# 5.2 Heating effect of an electric current

## Understanding

- Circuit diagrams
- Kirchhoff's laws
- Heating effect of an electric current and its consequences
- Resistance
- Ohm's law
- Resistivity
- Power dissipation

## Nature of science

Peer review – a process in which scientists repeat and criticize the work of other scientists – is an important part of the modern scientific method. It was not always so. The work of Ohm was neglected in England at first as Barlow, a much-respected figure in his day, had published contradictory material to that of Ohm. In present-day science the need for repeatability in data collection is paramount. If experiments or other findings cannot be repeated, or if they contradict other scientists' work, then a close look is paid to them before they are generally accepted.

## Applications and skills

- Drawing and interpreting circuit diagrams
- Indentifying ohmic and non-ohmic conductors through a consideration of the $V$–$I$ characteristic graph
- Investigating combinations of resistors in parallel and series circuits
- Describing ideal and non-ideal ammeters and voltmeters
- Describing practical uses of potential divider circuits, including the advantages of a potential divider over a series variable resistor in controlling a simple circuit
- Investigating one or more of the factors that affect resistivity
- Solving problems involving current, charge, potential difference, Kirchhoff's laws, power, resistance and resistivity

### Equations

- resistance definition: $R = \frac{V}{I}$
- electrical power: $P = VI = I^2 R = \frac{V^2}{R}$
- combining resistors:
  in series $R_{total} = R_1 + R_2 + R_3 ...$
- in parallel $\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$
- resisitivity definition: $\rho = \frac{RA}{l}$

## Effects of electric current

### Introduction

This is the first of three sub-topics that discusses some of the effects that occur when charge flows in a circuit. The three effects are:

- **heating effect**, when energy is transferred to a resistor as internal energy
- **chemical effect**, when chemicals react together to alter the energy of electrons and to cause them to move, or when electric current in a material causes chemical changes (Sub-topic 5.3)
- **magnetic effect**, when a current produces a magnetic field, or when magnetic fields change near conductors and induce an emf in the conductor (Sub-topic 5.4).

This sub-topic deals with the heating effect of a current after giving you some advice on setting up and drawing electric circuits.

## Drawing and using circuit diagrams

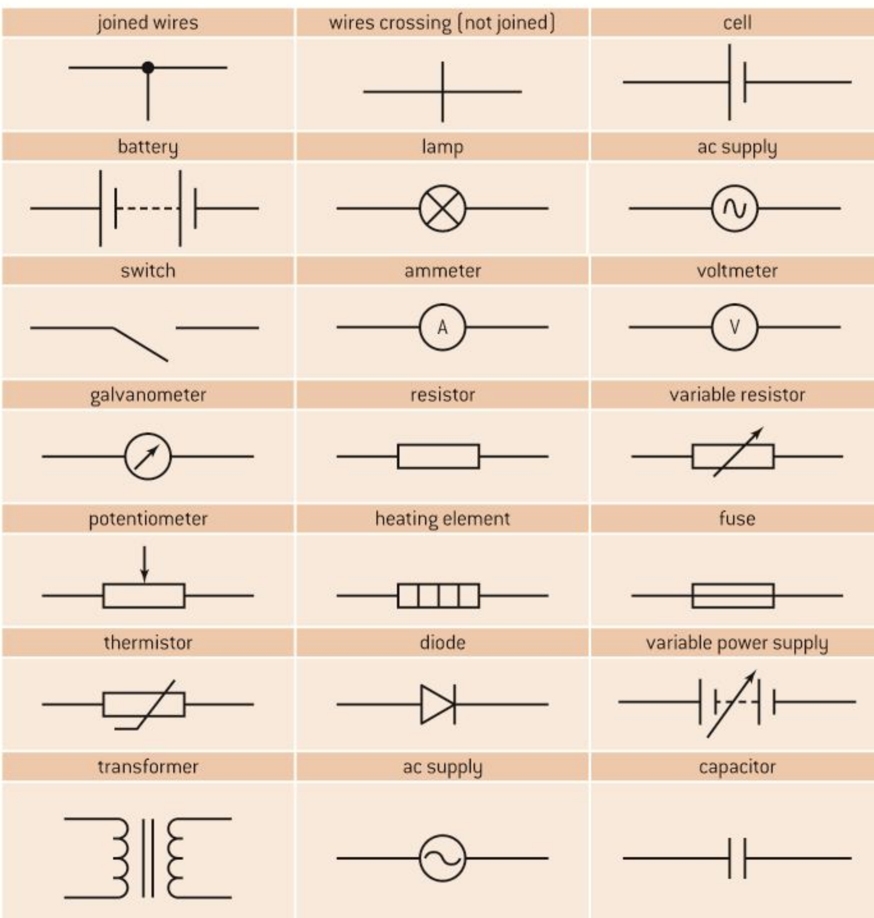
At some stage in your study of electricity you need to learn how to construct electrical circuits to carry out practical tasks.

This section deals with drawing, interpreting, and using circuit diagrams and is designed to stand alone so that you can refer to it whenever you are working with diagrams and real circuits.

You may not have met all the components discussed here yet. They will be introduced as they are needed.

### Circuit symbols

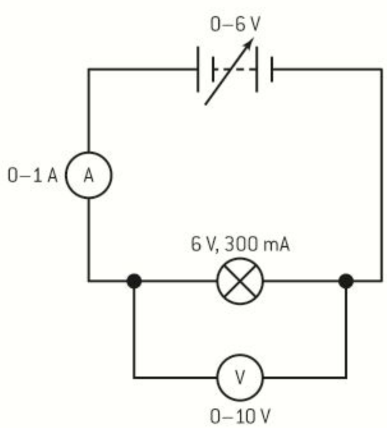
A set of agreed electrical symbols has been devised so that all physicists understand what is represented in a circuit diagram. The agreed symbols that are used in the IB Diploma Programme are shown in figure 1. Most of these are straightforward and obvious; some may be familiar to you already. Ensure that you can draw and identify all of them accurately.




▲ Figure 1 Circuit symbols.

There are points to make about the symbols.

- Some of the symbols here are intended for **direct current (dc)** circuits (cells and batteries, for example). Direct current refers to a circuit in which the charge flows in one direction. Typical examples of this in use would be a low-voltage flashlight or a mobile phone. Other types of electrical circuits use **alternating current (ac)** in which the current direction is first one way around the circuit and then the opposite. The time between changes is typically about 1/100th of a second. Common standards for the frequencies around the world include 50 Hz and 60 Hz. Alternating current is used in high-voltage devices (typically in the home and industry), where large amounts of energy transfer are required: kettles, washing machines, powerful electric motors, and so on. Alternating supplies can be easily transformed from one pd to another, whereas this is more difficult (though not impossible) for dc.
- There are separate symbols for cells and batteries. Most people use these two terms interchangeably, but there is a difference: a battery is a collection of cells arranged positive terminal to negative – the diagram for the battery shows how they are connected. A cell only contains one source of emf. Sub-topic 5.3 goes into more detail about cells and batteries.




▲ Figure 2 Circuit diagram.

## Circuit conventions

- It is not usual to write the name of the component in addition to giving its symbol, unless there is some chance of ambiguity or the symbol is unusual. However, if the value of a particular component is important in the operation of the circuit it is usual to write its value alongside it.
- Particular care needs to be taken when it is necessary to draw one connecting lead over another. The convention is that if two leads cross and are joined to each other, then a dot is placed at the junction. If there is no dot, then the leads are not connected to each other. In the circuit in figure 2 it is important to know the emf of the cell, the data for the lamp at its working temperature, and the **full scale deflections (fsd)** of the two meters in the circuit.

## Practical measurements of current and potential difference

We often need to measure the current in a circuit and the pd across components in the circuit. This can be achieved with the use of meters or sensors connected to computers (data loggers). You may well use both during the course. Schools use many types and varieties of meters and it is impossible to discuss them all here, but an essential distinction to make is between analogue and digital meters. Again, you may well use both as you work through the course.

**Analogue meters** have a mechanical system of a coil and a magnet. When charge flows through the coil, a magnetic field is produced that interacts with the field of the magnet and the coil swings round against a spring. The position reached by the pointer attached to the coil is a measure of the current in the meter.

**Digital meters** sample the potential difference across the terminals of the meter (or, for current, the pd across a known resistor inside the meter) and then convert the answer into a form suitable for display on the meter.

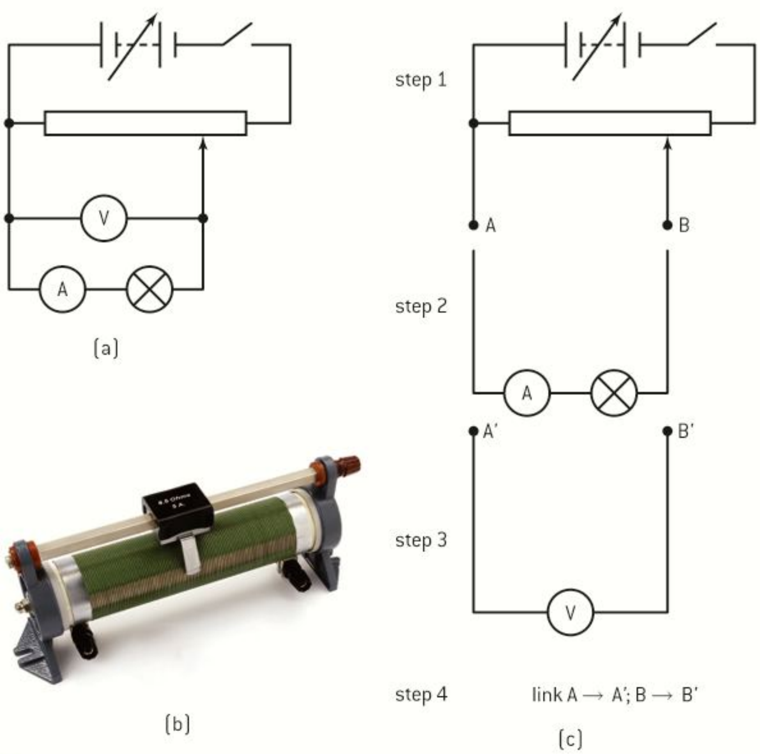
**Ammeters** measure the current in the circuit. As we want to know the size of the current in a component it is clear that the ammeter must have the same current. The ammeter needs to be in **series** with the circuit or component. An ideal ammeter will not take any energy from the electrons as they flow through it, otherwise it would disturb the circuit it is trying to measure. Figure 2 shows where the ammeter is placed to measure the current.

**Voltmeters** measure the energy converted per unit charge that flows in a component or components. You can think of a voltmeter as needing to compare the energy in the electrons before they enter a component to when they leave it, rather like the turnstiles (baffle gates) to a rail station that count the number of people (charges) going through as they give a set amount of money (energy) to the rail company. To do this the voltmeter must be placed across the terminals of the component or components whose pd is being measured. This arrangement is called **parallel**. Again, figure 2 shows this for the voltmeter.

## Constructing practical circuits from a diagram

Wiring a circuit is an important skill for anyone studying physics. If you are careful and work in an organized way then you should have no problems with any circuit no matter how complex.

As an example, this is how you might set up one of the more difficult circuits in this course.




▲ Figure 3 Variable resistors.

This is a potential divider circuit that it is used to vary the pd across a component, in this case a lamp.

The most awkward component to use here is the potential divider itself (a form of variable resistor, that is sometimes known as a **potentiometer**). In one form it has three terminals (figure 3(b)), in another type it has three in a rotary format. The three-terminal linear device has a terminal at one end of a rod with a wiper that touches the resistance windings, and another two terminals one at each end of the resistance winding itself.

Begin by looking carefully at the diagram figure 3(a). Notice that it is really two smaller circuits that are linked together: the top sub-circuit with the cell, and the bottom sub-circuit with the lamp and the two meters. The bottom circuit itself consists of two parts: the lamp/ammeter link together with the voltmeter loop.

The rules for setting up a circuit like this are:

- If you do not already have one, draw a circuit diagram. Get your teacher to check it if you are not sure that it is correct.
- Before starting to plug leads in, lay out the circuit components on the bench in the same position as they appear on the diagram.
- Connect up one loop of the circuit at a time.
- Ensure that components are set to give minimum or zero current when the circuit is switched on.
- Do not switch the circuit on until you have checked everything.

Figure 3(c) shows a sequence for setting up the circuit step-by-step.

Another skill you will need is that of troubleshooting circuits – this is an art in itself and comes with experience. A possible sequence is:

- Check the circuit – is it really set up as in your diagram?
- Check the power supply (try it with another single component such as a lamp that you know is working properly).
- Check that all the leads are correctly inserted and that there are no loose wires inside the connectors.
- Check that the individual components are working by substituting them into an alternative circuit known to be working.

## Resistance

We saw in Sub-topic 5.1 that, as electrons move through a metal, they interact with the positive ions and transfer energy to them. This energy appears as kinetic energy of the lattice, in other words, as internal energy: the metal wire carrying the current heats up.

However, simple comparison between different conductors shows that the amount of energy transferred can vary greatly from metal to metal. When there is the same current in wires of similar size made of tungsten or copper, the tungsten wire will heat up more than the

copper. We need to take account of the fact that some conductors can achieve the energy transfer better than others. The concept of **electrical resistance** is used for this.

The resistance of a component is defined as

$$\frac{\text{potential difference across the component}}{\text{current in the component}}$$

The symbol for resistance is $R$ and the definition leads to a well-known equation

$$R = \frac{V}{I}$$

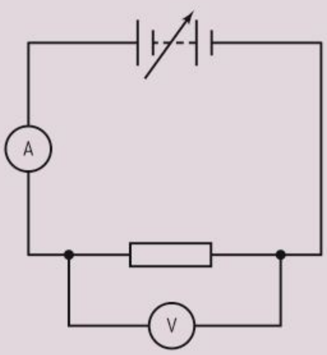
The unit of resistance is the ohm (symbol Ω; named after Georg Simon Ohm, a German physicist). In terms of its fundamental units, 1 Ω is 1 kg $m^2 s^{-3} A^{-2}$; using the ohm as a unit is much more convenient!

Alternative forms of the equation are: $V = IR$ and $I = \frac{V}{R}$

## Investigate!

### Resistance of a metal wire

- Take a piece of metal wire (an alloy called constantan is a good one to choose) and connect it in the circuit shown. Use a power supply with a variable output so that you can alter the pd across the wire easily.




▲ Figure 4

- If your wire is long, coil it around an insulator (perhaps a pencil) and ensure that the coils do not touch.
- Take readings of the current in the wire and the pd across it for a range of currents. Your teacher will tell you an appropriate range to use to avoid changing the temperature of the wire.
- For each pair of readings divide the pd by the current to obtain the resistance of the wire in ohms.

A table of results is shown in figure 5 for a metal wire 1 m in length with a diameter of 0.50 mm. For each pair of readings the resistance of the wire has been calculated by dividing the pd by the current. Although the resistance values are not identical (it is an experiment with real errors, after all), they do give an average value for the resistance of 2.54 Ω. This should be rounded to 2.5 Ω given the signficant figures in the data.

| current/A | pd/V | resistance/Ω |
|---|---|---|
| 0.05 | 0.13 | 2.55 |
| 0.10 | 0.26 | 2.60 |
| 0.20 | 0.50 | 2.50 |
| 0.30 | 0.76 | 2.53 |
| 0.40 | 1.01 | 2.53 |
| 0.50 | 1.27 | 2.54 |
| 0.60 | 1.53 | 2.55 |
| 0.70 | 1.78 | 2.54 |

▲ Figure 5 Variation of pd with current for a conductor.

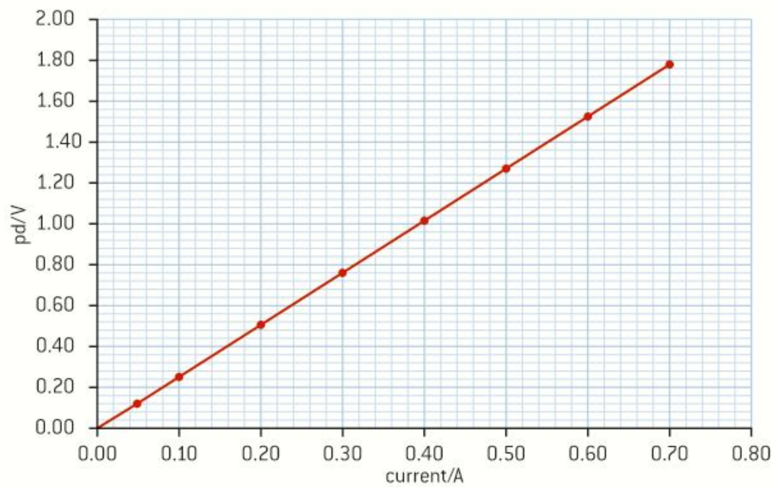
## Nature of science

### Edison and his lamp

The conversion of electrical energy into internal energy was one of the first uses of distributed electricity. Thomas Edison an inventor and entrepreneur, who worked in the US at around the end of the nineteenth century, was a pioneer of electric lighting. The earliest forms of light were provided by producing a current in a metal or carbon filament. These filaments heated up until they glowed. Early lamps were primitive but produced a revolution in the way that homes and public spaces were lit. The development continues today as inventors and manufacturers strive to find more and more efficient electric lamps such as the light-emitting diodes (LED). More developments will undoubtedly occur during the lifetime of this book.

## Ohm's law

Figure 6 shows the results from the metal wire when they are plotted as a graph of *V* against *I*.




▲ Figure 6 pd against current from the table.

A best straight line has been drawn through the data points. For this wire, the resistance is the same for all values of current measured. Such a resistor is known as **ohmic**. An equivalent way to say this is that the potential difference and the current are proportional (the line is straight and goes through the origin). In the experiment carried out to obtain these data, the temperature of the wire did not change.

This behaviour of metallic wires was first observed by Georg Simon Ohm in 1826. It leads to a rule known as **Ohm's law**.

Ohm's law states that the potential difference across a metallic conductor is directly proportional to the current in the conductor providing that the physical conditions of the conductor do not change.

By physical conditions we mean the temperature (the most important factor as we shall see) and all other factors about the wire. But the temperature factor is so important that the law is sometimes stated replacing the term "physical conditions" with the word "temperature".

### Worked example

The current in a component is 5.0 mA when the pd across it is 6.0 V.

Calculate:

**a)** the resistance of the component

**b)** the pd across the component when the current in it is 150 μA.

### Solution

**a)** $R = \frac{V}{I} = \frac{6}{5 \times 10^{-3}} = 1.2\ \text{k}\Omega$

**b)** $V = IR = 1.5 \times 10^{-4} \times 1.2 \times 10^{3} = 0.18$ A

## Nature of science

### Ohm and Barlow

Ohm's law has its limitations because it only tells us about a material when the physical conditions do not change. However, it was a remarkable piece of work that did not find immediate favour. Barlow was an English scientist who was held in high respect for his earlier work and had recently published an alternative theory on conduction. People simply did not believe that Barlow could be wrong.

This immediate acceptance of one scientist's work over another would not necessarily happen today. Scientists use a system of peer review. Work published by one scientist or scientific group must be set out in such a way that other scientists can repeat the experiments or collect the same data to check that there are no errors in the original work. Only if the scientific community as a whole can verify the data is the new work accepted as scientific "fact".

Towards the end of the 20th century a research group thought that it had found evidence that nuclear fusion could occur at low temperatures (so-called "cold fusion"). Repeated attempts by other research groups to replicate the original results failed, and the cold fusion ideas were discarded.

## Investigate!

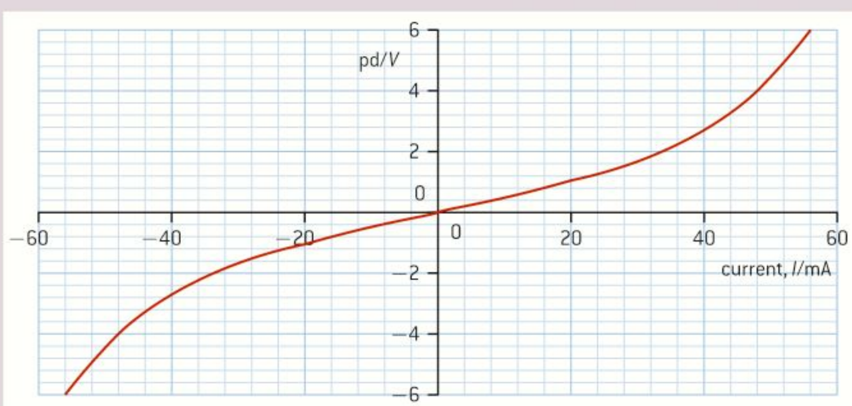
### Variation of resistance of a lamp filament

- Use the circuit you used in the investigation on resistance of a metal wire, but repeat the experiment with a filament lamp instead of the wire.

  Your teacher will advise you of the range of currents and pds to use.

- This time do the experiment twice, the second time with the charge flowing through the lamp in the opposite direction to the first. There are two ways to achieve this: the first is to reverse the connections to the power supply, also reversing the connections to the ammeter and voltmeter (if the meters are analogue). The second way is somewhat easier, simply reverse the lamp and call all the readings negative because they are in the opposite direction through the lamp.

- Plot a graph of $V$ ($y$-axis) against $I$ ($x$-axis) with the origin (0,0) in the centre of the paper. Figure 7 shows an example of a $V$–$I$ graph for a lamp.




▲ Figure 7

TOK

**But is it a law?**

This rule of Ohm is always called a law – but is it? In reality it is an experimental description of how a group of materials behave under rather restricted conditions. Does that make it a law? You decide.

There is also another aspect to the law that is often misunderstood.

Our definition of resistance is that $R = \frac{V}{I}$ or $V = IR$. Ohm's law states that

$$V \propto I$$

and, including the constant of proportionality $k$,

$$V = kI$$

We therefore define $R$ to be the same as $k$, but the definition of resistance does not correspond to Ohm's law (which talks only about a proportionality). $V = IR$ is emphatically not a statement of Ohm's law and if you write this in an examination as a statement of the law you will lose marks.

The graph is not straight (although it goes through the origin) so $V$ and $I$ are not proportional to each other. The lamp does not obey Ohm's law and it is said to be **non-ohmic**. However, this is not a fair test of ohmic behaviour because the filament is not held at a constant temperature. If it were then, as a metal, it would probably obey the law.

When the resistance is calculated for some of the data points it is not constant either. The table shows the resistance values at each of the positive current points.

| Current/mA | Resistance/Ω |
|---|---|
| 20 | 50 |
| 34 | 59 |
| 41 | 73 |
| 47 | 85 |
| 52 | 96 |
| 55 | 109 |

Tip

Notice that these resistances were calculated for each individual data point using $\frac{V}{I}$. They were not evaluated from the tangent to the graph at the current concerned. The definition of resistance is in terms of $\frac{V}{I}$ *not* in terms of $\frac{\Delta V}{\Delta I}$ which is what the tangent would give.

The data show that resistance of the lamp increases as the current increases. At large currents, it takes greater changes in pd to change the current by a fixed amount. This is exactly what you might have predicted. As the current increases, more energy is transferred from the electrons every second because more electrons flow at higher currents. The energy goes into increasing the kinetic energy of the lattice ions and therefore the temperature of the bulk material. But the more the ions vibrate in the lattice, the more the electrons can collide with them so at higher temperatures even more energy is transferred to the lattice by the moving charges.

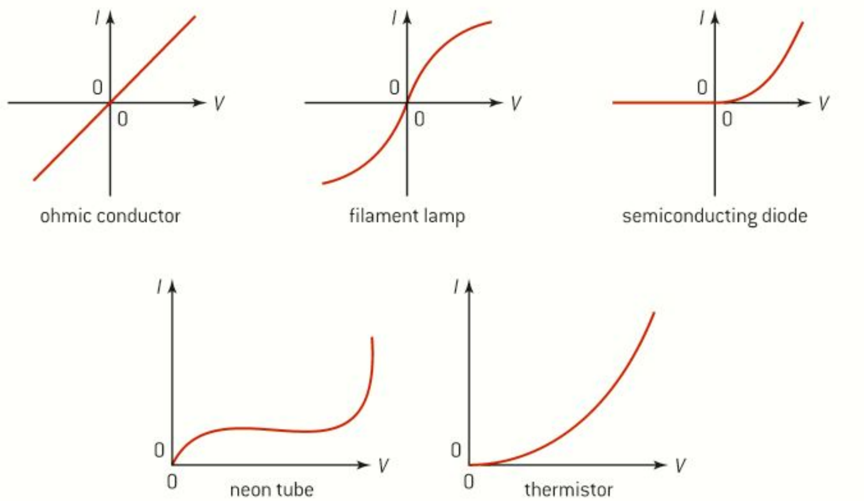
Other non-ohmic conductors include semiconducting diodes and thermistors; these are devices made from a group of materials known as semiconductors.

## Investigate!

### Diodes and thermistors

- You can easily extend the *Investigate!* lamp experiment to include these two types of device. There are some extra practical points however:
- The diode will require a protecting resistor in series with it (100 Ω is usually appropriate) as the current can become very large at even quite small potential differences; this will cause the diode to melt as large amounts of energy are transferred to it. The resistor limits the current so that the diode is not damaged.
- The thermistor also needs care, because as it heats up its resistance decreases and if too much current is used, the thermistor can also be destroyed.

The results of these experiments and some others are summarized in the *I–V* graphs in figure 8.




▲ Figure 8 *I–V* graphs for various conductors.

## Semiconducting diodes

Semiconducting diodes are designed only to allow charges to flow through them in one direction. This is seen clearly in the graph. For negative values of *V* there is actually a very small current flowing in the negative direction but it is far less than the forward current. The nature of semiconductor material also means that there is no significant current in the forward direction until a certain forward pd is exceeded.

## Thermistors

Thermistors are made from one of the two elements that are electrical semiconductors: silicon and germanium. There are several types of thermistor, but we will only consider the negative temperature coefficient type (ntc). As the temperature of an ntc thermistor increases, its resistance falls. This is the opposite behaviour to that of a metal.

Semiconductors have many fewer free electrons per cubic metre compared with metals. Their resistances are typically $10^5$ times greater than similar samples of metals. However, unlike in a metal, the charge density in semiconductors depends strongly on the temperature. The higher the temperature of the semiconductor, the more charge carriers are made available in the material.

As the temperature rises in the germanium:

- The lattice ions vibrate more and impede the movement of the charge carriers. This is exactly the same as in metals and also leads to an *increase* in resistance.
- More and more charge carriers become available to conduct because the increase in temperature provides them with enough energy to break away from their atoms. This leads to a large *decrease* in resistance.
- The second effect is much greater than the first and so the net effect is that conduction increases (resistance falls) as the temperature of the semiconductor rises.

## Investigate!

### How resistance depends on size and shape

- You will need a set of wires made from the same metal or metal alloy. The wires should have a circular cross-section and be available in a range of different diameters. You will also need to devise a way to vary the length of one of the wires in the circuit.
- Use the circuit in figure 4 on p197 to answer the following questions:
  - How does the resistance $R$ of one of the wires vary with length $l$?
  - How does the resistance $R$ of the wires vary with diameter $d$ when the wires all have the same length?
- Try to make things easy for your analysis. Double and halve the values of length to see what difference this makes to the resistance – is there an obvious relationship? The diameter of the wire may be more difficult to test in this way, but a graph of resistance against diameter may give you a clue.
- Once you have an idea what is going on, you may decide that the best way to answer these questions is to plot graphs of

  $R$ against $l$, and

  $R$ against $\frac{1}{d^2}$

## Resistivity

The resistance of a sample of a material depends not only on what it is made of, but also on the physical dimensions (the size and shape) of the sample itself.

The graphs suggested in the *Investigate!* should give straight lines that go through the origin and can be summed up in the following rule.

The resistance of a conductor is:

- proportional to its length $l$
- inversely proportional to its cross-sectional area $A$ (which is itself proportional to $d^2$).

So

$$R \propto \frac{l}{A}$$

This leads to a definition of a new quantity called **resistivity**.

Resistivity $\rho$ is defined by

$$\rho = \frac{RA}{l}$$

The unit of resistivity is the ohm-metre (symbol $\Omega$ m). Take care here: this is ohm metre. It is not ohm metre$^{-1}$ – a mistake frequently made by students in examinations. The meaning of ohm metre$^{-1}$ is the resistance of one metre length of a particular conductor, which is a relevant quantity to know, but is not the same as resistivity.

Resistivity is a quantity of considerable use. The resistance of a material depends on the shape of the sample as well as what it is made from. Even a constant volume of a material will have values of resistance that depend on the shape. However, the value of the resistivity is the same for all samples of the material. Resistivity is independent of shape or size just like quantities such as *density* (where the size of a material has been removed by the use of volume) or *specific latent heat* (where the value is related to unit mass of the material).

### Worked examples

**1** A uniform wire has a radius of 0.16 mm and a length of 7.5 m. Calculate the resistance of the wire if its resistivity is $7.0 \times 10^{-7}\ \Omega$ m.

#### Solution

Unless told otherwise, assume that the wire has a circular cross-section.

area of wire $= \pi(1.6 \times 10^{-4})^2 = 8.04 \times 10^{-8}\ \text{m}^2$

$$\rho = \frac{RA}{l}$$

and $R = \dfrac{\rho l}{A} = \dfrac{7.0 \times 10^{-7} \times 7.5}{8.04 \times 10^{-8}} = 65\ \Omega$

**2** Calculate the resistance of a block of copper that has a length of 0.012 m with a width of 0.75 mm and a thickness of 12 mm. The resistivity of copper is $1.7 \times 10^{-8}\ \Omega$ m.

#### Solution

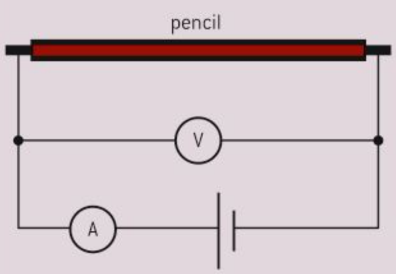
The cross-sectional area of the block is $7.5 \times 10^{-4} \times 1.2 \times 10^{-2} = 9.0 \times 10^{-6}\ \text{m}^2$

The relevant dimension for the length is 0.012 m, so

$R = \dfrac{\rho l}{A} = \dfrac{1.7 \times 10^{-8} \times 0.012}{9.0 \times 10^{-6}} = 0.023\ \text{m}\Omega$

## Investigate!

### Resistivity of pencil lead




▲ Figure 9

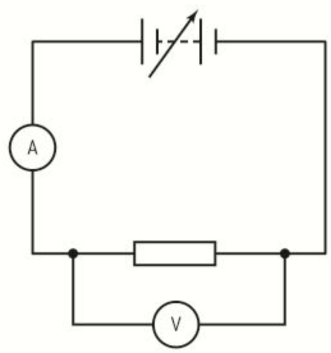
- Graphite is a semi-metallic conductor and is a constituent of the lead in a pencil. Another constituent in the pencil lead is clay. It is the ratio of graphite to clay that determines the "hardness" of the pencil. This experiment enables you to estimate the resistivity of the graphite.
- Take a B grade pencil (sometimes known as #1 grade in the US) and remove about 1.5 cm of the wood from each end leaving a cylinder of the lead exposed. Attach a crocodile (alligator) clip firmly to each end. Use leads attached to the crocodile clips to connect the pencil into a circuit in order to measure the resistance of the lead. Expect the resistance of the pencil lead to be about 1 Ω for the purpose of choosing the power supply and meters.
- Determine the resistance of the lead.
- Measure the length of pencil lead between the crocodile clips.
- Measure the diameter of the lead using a micrometer screw gauge or digital callipers and calculate the area of the lead.
- Using the data, calculate the resistivity of the lead. The accepted value of the resistivity of graphite is about $3 \times 10^{-5}\ \Omega$ m but you will not expect to get this value given the presence of clay in the lead as well.
- Take the experiment one step further with a challenge. Use your pencil to uniformly shade a 10 cm by 2 cm area on a piece of graph paper. This will make a graphite resistance film on the paper. Attach the graphite film to a suitable circuit and measure the resistance of the film. Knowing the resistance and the dimensions of your shaded area should enable you to work out how thick the film is. (*Hint:* in the resistivity equation, the length is the *distance across the film*, and the area is *width of the film* × *thickness of the film*.)

## Practical resistors

Resistors are of great importance in the electronics and electrical industries. They can be single value (fixed) devices or they can be variable. They can be manufactured in bulk and are readily and cheaply available.

Resistors come in different sizes. Small resistors can have a large resistance but only be able to dissipate (lose) a modest amount of energy every second. If the power that is being generated in the resistor is too large, then its temperature will increase and in a worst case, it could become a thermal fuse! Resistors are rated by their manufacturers so that, for example, a resistor could have a resistance of 270 Ω with a power rating of 0.5 W. This would mean that the maximum current that the resistor can safely carry is $\sqrt{\frac{0.5}{270}} = 43$ mA.

## Combining resistors



▲ Figure 10

Electrical components can be linked together in two ways in an electrical circuit: **in series**, where the components are joined one after another as the ammeter, the cell and the resistor in figure 10, or **in parallel** as are the resistor and the voltmeter in the same figure.

Two components connected in series have the same current in each. The number of free electrons leaving the first component must equal the number entering the second component; if electrons were to stay in the first component then it would become negatively charged and would repel further electrons and prevent them from entering it. The flow of charges would rapidly grind to a halt. This is an important rule to understand.

In series the potential differences (pds) add. To see this, think of charge as it travels through two components, the total energy lost is equal to the sum of the two separate amounts of energy in the components. Because the charge is the same in both cases, therefore, the sum of the pds is equal to the total pd dropped across them.

Components in parallel, on the other hand, have the same pd across them, but the currents in the components differ when the resistances of the components differ. Consider two resistors of different resistance values, in parallel with each other and connected to a cell. If one of the resistors is temporarily disconnected then the current in the remaining resistor is given by the emf of the cell divided by the resistance. This will also be true for the other resistor if it is connected alone. If both resistors are now connected in parallel with each other, both resistors have the same pd across them because a terminal of each resistor is connected to one of the terminals of the cell. The cell will have to supply more current than if either resistor were there alone – to be precise, it has to supply the sum of the separate currents.

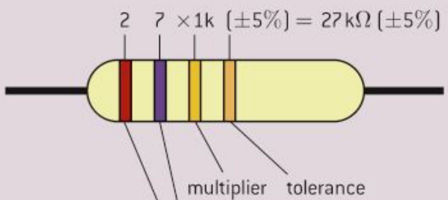
To sum up

| | Currents ... | Potential differences ... |
|---|---|---|
| In series | ...are the same | ...add |
| In parallel | ...add | ...are the same |

## Investigate!

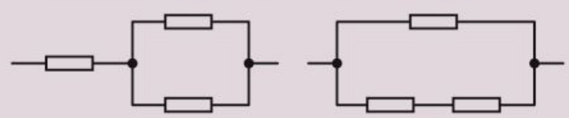
### Resistors in series and parallel

- For this experiment you need six resistors, each one with a tolerance of ±5%. Two of these resistors should be the same. The tolerance figure means that the manufacturer only guarantees the value to be within 5% of the nominal value – "nominal" means the printed value. Your resistors may have the nominal value written on them or you may have to use the colour code printed on them. The code is easy to decipher.




| colour | digit | multiplier | tolerance |
|---|---|---|---|
| silver | | 0.01 | 10% |
| gold | | 0.1 | 5% |
| black | 0 | 1 | |
| brown | 1 | 10 | 1% |
| red | 2 | 100 | 2% |
| orange | 3 | 1 k | |
| yellow | 4 | 10 k | |
| green | 5 | 100 k | 0.5% |
| blue | 6 | 1 M | |
| violet | 7 | 10 M | |
| grey | 8 | | |
| white | 9 | | |

- You also need a multimeter set to measure resistance directly and a way to join the resistors together and to connect them to the multimeter.
- First, measure the resistance of each resistor and record this in a table.
- Take the two resistors that have the same nominal value and connect them in series. Measure the resistance of the combination. Can you see a rule straight away for the combined resistance of two resistors?
- Repeat with five of the possible combinations for connecting resistors in series.
- Now measure the combined resistance of the two resistors with the same nominal value when they are in parallel. Is there an obvious rule this time?
- One way to express the rule for combining two resistors $R_1$ and $R_2$ in parallel is as $\frac{R_1 R_2}{R_1 + R_2}$. Test this relationship for five combinations of parallel resistors.
- Test your two rules together by forming combinations of three resistors such as:



These ideas provide a set of rules for the combination of resistors in various arrangements:

### Resistors in series

Suppose that there are three resistors in series: $R_1$, $R_2$ and $R_3$ (figure 11(a)). What is the resistance of the single resistor that could replace them so that the resistance of the single resistor is equivalent to the combination of three?

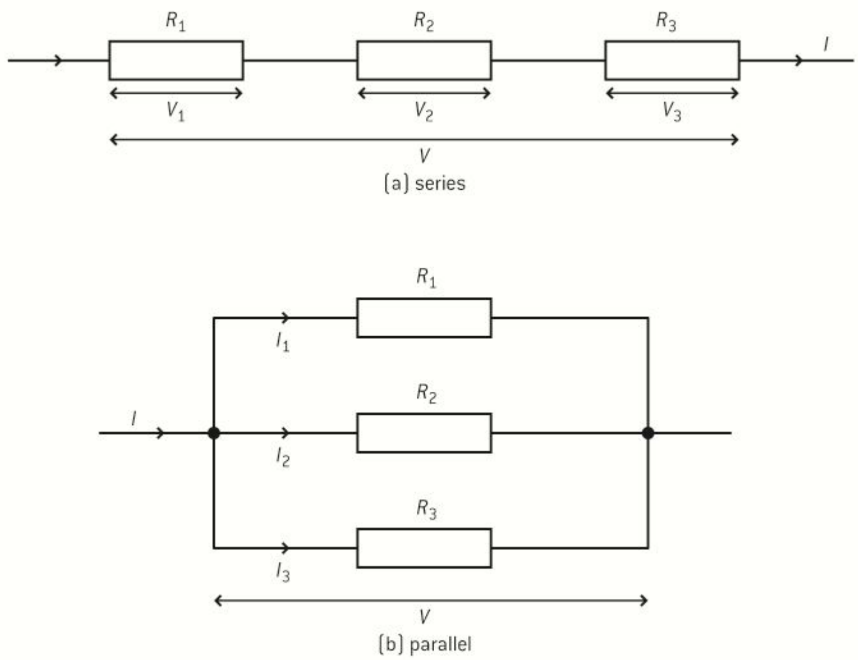
The resistors are in series (figure 11(a)) and therefore the current $I$ is the same in each resistor.

Using the definition of resistance, the pd across each resistor, $V_1$, $V_2$ and $V_3$ is

$$V_1 = IR_1 \qquad V_2 = IR_2 \qquad \text{and} \qquad V_3 = IR_3$$

The single resistor with a resistance $R$ has to be indistinguishable from the three in series. In other words, when the current through this single resistor is the same as that through the three, then it must have a pd $V$ across it such that

$$V = IR$$




▲ Figure 11 Resistors in series and parallel.

As this is a series combination, the potential differences add, so

$$V = V_1 + V_2 + V_3$$

Therefore

$$IR = IR_1 + IR_2 + IR_3$$

and cancelling $I$ leads to, in **series**

$$\boldsymbol{R = R_1 + R_2 + R_3}$$

When resistors are combined in series, the resistances add to give the total resistance.

### Resistors in parallel

Three resistors in parallel (figure 11(b)) have the same pd $V$ across them (figure 11(b)). The currents in the three separate resistors add to give the current in the connecting leads to and from all three resistors, therefore

$$I = I_1 + I_2 + I_3$$

Each current can be written in terms of $V$ and $R$ using the definition of resistance:

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

This time $V$ cancels out and, in **parallel**,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In parallel combinations of resistors, the reciprocal of the total resistance is equal to the sum of the reciprocals of the individual resistances.

The parallel equation needs some care in calculations. The steps are:

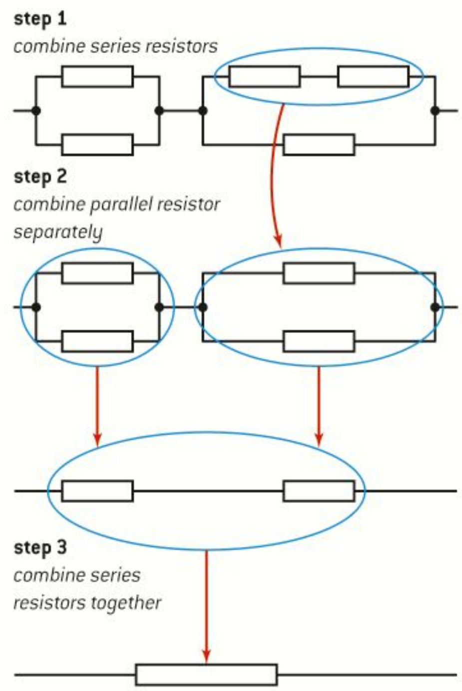
- calculate the reciprocals of each individual resistor
- add these reciprocals together
- take the reciprocal of the answer.

It is common to see the last step ignored so that the answer is incorrect; a worked example below shows the correct approach.

## More complicated networks

When the networks of resistors are more complicated, then the individual parts of the network need to be broken down into the simplest form.

Figure 12 shows the order in which a complex resistor network could be worked out.




▲ Figure 12

## Worked examples

**1** Three resistors of resistance, 2.0 Ω, 4.0 Ω, and 6.0 Ω are connected. Calculate the total resistance of the three resistors when they are connected:

**a)** in series

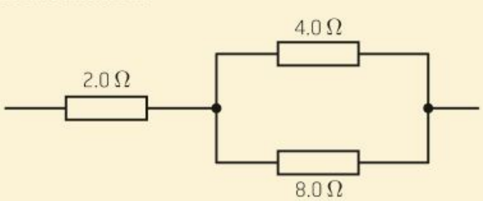
**b)** in parallel.

### Solution

**a)** In series, the resistances are added together, so $2 + 4 + 6 = 12\ \Omega$

**b)** In parallel, the reciprocals are used:
$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{6+3+2}{12} = \frac{11}{12}$
The final step is to take the reciprocal of the sum, $R = \frac{12}{11} = 1.1\ \Omega$

**2** 2.0 Ω, 4.0 Ω, and 8.0 Ω resistors are connected as shown. Calculate the total resistance of this combination.



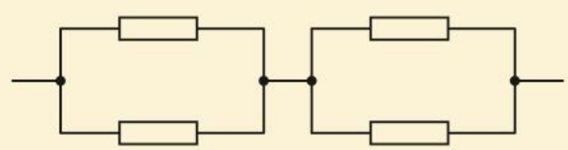

### Solution

The two resistors in parallel have a combined resistance of $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$

$$R = \frac{8}{3} = 2.67\ \Omega$$

This 2.67 Ω resistor is in series with 2.0 Ω, so the total combined resistance is $2.67 + 2.0 = 4.7\ \Omega$.

**3** Four resistors each of resistance 1.5 Ω are connected as shown. Calculate the combined resistance of these resistors.



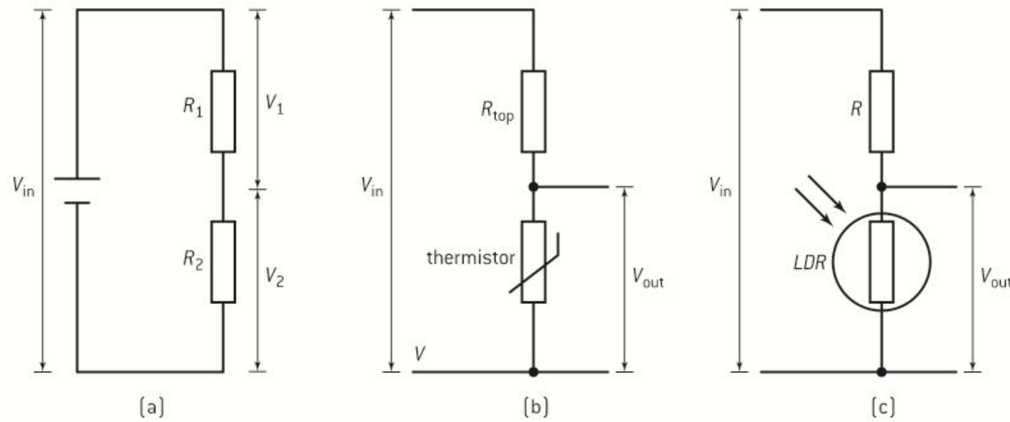
### Solution

Two 1.5 Ω resistors in parallel have a resistance given by $\frac{1}{R} = \frac{1}{1.5} + \frac{1}{1.5} = \frac{2}{1.5}$. So $R = 0.75\ \Omega$.

Two 0.75 Ω resistors in series have a combined resistance of $0.75 + 0.75 = 1.5\ \Omega$

## Potential divider

In the section on circuit diagrams, a potential divider circuit was shown. This is a circuit commonly used with sensors and also to produce variable potential differences. It has some advantages over the simpler variable resistor circuit even though it is more complicated to set up.




▲ Figure 13 The potential divider.

The most basic potential divider consists of two resistors with resistances $R_1$ and $R_2$ in series with a power supply. This arrangement is used to provide a fixed pd at a value somewhere between zero and the emf of the power supply. Figure 13(a) shows the arrangement.

The two resistors have the same current in them, and the sum of the pds across the resistors is equal to the source emf.

So

The current $I$ in both resistors is

$$I = \frac{\textit{total pd across both resistors}}{\textit{total resistance}} = \frac{V}{R}$$

As usual,

$$V_1 = IR_1 \text{ and } V_2 = IR_2$$

The resistors are in series so

$$R = R_1 + R_2$$

This leads to equations for the pd across each resistor

$$V_1 = \frac{R_1}{(R_1 + R_2)} V_{in} \qquad \text{and} \qquad V_2 = \frac{R_2}{(R_1 + R_2)} V_{in}$$

### Using a potential divider with sensors

It is a simple matter to extend the fixed pd arrangement to a circuit that will respond to changes in the external conditions. Such an arrangement might be used by a computer that can sense changes in pd and respond accordingly, for example, by turning on a warning siren if a refrigerator becomes too warm, or turning on a light when it becomes dark.

Typical circuits are shown in figures 13(b) and13(c).

In figure 13(b) a thermistor is used instead of one of the fixed resistors.

### Worked example

**1** A potential divider consists of two resistors in series with a battery of 18 V. The resistors have resistances 3.0 Ω and 6.0 Ω. Calculate, for each resistor:

**a)** the pd across it

**b)** the current in it.

### Solution

**a)** The pd across the 3.0 Ω resistors $\frac{V_{in} \times R_1}{(R_1 + R_2)} = \frac{18 \times 3}{3 + 6}$ = 6.0 V. The pd across the 6.0 Ω is then 18 – 6.0 V = 12 V.

**b)** The current is the same in both resistors because they are in series. The total resistance is 9.0 Ω and the emf of the battery is 18 V. The current = $\frac{V}{R} = \frac{18}{9} = 2.0$ A.

Recall that when a thermistor is at a high temperature its resistance is small, and that the resistance increases when the temperature falls.

Rather than calculating the values, for this example we will use our knowledge of how pd and current are related to work out the behaviour of the circuit from first principles.

Suppose the temperature is low and that the thermistor resistance is high relative to that of the fixed-value resistor. Most of the pd will be dropped across the thermistor and very little across the fixed resistor. If you cannot see this straight away, remember the equations from the previous section:

$$V_{\text{thermistor}} = \frac{V_{\text{in}} R_{\text{thermistor}}}{(R_{\text{thermistor}} + R_{\text{resistor}})} \text{ and } V_{\text{resistor}} = \frac{V_{\text{in}} R_{\text{resistor}}}{(R_{\text{thermistor}} + R_{\text{resistor}})}$$

If $R_{\text{thermistor}} \gg R_{\text{resistor}}$ ($\gg$ means "much greater than")
then $R_{\text{thermistor}} \approx (R_{\text{thermistor}} + R_{\text{resistor}})$ and $V_{\text{thermistor}} \approx V_{\text{in}}$.

So the larger resistance (the thermistor at low temperatures) has the larger pd across it.

If the thermistor temperature now increases, then the thermistor resistance will fall. Now the fixed-value resistor will have the larger resistance and the pds will be reversed with the thermistor having the small voltage drop across it. A voltage sensor connected to a computer can be set to detect this voltage change and can activate an alarm if the thermistor has too high a temperature.

You may be asking what the resistance of the fixed resistor should be. The answer is that it is normally set equal to that of the thermistor when it is at its optimum (average) temperature. Then any deviation from the average will change the potential difference and trigger the appropriate change in the sensing circuit.

The same principle can be applied to another sensor device, a light-dependent resistor (LDR). This, like the thermistor, is made of semiconducting material but this time it is sensitive to photons incident on it. When the light intensity is large, charge carriers are released in the LDR and thus the resistance falls. When the intensity is low, the resistance is high as the charge carriers now re-combine with their atoms.

Look at figure 13(c). You should be able to explain that when the LDR is in bright conditions then the pd across the fixed resistor is high.

## Using a potential divider to give a variable pd

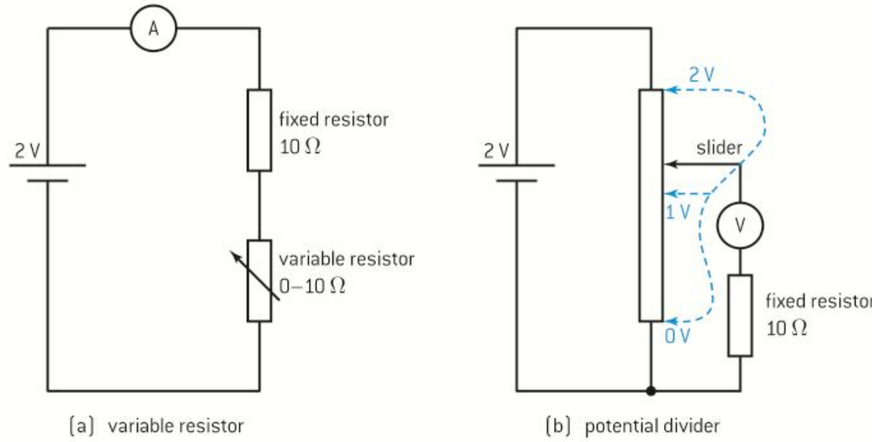
A variable resistor circuit is shown in figure 14(a). It consists of a power supply, an ammeter, a variable resistor and a resistor. The value of each component is given on the diagram.

We can predict the way this circuit will behave.

When the variable resistor is set to its minimum value, 0 Ω, then there will be a pd of 2 V across the resistor and a current of 0.2 A in the circuit.

When the variable resistor is set to its maximum value, 10 Ω, then the total resistance in the circuit is 20 Ω, and the current is 0.1 A.

This means that with 0.1 A in the 10 Ω fixed resistor, only 1 V is dropped across it. Therefore the range of pd across the fixed resistor can only vary from 1 V to 2 V – half of the available pd that the power supply can in principle provide. You should now be able to predict the range across the variable resistor.




▲ Figure 14

The limited range is a significant limitation in the use of the variable resistor. To achieve a better range, we could use a variable resistor with a much higher range of resistance. To get a pd of 0.1 V across the fixed resistor the resistance of the variable resistor has to be about 200 Ω. If the fixed resistor had a much greater resistance, then the variable resistor would need an even higher value too and this would limit the current.

The potential divider arrangement (figure 14(b)) allows a much greater range of pd to the component under test than does a variable resistor in series with the component. In a potential divider, the same variable resistor can be used but the set up is different and involves the use of the three terminals on the variable resistor. (A variable resistor is also sometimes called a **rheostat**.) One terminal is connected to one side of the cell, and the other end of the rheostat resistor is connected to the other terminal of the cell.

The potential at any point along the resistance winding depends on the position of the slider (or wiper) that can be swept across the windings from one end to the other. Typical values for the potentials at various points on the windings are shown for the three blue slider positions on figure 14(b). The component that is under test (again, a resistor in this case) is connected in a secondary circuit between one terminal of the resistance winding and the slider on the rheostat. When the slider is positioned at one end, the full 2 V from the cell is available to the resistor under test. When at the other end, the pd between the ends of the resistor is 0 V (the two leads to the resistor are effectively connected directly to each other at the variable resistor).

You should know how to set this arrangement up and also how to draw the circuit and explain its use.

### Worked examples

**1** A light sensor consists of a 6.0 V battery, a 1800 Ω resistor and a light-dependent resistor in series. When the LDR is in darkness the pd across the resistor is 1.2 V.

**a)** Calculate the resistance of the LDR when it is in darkness.

**b)** When the sensor is in the light, its resistance falls to 2400 Ω. Calculate the pd across the LDR.

#### Solution

**a)** As the pd across the resistor is 1.2 V, the pd across the LDR must be 6 – 1.2 = 4.8 V.

The current in the circuit is
$I = \frac{V}{R} = \frac{1.2}{1800} = 0.67$ mA.
The resistance of the LDR is
$\frac{V}{I} = \frac{4.8}{0.67 \times 10^{-3}} = 7200\ \Omega$.

**b)** The ratio of $\frac{\text{resistance across LDR}}{\text{resistance across } 1800\ \Omega} = \frac{2400}{1800} = 1.33$.
This is the same value as $\frac{\text{pd across LDR}}{\text{pd across } 1800\ \Omega}$. For the ratio of pds to be 1.33, the pds must be 2.6 V and 3.4 V with the 3.4 V across the LDR.

**2** A thermistor is connected in series with a fixed resistor and a battery. Describe and explain how the pd across the thermistor varies with temperature.

#### Solution

As the temperature of the thermistor rises, its resistance falls. The ratio of the pd across the fixed resistor to the pd across the thermistor rises too because the thermistor resistance is dropping. As the pd across the fixed resistor and thermistor is constant, the pd across the thermistor must fall.

The change in resistance in the thermistor occurs because more charge carriers are released as the temperature rises. Even though the movement of the charge carriers is impeded at higher temperatures, the release of extra carriers means that the resistance of the material decreases.

### Investigate!

#### Variable resistor or potential divider?

- Set up the two circuits shown in figure 14. Match the value of the fixed resistor to the variable resistor – they do not need to be exactly the same but should be reasonably close. Add a voltmeter connected across the fixed resistor to check the pd that is available across it.
- Make sure that the maximum current rating for the fixed resistor and the variable resistor cannot be exceeded.
- Check the pd available in the two cases and convince yourself that the potential divider gives a wider range of voltages.

## Heating effect equations

We saw earlier that the power $P$ dissipated in a component is related to the pd $V$ across the component and the current $I$ in it:

$$P = IV$$

The energy $E$ converted in time $\Delta t$ is

$$E = IV\Delta t$$

When either $V$ or $I$ are unknown, then two more equations become available:

$$P = IV = I^2R = \frac{V^2}{R}$$

and

$$E = IV\Delta t = I^2R\Delta t = \frac{V^2}{R}\,\Delta t$$

These equations will allow you to calculate the energy converted in electrical heaters and lamps and so on. Applications that you may come across include heating calculations, and determining the consumption of energy in domestic and industrial situations.

### Worked examples

**1** Calculate the power dissipated in a 250 Ω resistor when the pd across it is 10 V.

### Solution

$$P = \frac{V^2}{R} = \frac{10^2}{250} = 0.40\ \text{W}$$

**2** A 9.0 kW electrical heater for a shower is designed for use on a 250 V mains supply. Calculate the current in the heater.

### Solution

$$P = IV \text{ so } I = \frac{P}{V} = \frac{9000}{250} = 36\ \text{A}$$

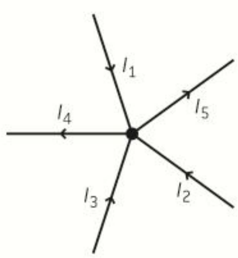
**3** Calculate the resistance of the heating element in a 2.0 kW electric heater that is designed for a 110 V mains supply.

### Solution

$$P = \frac{V^2}{R};\ R = \frac{V^2}{P} = \frac{110^2}{2000} = 6.1\ \text{A}.$$

## Kirchhoff's first and second laws

This section contains no new physics, but it will consolidate your knowledge and put the electrical theory you have learnt so far into a wider physical context.



▲ Figure 15 Kirchhoff's first law.

We saw that the charge carriers in a conductor move into and out of the conductor at equal rates. If $10^6$ flow into a conductor in one second, then $10^6$ must flow out during the same time to avoid the buildup of a static charge.

We also considered what happens when current splits into two or more parts at the junction where a parallel circuit begins. This can be taken one step further to a situation where there is more than one incoming current at the junction too.

Figure 15 shows a junction with three incoming currents and two outgoing ones. Our rule about the incoming charge equating to the outgoing charge must apply here too, so algebraically:

$$I_1 + I_2 + I_3 = I_4 + I_5$$

A general way to write this is to use the Σ sign (meaning "add up everything"), so that

**for any junction $\sum I = 0$**

When using the Σ notation, remember to get the signs of the currents correct. Call any current in *positive*, and any current out *negative*.

In words this can be written as

*the sum of the currents into a junction equals the sum of the currents away from a junction*
or
*total charge flowing into a junction equals the total charge flowing away from the junction.*

This important rule was first quoted by the Prussian physicist Gustav Kirchhoff in 1845. It is known as **Kirchhoff's first law**.

It is equivalent to a statement of **conservation of charge**.

Kirchhoff devised a second law which is also a conservation rule this time of energy.

In any electric circuit there are sources of emf (often a cell or a battery of cells for dc) and sinks of pd (typically, lamps, heating coils, resistors, and thermistors). A general rule in physics is that energy is conserved. Electrical components have to obey this too. So, in any electrical circuit, the energy being converted into electrical energy (in the sources of emf) must be equal to the energy being transferred from electrical to internal, by the sinks of pd.

This is **Kirchhoff's second law** equivalent to **conservation of energy**.

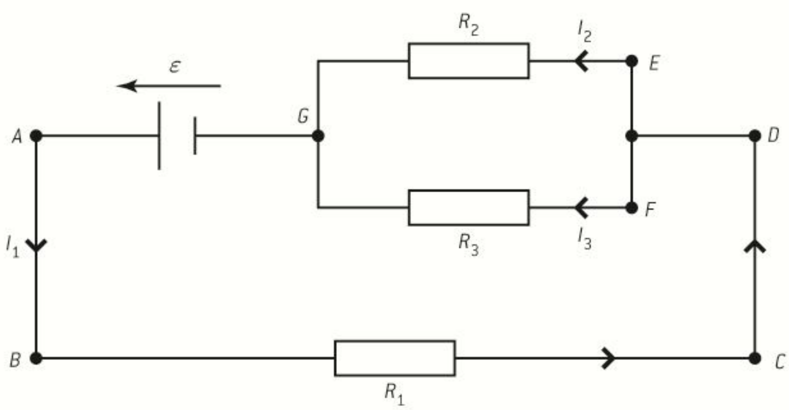
This second law applies to all closed circuits – both simple and complex.

In words, Kirchhoff's second law can be written as

*in a complete circuit loop, the sum of the emfs in the loop is equal to the sum of the potential differences in the loop*
or
*the sum of all variations of potential in a closed loop equals zero*

In symbols

*for any closed loop in a circuit* $\sum \varepsilon = \sum IR$




▲ Figure 16 Kirchhoff's second law.

Look at the circuit in figure 16, it has parallel and series elements in it. A number of possible loops are drawn and analysed:

***Loop GABCDEG travelling anticlockwise round the loop***

This loop begins at the cell and goes around the circuit, through resistor $R_1$ and resistor $R_2$ finally ending at the cell again. In this loop there is one source of emf and two sinks of pd (ignoring the leads, which we assume have zero resistance).

So

$\varepsilon$ (*the emf of the cell*) $= I_1R_1 + I_2R_2$ (*the total pds aross the resistors*)

The direction of loop travel and the current direction are in all cases the same. We give a positive sign to the currents when this is the case. The emf of the cell is driving in the same direction as the loop travel direction; it gets a positive sign as well. If the loop direction and the current or emf were to be opposed then they would be given a negative sign.

***loop EFGE travelling clockwise round the loop***

This loop goes first through resistor $R_3$ and the loop direction is in the same direction as the conventional current. Next the loop goes through resistor $R_1$ but this time the current direction and the loop are different so there has to be a negative sign. There is no source of emf in the loop so the Kirchhoff equation becomes

$$0 = I_3R_3 - I_2R_2$$

Kirchhoff's first law can be applied at point G. The total current into point G is $I_2 + I_3$; the total out is $I_1$. The application of the law is $I_1 = I_2 + I_3$.

There are now three separate equations with three unknowns and these equations can be solved to work out the currents in each part of the circuit assuming that we know the value for the emf of the cell and the values of the resistances in the circuit.

By setting up a series of loops it is possible to work out the currents and pds for complicated resistor networks, more complicated than could be done using the resistor series and parallel rules alone.

## Ideal and non-ideal meters

So far in this topic we have assumed (without mentioning it!) that the meters used in the circuits were ideal. Ideal meters have no effect on the circuits that they are measuring. We would always want this to be true but, unfortunately, the meters we use in real circuits in the laboratory are not so obliging and we need to know how to allow for this.

Ammeters are placed in series with components so the ammeter has the same current as the components. It is undesirable for the ammeter to change the current in a circuit but, if the ammeter has a resistance of its own, then this is what will happen. An ideal ammeter has zero resistance – clearly not attainable in practice as the coils or circuits inside the ammeter have resistance.

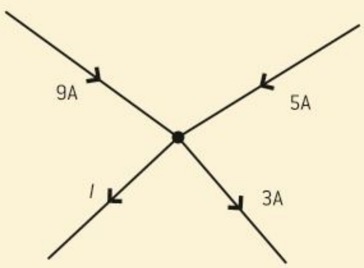
Voltmeters are placed in parallel with the device or parts of a circuit they are measuring. In an ideal world, the voltmeter will not require any energy for its coil to move (if it is a moving-coil type) or for its analogue to digital conversion (if it is a digital meter). The way to avoid current in the voltmeter is for the meter to have an infinite resistance – again, not an attainable situation in practice.

Some modern digital meters can get very close to these ideals of zero ohm for ammeters and infinite ohms for voltmeters. Digital meters are used more and more in modern science.

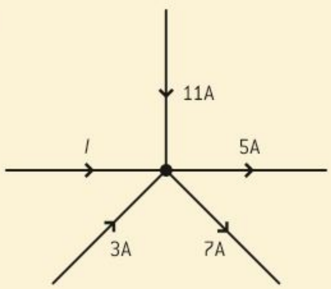
## Worked examples

**1** Calculate the unknown branch current in the following junctions.

**a)**




**b)**




### Solution

**a)** Current into the junction = 9 + 5A

Current out of the junction = 3 + $I$A

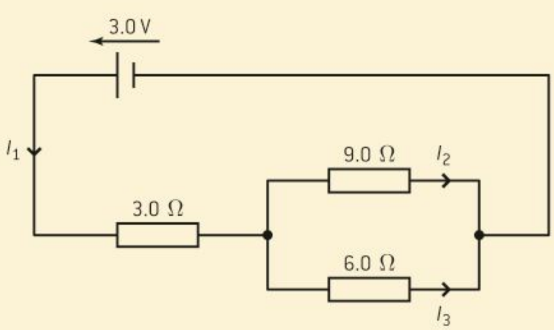
$I = 9 + 5 - 3 = 11$ A out of the junction.

**b)** Current into the junction = $I + 11 + 3 = I + 14$A

Current out of the junction = 5 + 7 = 12A

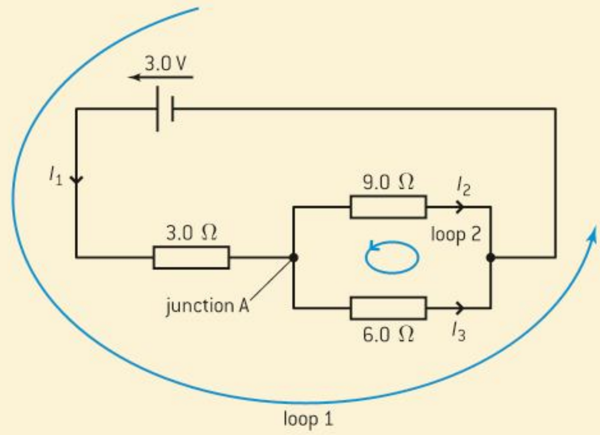
$I = 12 - 14 = -2$ A, so the current I is directed away from the junction.

**2** Calculate the currents in the circuit shown.




### Solution

(This circuit can be analysed using the resistors in series and parallel equations. It is given here to show the technique and to convince you that it works.)




Current directions have been assigned and two loops 1 and 2 and junction A defined in the diagram.

For loop 1

$$3 = 3I_1 + 9I_2 \qquad \textit{[equation 1]}$$

(the emf in the loop is 3 V)

for loop 2

$$0 = 6I_3 - 9I_2 \qquad \textit{[equation 2]}$$

(there is no source of emf in this loop, current $I_2$ is in the opposite direction to the loop direction 0.

For junction A

$$I_1 = I_2 + I_3$$

so

$$0 = 9I_2 - 6I_1 + 6I_2$$

$$0 = 15I_2 - 6I_1$$

And from equation 1

$$6 = 6I_1 + 18I_2$$

Adding the equations

$$6 = 33I_2$$

$$I_2 = 0.18 \text{ A}$$

Substituting gives:

$$I_1 = 0.45 \text{ A}$$

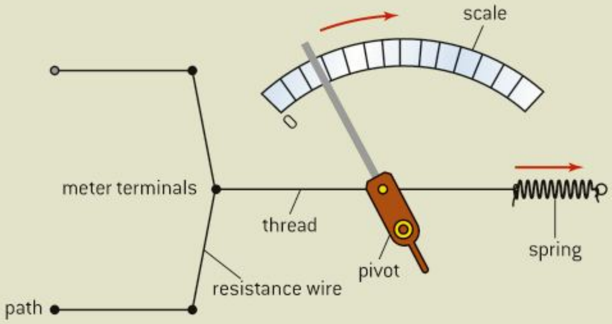
and

$$I_3 = 0.27 \text{ A}$$

## Nature of science

### How times change!

Digital meters are a good way to get very close to the ideal meter. The resistance between the input terminals can be made to be very large ($10^{12}$ Ω or more) for a voltmeter and to have a very small value for an ammeter. The meters themselves are easy to read without any judgements required about what the pointer indicates. It was not always like this.




▲ Figure 17

An early form of meter was the hot-wire ammeter. This uses the heating effect of a current directly to increase the temperature of a metal wire (the current flows through the wire). There are a number of forms of the ammeter, but in the type shown here, a spring keeps the wire under tension. As the wire expands with the increase in temperature, any point on the wire moves to the right. A pivoted pointer is attached to the insulated thread. As the wire expands, the spring pulls the thread causing the pointer to rotate about the pivot; a reading of current can now be made. The scale is usually extremely non-linear.

### Worked examples

**1** A 250 Ω resistor is connected in series with a 500 Ω resistor and a 6.0 V battery.

**a)** Calculate the pd across the 250 Ω resistor.

**b)** Calculate the pd that will be measured across the 250 Ω resistor if a voltmeter of resistance 1000 Ω is connected in parallel with it.

#### Solution

**a)** The pd across the 250 Ω resistor

$$= \frac{V_{in} \times R_1}{(R_1 + R_2)} \frac{6 \times 250}{(250 + 500)} = 2.0 \text{ V}.$$

**b)** When the voltmeter is connected, the resistance of the parallel combination is

$$R = \frac{R_1 R_2}{(R_1 + R_2)} = \frac{250 \times 1000}{1250} = 200\ \Omega$$

$$V = \frac{200 \times 6}{700} = 1.7 \text{ V}$$

**2** An ammeter with a resistance of 5.0 Ω is connected in series with a 3.0 V cell and a lamp rated at 300 mA, 3 V. Calculate the current that the ammeter will measure.

#### Solution

Resistance of lamp $= \frac{V}{I} = \frac{3}{0.3} = 10\ \Omega$ . Total resistance in circuit $= 10 + 5 = 15\ \Omega$. So current in circuit $= \frac{V}{R} = \frac{3}{15} = 200$ mA. This assumes that the resistance of the lamp does not vary between 0.2 A and 0.3 A