car is 12m/s, and it travels for 5 seconds. What is the distance traveled in that time by the car? (note that rate refers to velocity in ms^{-1})

PRACTICE: Decide if the formulas are dimensionally consistent. The information you need is that v is measured in m/s, a is in m/s^2, x is in m and t is in s.

   (a) $v = at^2$ (b) $v^2 = 3ax$ (c) $x = at^2$

Unit Conversions and Powers of 10

- Since the United States (and England) use a different set of units for their measurement systems it is frequently necessary to convert between these units and the metric system. It is also necessary to perform unit conversions between different units when they are expressed in very large or very small units of time.

- Particularly, time is measured as base seconds but 1min=60seconds and 1hr=60mins and 1day=24hrs and 1year=365days, etc.

Then make the following conversion:

Ex: how many minutes are in 1 year?

Ex: how many seconds are in the average human lifespan? (approx. 75yrs)

- Physics encompasses a vast range of values though. Distances on earth are commonly measured using meters or, at most, units which consist of 1000m, the kilometer. In space however, distances are so large (the distance between stars for example) that it is near meaningless to use meters to describe these distances so the distance light travels in a year the lightyear is used instead.

Ex: If light moves at $3(10^8)ms^{-1}$ calculate how many meters are in 1 lightyear (ly).

- In order to make writing and working with these numbers more efficient a base 10 prefix system has been developed to make writing and converting between units more concise. Each prefix is an abbreviation for a power of 10 multiplier.

Prefix System

Power of 10	Prefix Name	Symbol
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	µ
10^{-3}	milli	m
10^{-2}	centi	c
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T

Ex: $9,000,000m = 9(10^6)m = 9Mm$
Ex: $0.000000136s = 1.36(10^{-7})s = 1.36(10^{-6})(10^{-1})s = .136\mu s$
OR
$0.000000136s = 1.36(10^{-7})s = 1.36(10^{-9})(10^2)s = 136(10^{-9})s = 136ns$

Memorize this table! You will be responsible for these prefixes on quizzes and exams.

Then we can convert between these prefixes by division and multiplication by powers of 10.
Ex: Convert 25mm into cm.
Ex: Convert 2Gbytes into kbytes
Homework Question: Why is the base unit for mass referred to as the kilogram, instead of the gram and what problems does this introduce? **Be prepared to answer for a bellwork quiz.**

Scientific Notation

- Given the very large and very small values in physics it is also useful to represent numbers as their significant figures multiplied by their powers of 10.

- This practice greatly helps to keep track of significant figures throughout a calculation and makes the mathematics easier for working with extremely large and small quantities in a problem.

- Form: $A(10^B)$ where A is some number coefficient with a certain number of significant figures and B is an integer.

- Note, do **not** write your scientific notation using $Ax10^B$.

Ex: Put the number 345,000,000mol into scientific notation, then use the proper prefix.

Ex: Put the number 100400m into scientific notation, then use the proper prefix.

Standard decimal notation	Normalized scientific notation
2 s	2×10^0 s
300 s	3×10^2 s
4,321.768 s	$4.321\,768 \times 10^3$ s
−53,000 s	$−5.3 \times 10^4$ s
6,720,000,000 s	6.72×10^9 s
0.2 s	2×10^{-1} s
0.000 000 007 51 s	7.51×10^{-9} s

Significant Figures and Measuring Devices

- Error in measurement is expected because of the imperfect nature of our measuring devices.

- **Significant figures** are the reasonable number of digits that a measurement or calculation should have.

- Ex: A typical meter stick has marks at every millimeter (10^{-3} m or 1/1000 m). Thus the best measurement you can get from a typical meter stick is to the nearest mm.

- Such is always true of analog devices or devices whose measurements depend on a physical observation or marking which is manually read.

- A **digital** measuring device, on the other hand, is only "good" to the least significant digit's place.

How long is this line?
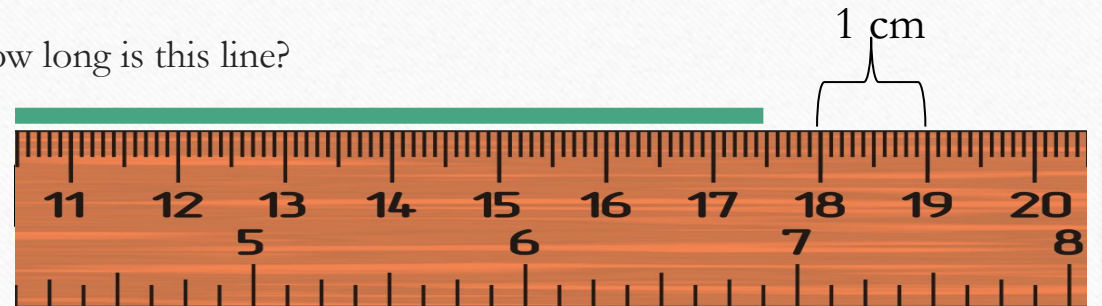
What precision do we have using this scale? Then how many significant figures can our measurement have?

What is the number of significant figures on this digital multimeter?

Rules for Significant Figures

(**1**) *All non-zero digits are significant.*

438 g	3
26.42 m	4
0.75 cm	2

(**2**) *All zeros between non-zero digits are significant.*

12060 m	4
900.43 cm	5

(**3**) *Filler zeros to the left of an understood decimal place are not significant.*

220 L	2
60 g	1
30. cm	2

(**4**) *Filler zeros to the right of a decimal place are not significant.*

0.006 L	1
0.08 g	1

(**5**) *All non-filler zeros to the right of a decimal place are significant.*

8.0 L	2
60.40 g	4

EXAMPLE   **CALCULATOR** SIG. FIGS

• **Multiplication and division** – *round your answer to the same number of <u>significant digits</u> as the quantity with the fewest number of significant digits.*

$(1.2 \text{ cm})(2 \text{ cm})$	2.4 cm^2	2 cm^2
$\pi(2.75 \text{ cm})^2$	7.5625 cm^2	7.56 cm^2
5.350 m/2.752 s	1.944040698 m/s	1.944 m/s
(0.0075 N)(6 m)	0.045 Nm	0.04 Nm

• **Addition and subtraction** – *round your answer to the same number of <u>decimal places</u> as the quantity with the fewest number of decimal places.*

1.2 cm + 2 cm	3.2 cm	3 cm
2000 m+2.1 m	2002.1 m	2000 m
0.00530 m – 2.10 m	-2.0947 m	-2.09 m

Orders of Magnitude and Estimation

- As previously discussed there is tremendous difference in the size of quantities observed in the universe.

Mass of universe	10^{50}	kg
Diameter of universe	10^{25}	m
Diameter of galaxy	10^{21}	m
Planck Length	10^{-35}	m
Speed of light	10^{8}	m s^{-1}
Diameter of atom	10^{-10}	m
Diameter of nucleus	10^{-15}	m
Diameter of quark	10^{-18}	m
Mass of proton	10^{-27}	kg
Mass of quark	10^{-30}	kg
Mass of electron	10^{-31}	kg

Ex: Given that the smallest length in the universe is the Planck length of 10^{-35} meters and that the fastest speed in the universe is that of light at 10^{8} meters per second, find the smallest time interval in the universe.

Ex: Find the difference in order of magnitude of the mass of the universe to the mass of a quark.

Ex: For nuclear diameter 10^{-15} and atomic diameter 10^{-10} answer the following: The ratio $\dfrac{\text{diameter of a nucleus}}{\text{diameter of an atom}}$ is approximately equal to

A. 10^{-15}. B. 10^{-8}. C. 10^{-5}. D. 10^{-2}.

Uncertainty in Measurement

- Scientists aim towards designing experiments that can give a "true value" from their measurements, but due to the limited precision in measuring devices, they often quote their results with some form of uncertainty.

- Uncertainties: "All scientific knowledge is uncertain. When the scientist tells you he does not know the answer, he is an ignorant man. When he tells you he has a hunch about how it is going to work, he is uncertain about it. When he is pretty sure of how it is going to work, he still is in some doubt. And it is of paramount importance, in order to make progress, that we recognize this ignorance and this doubt. Because we have the doubt, we then propose looking in new directions for new ideas."

  – Feynman, Richard P. 1998. The Meaning of It All: Thoughts of a Citizen-Scientist. Reading, Massachusetts, USA. Perseus. P 13.

- "One aim of the physical sciences has been to give an exact picture of the material world. One achievement of physics in the twentieth century has been to prove that this aim is unattainable."

  – Jacob Bronowski

ToK and Aims

Theory of knowledge: To what extent can we, as scientists and individuals, ever be certain?

Utilization:

- Students studying more than one group 4 subject will be able to use these skills across all subjects

Aims:

- Aim 4: it is important that students see scientific errors and uncertainties not only as the range of possible answers but as an integral part of the scientific process

- Aim 9: the process of using uncertainties in classical physics can be compared to the view of uncertainties in modern (and particularly quantum) physics

Section 2 Objectives

Core Principles

- Random and systematic errors
- Absolute, fractional and percentage uncertainties
- Error bars
- Uncertainty of gradients and intercepts

Applications and skills:

• Explaining how random and systematic errors can be identified and reduced

• Collecting data that include absolute and/or fractional uncertainties and stating these as an uncertainty range (expressed as: best estimate ± uncertainty range)

• Propagating uncertainties through calculations involving addition, subtraction, multiplication, division and raising to a power

• Determining the uncertainty in gradients and intercepts

Data Book References

Note: Analysis of uncertainties will not be expected for trigonometric or logarithmic functions in examinations

Data booklet reference: (grow comfortable using these data booklets for quick reference, you will have to use them during the IB exam)
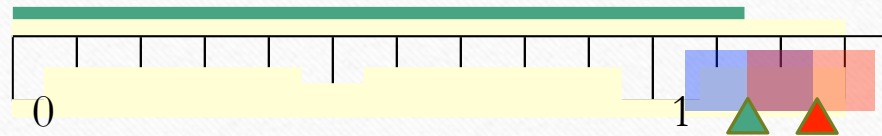
If $y = a \pm b$ then $\Delta y = \Delta a + \Delta b$

If $y = a \cdot b / c$ then $\Delta y / y = \Delta a / a + \Delta b / b + \Delta c / c$

If $y = a^n$ then $\Delta y / y = | n \cdot \Delta a / a |$

Good Uncertainty Example

- Ex: Suppose **Bob** measures $L = 11$ mm ± 1 mm and **Ann** measures $L = 12$ mm ± 1 mm with the ruler below. Because we use a measuring device with a least increment of 1mm or 10^{-3}m, we say the **precision** or **uncertainty** in our measurement is ± 1 mm.

- Then **Bob** guarantees that the line falls between 10 mm and 12 mm.

- **Ann** guarantees it is between 11 mm and 13 mm.

- **Both** are absolutely correct.

Bad Uncertainty Example

The diagram below shows the position of the meniscus of the mercury in a mercury-in-glass thermometer.
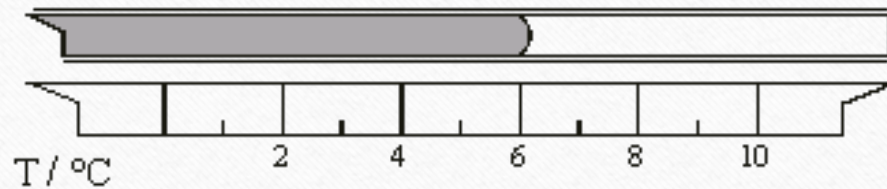
Which of the following best expresses the indicated temperature with its uncertainty?

A. $(6.0 \pm 0.5)°C$ C. $(6.2 \pm 0.2)°C$

B. $(6.1 \pm 0.1)°C$ D. $(6.2 \pm 0.5)°C$

SOLUTION GIVEN:

• 6.2 is the nearest reading.

• The uncertainty is certainly less than 0.5.

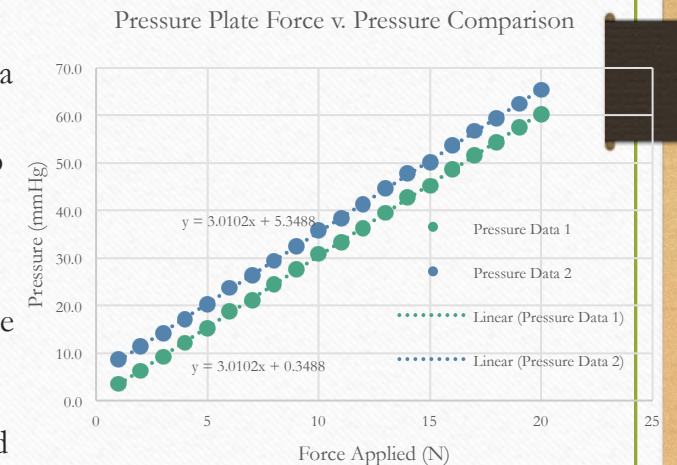Problems: What is wrong with this?

Types of Error

Random

- Random error originates from imprecision of measurement and observation of measurement

- Compounded by rapidly fluctuating systems.

- Most commonly observed error in data.

- Random error is visualized by the error bars of a scatter plot.

- Minimized by making repeated measures to find an average and its standard deviation.

Scatterplot with Errorbars
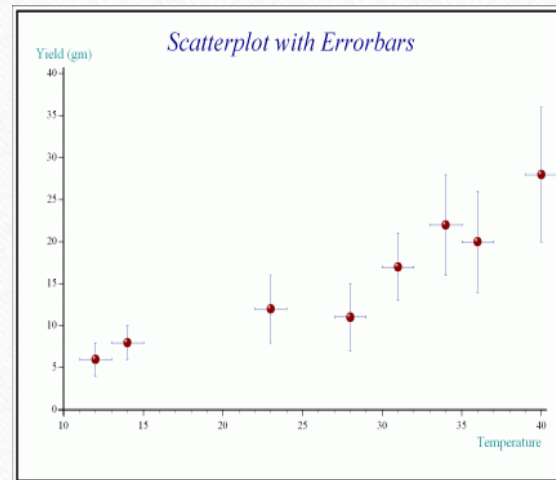
Systematic

- Systematic error originates from a baseline or calibration error.

- Often due to experimental set up or measurement device offset.

- More difficult to observe unless the observer has a very good model or verified data to compare results.

- Graphically can be observed in a consistently higher or lower trend line than your model function.

- Systematic error can be corrected for by using data normalization techniques

Pressure Plate Force v. Pressure Comparison

$y = 3.0102x + 5.3488$

$y = 3.0102x + 0.3488$

Pressure Data 1

Pressure Data 2

Linear (Pressure Data 1)

Linear (Pressure Data 2)
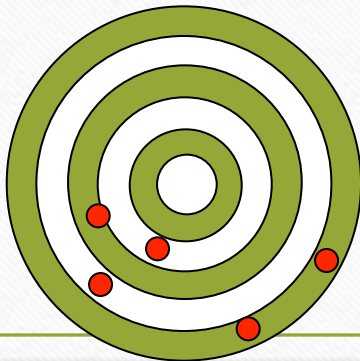
Precision v. Accuracy

Accurate

- Refers to the closeness of the data/result to the expected true value.
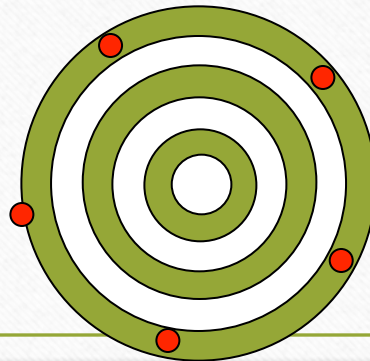
Precise

- Refers to the closeness of the experimental data to other data points within the sample.

Ex: Dart board. The objective is to throw 5 darts and hit a bullseye. The results of 4 rounds are plotted below, discuss the accuracy v. precision of each round and then relate to random v. systematic error.
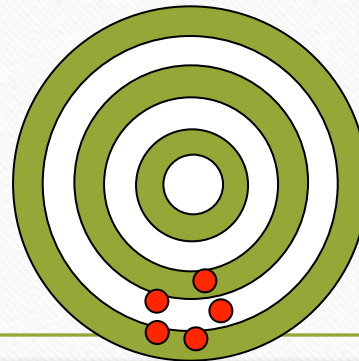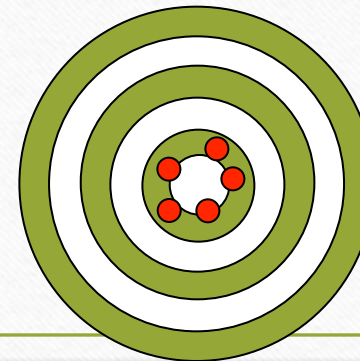
Trial 1   Trial 2   Trial 3   Trial 4

Sample Questions

An ammeter has a zero offset error. This fault will affect
A. neither the precision nor the accuracy of the readings.
B. only the precision of the readings.
C. only the accuracy of the readings.
D. both the precision and the accuracy of the readings.

The reading of a constant potential difference is made four times by a student. The readings are

1.176 V
1.178 V
1.177 V
1.176 V.

The student averages these readings but does not take into account the zero error on the voltmeter.

The average measurement of the potential difference is

A. precise and accurate.

B. precise but not accurate.

C. accurate but not precise.

D. not accurate and not precise.

When a voltage V of 12.2 V is applied to a DC motor, the current I in the motor is 0.20 A. Which **one** of the following is the output power VI of the motor given to the correct appropriate number of significant digits?

A. 2 W

B. 2.4 W

C. 2.40 W

D. 2.44 W

The masses and weights of different objects are independently measured. The graph is a plot of weight versus mass that includes error bars.
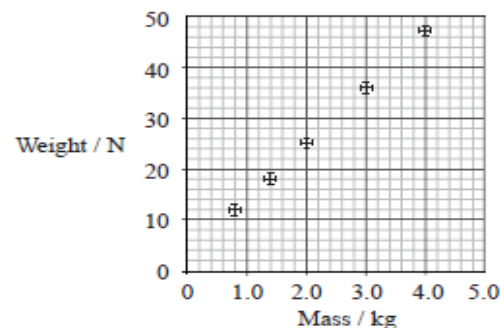
These experimental results suggest that

A. the measurements show a significant systematic error but small random error.

B. the measurements show a significant random error but small systematic error.

C. the measurements are precise but not accurate.

D. the weight of an object is proportional to its mass.

Representing Uncertainty/Error

- 3 main ways: Absolute uncertainty, fractional uncertainty, and percentage uncertainty

- **Absolute error** is the raw uncertainty or precision of your measurement. Ex: 15cm $\pm$ 2cm; 2cm is the absolute uncertainty. The expected true value of the measurement could be anywhere from13 to 17cm. If you are provided no knowledge of your instrument but only some experimental data values you can determine the raw error by taking the range of the data (the largest value –the smallest value)/2 .

- **Fractional error** is the ratio of the absolute or raw error to the measurement. Ex: 15cm $\pm$ 2cm is the absolute error, then $2/15 = 0.13$ is the fractional uncertainty.

- **Percent error** is the fractional uncertainty represented as a percentage of the measurement. Ex: 15cm $\pm$ 2cm is the raw error, $2/15 = 0.13$ is the fractional error, then $2/15 \times 100\% = 13\%$ is the percent uncertainty

ToK Question: why is the percent error useful compared to the other two?

Error Propagation Calculations

- Error never decreases in a system. When a formula involving arithmetic operations occurs on values with error that error will increase through the operation in a way described by the following formulas:

If $y = a \pm b$ then $\Delta y = \Delta a + \Delta b$

If $y = a \cdot b / c$ then $\Delta y / y = \Delta a / a + \Delta b / b + \Delta c / c$

If $y = a^n$ then $\Delta y / y = | n \cdot \Delta a / a |$

To find the **uncertainty in a sum or difference** you just <u>add</u> the uncertainties of all the ingredients.

To find the **uncertainty in a product or quotient** you just add the percentage or fractional uncertainties of all the ingredients.

Remember that exponents are merely a concise form of multiplication then your formula is also a concise version of the product uncertainty formula.

Ex: If A=3.5±0.5m; B=6.2±0.6m; C=10.1±1.5m; D=5.5±0.2m, calculate the resulting value including uncertainty in raw, fractional, and percent forms for the resulting value using the formula provided above.

1) A+B-C
2) C·D÷B
3) A^3
4) B-A·C^2

More Practice

A student measures a distance several times. The readings lie between 49.8 cm and 50.2 cm. This measurement is best recorded as

A.   49.8 ± 0.2 cm.   C.   50.0 ± 0.2 cm.

B.   49.8 ± 0.4 cm.   D.   50.0 ± 0.4 cm.

A 9.51 ± 0.15 meter rope ladder is hung from a roof that is 12.56 ± 0.07 meters above the ground. How far is the bottom of the ladder from the ground?

A car travels 64.7 ± 0.5 meters in 8.65 ± 0.05 seconds. What is its speed?

The power dissipated in a resistor of resistance R carrying a current I is equal to I^2R. The value of I has an uncertainty of ±2% and the value of R has an uncertainty of ±10%. The value of the uncertainty in the calculated power dissipation is

A.   ±8%.   B.   ±12%.   C.   ±14%.   D.   ±20%.

The radius of a loop is measured to be (10.0 ± 0.5) cm. Which of the following is the best estimate of the uncertainty in the calculated area of the loop?

A.   0.25%   B.   5%   C.   10%   D.   25%

When a force F of (10.0 ± 0.2) N is applied to a mass m of (2.0 ± 0.1) kg, the percentage uncertainty attached to the value of the calculated acceleration $\dfrac{F}{m}$ is

A.   2%.   B.   5%.   C.   7%.   D.   10%.

Analytical Methods for Data

Experimental rigor:

- IB has a requirement that when you conduct an experiment of your own design, you must have **five variations** in your **independent variable**.

- And for **each** variation of your independent variable you must conduct **three trials** to gather the values of the **dependent variable**. The three values for each dependent variable will then be averaged.

- Note that if you need to employ statistical analysis on data such as standard deviation you must use no fewer than **five trials** and for using data comparisons which determine significant difference such as student's t-tests, you must have no fewer than **10 trials**.

Analytics and Tables

•This is a well designed table containing all of the information and data points required by IB:

- Data tables must record both raw and mean data.
- Data tables must have informative and descriptive column headers including units where necessary and uncertainty.

Sheets n / no units $\Delta n = \pm 0$	Rebound Height h_i / cm $\Delta h_i = \pm 0.2$ cm			Average Rebound Height h / cm $\Delta h = \pm 2.0$ cm
	Trial 1	Trial 2	Trial 3	
0	54.8	55.1	54.6	55
2	53.4	52.5	49.6	52
4	50.7	48.7	48.6	49
6	49.0	47.1	48.5	48
8	45.9	45.0	44.6	45

Independent (manipulated) 3 Trials Dependent (responding)

Analytics and Tables Continued

In order to determine the uncertainty in the dependent variable we reproduce the first two rows of the previous table:

The uncertainty in the average height h was taken to be **half the largest range** in the trial data, which is in the row for $n = 2$ sheets: $\frac{53.4 - 49.6}{2} = 2.0.$

Sheets n / no units $\Delta n = \pm 0$	Rebound Height h_i / cm $\Delta h_i = \pm 0.2$ cm			Average Rebound Height h / cm $\Delta h = \pm 2.0$ cm
	Trial 1	Trial 2	Trial 3	
0	54.8	55.1	54.6	55
2	53.4	52.5	49.6	52

If there is a necessary calculation performed on the raw data to produce your dependent variable (for instance if the dependent variable were to be potential energy for a ball, which was PE=m·g·h we would need the constant values for mass ± uncertainty in mass and gravity (assumed a constant 9.8). Then we would have to use the fractional uncertainty formula to calculate our error through the calculation.
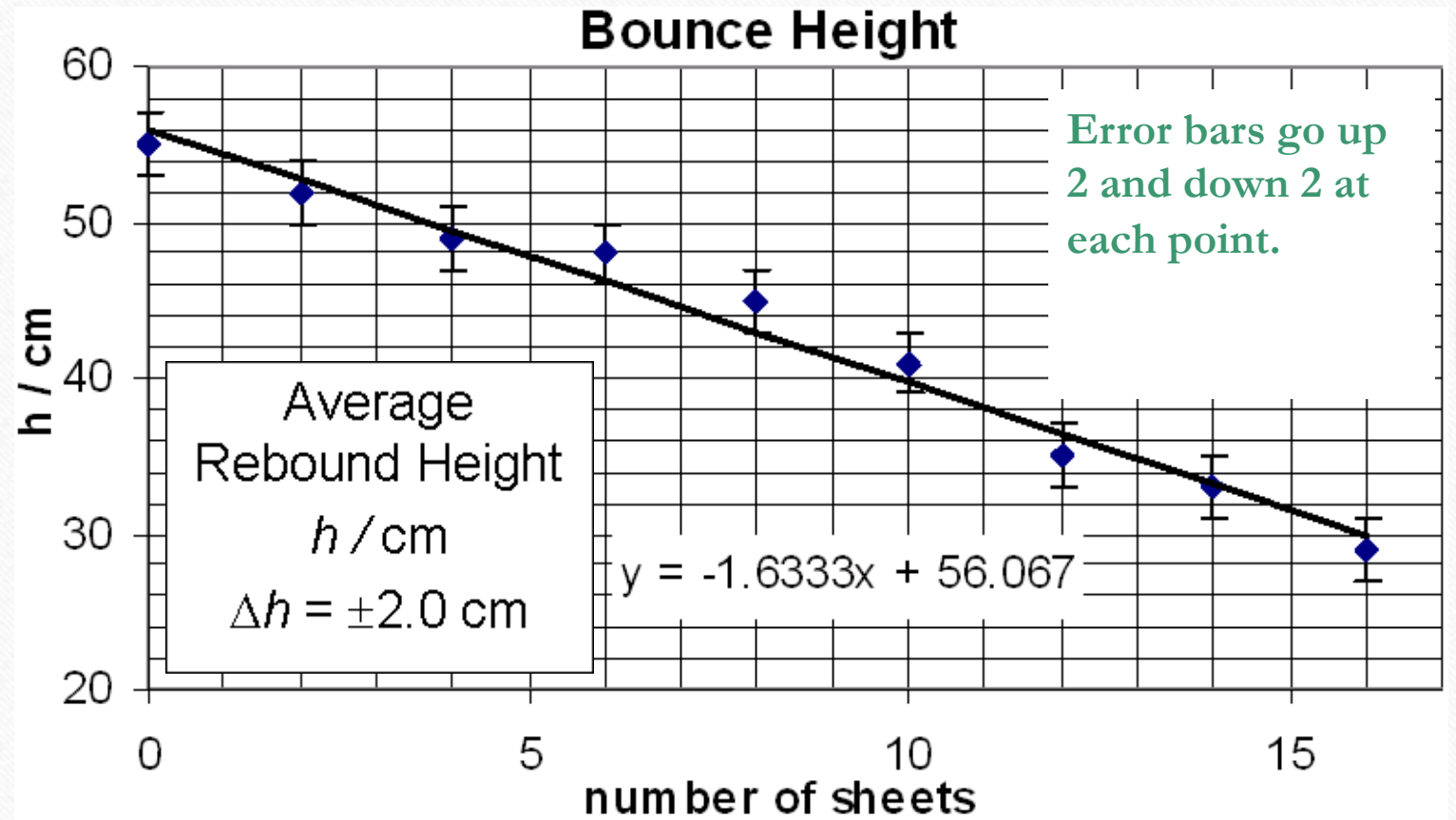
Analytics and Graphing

When making a graph it is required that you plot the data as scatter points.

You must also plot the error in the dependent variable using properly designed error bars. Error can either be generated by using the range/2 formula or by more rigorous methods if you have sufficient data.

Lastly, if possible you should have a trend line and a formula whose meaning is explicitly known.

Golden rule: the fit line must pass through all error bars to be valid, as well as be to the lowest possible power.

Bounce Height

Error bars go up 2 and down 2 at each point.

Average Rebound Height

h / cm

$\Delta h = \pm 2.0$ cm

$y = -1.6333x + 56.067$

h / cm (vertical axis)

number of sheets (horizontal axis)

Analytics and Graphing Error in Slopes and Intercepts

Uncertainty of gradient and intercepts

- To determine the uncertainty in the gradient and intercepts of a best fit line we look only at the first and last error bars, as illustrated here:

Recall: $y = mx + b$
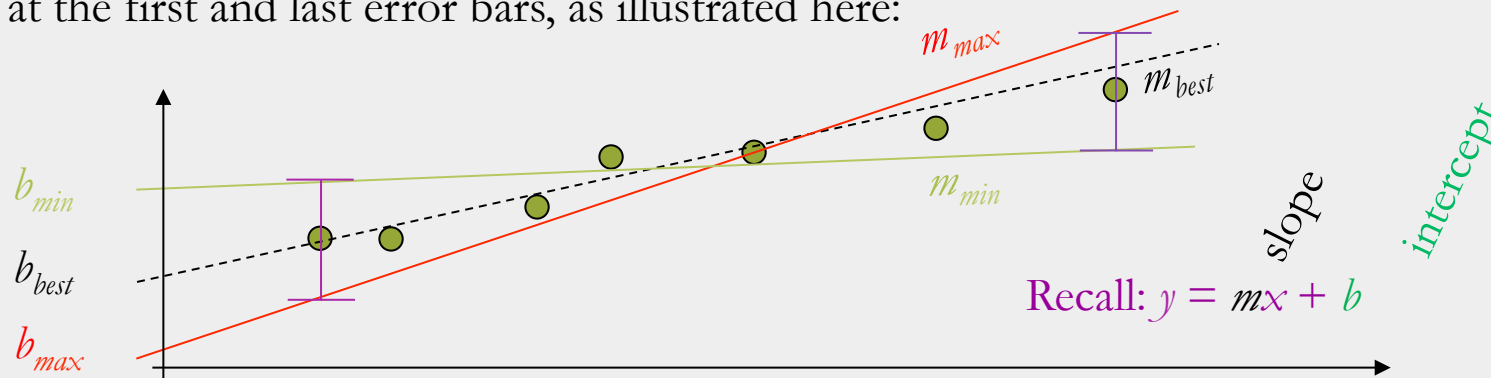
$$m_{best} \pm \Delta m = m_{best} \pm \left(\frac{m_{max} - m_{min}}{2} \right) \quad \text{slope \& uncertainty}$$

$$b_{best} \pm \Delta b = b_{best} \pm \left(\frac{b_{max} - b_{min}}{2} \right) \quad \text{intercept \& uncertainty}$$

Practice: Graphing Error in Slopes and Intercepts

Calculate the error in the trend line slope and intercept using the provided formula and first/last error bars.

The minimum slope is calculated from the bottom of the first bar to the top of the last: What are these resulting values?

Then calculate the resulting uncertainty in slope based on the data:

$$\Delta m = \frac{m_{max} - m_{min}}{2}$$

Then the reported slope is
-1.633±0.25

Now do the same for the y intercept.

$$\Delta b = \frac{b_{max} - b_{min}}{2}$$

Then the reported y-intercept is
56.067±2

Bounce Height

$y = -1.6333x + 56.067$