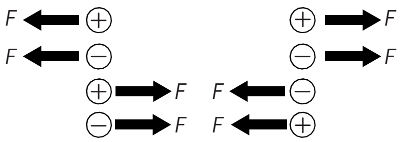
Electric charge and Coulomb's la

CONSERVATION OF CHARGE

Two types o charge exist – positive and negative. Equal amounts o positive and negative charge cancel each other. Matter that contains no charge, or matter that contains equal amounts o positive and negative charge, is said to be electrically **neutral**.

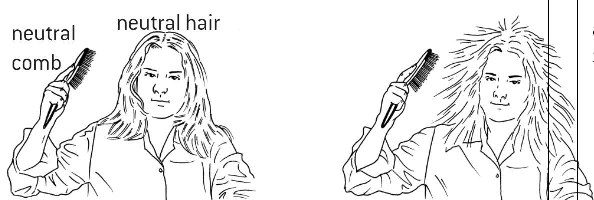
Charges are known to exist because o the orces that exist between all charges, called the **electrostatic force**: like charges repel, unlike charges attract.



A very important experimental observation is that charge is always conserved.

Charged objects can be created by riction. In this process electrons are physically moved rom one object to another. In order or the charge to remain on the object, it normally needs to be an insulator.

before



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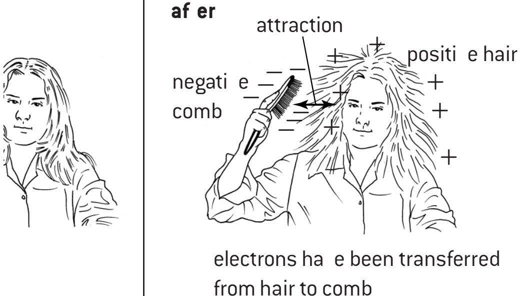
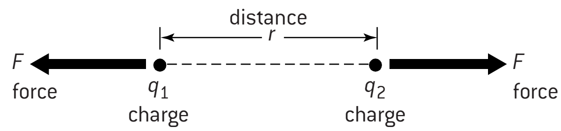


The total charge be ore any process must be equal to the total charge a terwards. It is impossible to create a positive charge without an equal negative charge. This is the law o conservation o charge.

COULOMB'S LAW

The diagram shows the orce between two point charges that are ar away rom the influence o any other charges.



The directions o the orces are along the line joining the charges. I they are like charges, the orces are away rom each other – they repel. I they are unlike charges, the orces are towards each other – they attract.

Each charge must eel a orce o the same size as the orce on the other one.

Experimentally, the orce is proportional to the size o both charges and inversely proportional to the square o the distance between the charges.

$$F = \frac{kq_1q_2}{r^2} = k\frac{q_1q_2}{r^2}$$

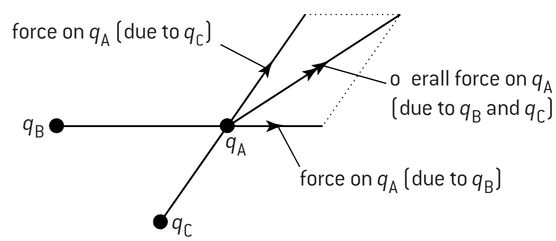
This is known as Coulomb's law and the constant k is called the Coulomb constant. In act, the law is o ten quoted in a slightly di erent orm using a di erent constant or the medium called the permittivity, ε.

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

value of rst charge; value of second charge; force between two point charges; distance between the charges; constants; permittivity of free space (a constant)

$$k = \frac{1}{4\pi\varepsilon_0}$$

I there are two or more charges near another charge, the overall orce can be worked out using vector addition.



Vec or addi ion of elec ros a ic forces

CONDUCTORS AND INSULATORS

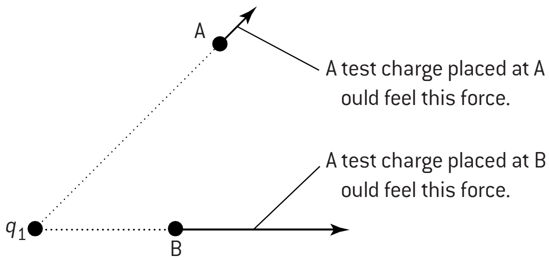
A material that allows the flow o charge through it is called an electrical **conductor**. I charge cannot flow through a material it is called an electrical **insulator**. In solid conductors the flow o charge is always as a result o the flow o electrons rom atom to atom.

Electrical conductors	Electrical insulators
all metals	plastics
e.g. copper	e.g. polythene
aluminium	nylon
brass	acetate
graphite	rubber
	dry wood
	glass
	ceramics

Elec ric fields

ELECTRIC FIELDS – DEFINITION

A charge, or combination o charges, is said to produce an **electric field** around it. I we place a **test charge** at any point in the field, the value o the orce that it eels at any point will depend on the value o the test charge only.



A test charge would eel a di erent orce at di erent points around a charge q_1.

In practical situations, the test charge needs to be small so that it doesn't disturb the charge or charges that are being considered.

The definition o electric field, E, is

$$E = \frac{F}{q_2} = \text{ orce per unit positive point test charge.}$$

Coulomb's law can be used to relate the electric field around a point charge to the charge producing the field.

$$E = \frac{q_1}{4\pi\varepsilon_0 \ {}^2}$$

When using these equations you have to be very care ul:

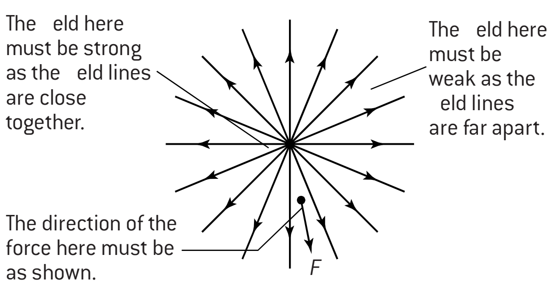
- not to muddle up the charge producing the field and the charge sitting in the field (and thus eeling a orce)
- not to use the mathematical equation or the field around a point charge or other situations (e.g. parallel plates).

REPRESENTATION OF ELECTRIC FIELDS

This is done using field lines.

At any point in a field:

- the direction o field is represented by the direction o the field lines closest to that point
- the magnitude o the field is represented by the number o field lines passing near that point.



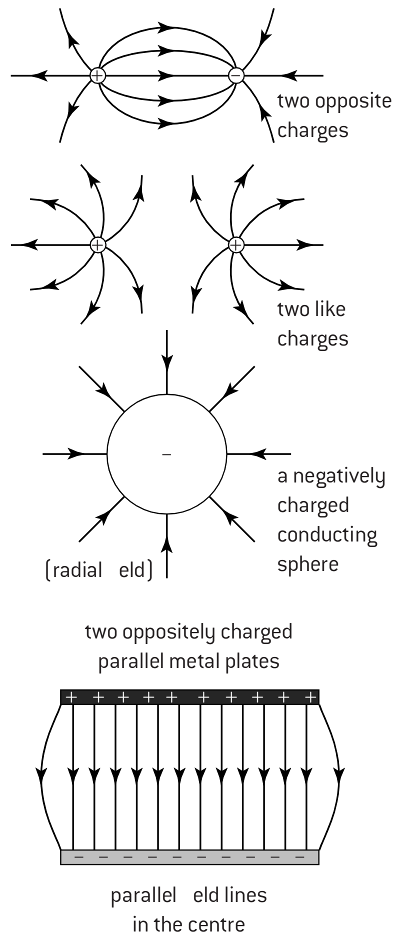
Field around a positive point charge

The resultant electric field at any position due to a collection o point charges is shown to the right.

The parallel field lines between two plates mean that the electric field is uni orm.

Electric field lines:

- begin on positive charges and end on negative charges
- never cross
- are close together when the field is strong.

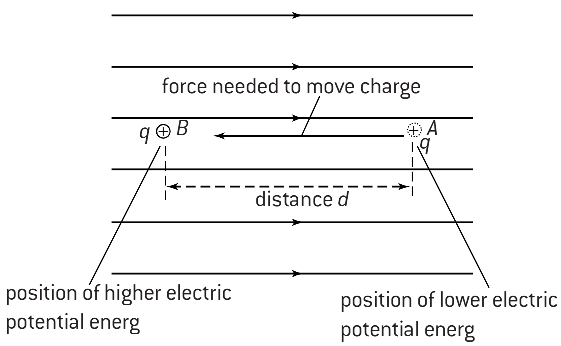


Patterns o electric fields

Electric potential energ   and electric potential difference

ENERGY DIFFERENCE IN AN ELECTRIC FIELD

When placed in an electric field, a charge eels a orce. This means that i it moves around in an electric field work will be done. As a result, the charge will either gain or lose electric potential energy. Electric potential energy is the energy that a charge has as a result o its position in an electric field. This is the same idea as a mass in a gravitational field. I we li t a mass up, its gravitational potential energy increases. I the mass alls, its gravitational potential energy decreases. In the example below a positive charge is moved rom position A to position B. This results in an increase in electric potential energy. Since the field is uni orm, the orce is constant. This makes it very easy to calculate the work done.



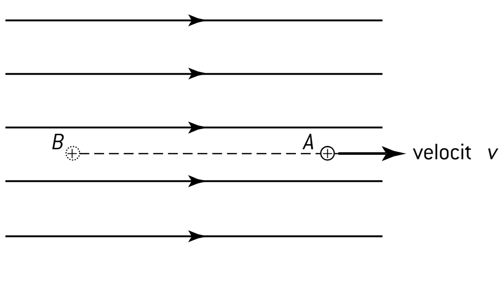
Charge moving in an electric field

Change in electric potential energy = orce × distance

$$= E\,q \times d$$

See page 52 or a definition o electric field, E.

In the example above the electric potential energy at B is greater than the electric potential energy at A. We would have to put in this amount o work to push the charge rom A to B. I we let go o the charge at B it would be pushed by the electric field. This push would accelerate it so that the loss in electrical potential energy would be the same as the gain in kinetic energy.



A positive charge released at B will be accelerated as it travels to point A.

gain in kinetic energy = loss in electric potential energy

$$\tfrac{1}{2}mv^2 = Eqd$$

$$mv^2 = 2Eqd$$

$$\therefore\ v = \sqrt{\frac{2Eqd}{m}}$$

ELECTRIC POTENTIAL DIFFERENCE

In the example on the le t, the actual energy di erence between A and B depended on the charge that was moved. I we doubled the charge we would double the energy di erence. The quantity that remains fixed between A and B is the energy di erence **per unit charge**. This is called the **potential difference**, or **pd**, between the points.

Potential di erence between two points = energy di erence per unit charge moved

$$= \frac{\text{energy di erence}}{\text{charge}} = \frac{\text{work done}}{\text{charge}}$$

$$V = \frac{W}{q}$$

The basic unit or potential di erence is the joule/coulomb, $J\,C^{-1}$. A very important point to note is that or a given electric field, the potential di erence between any two points is a single fixed scalar quantity. The work done between these two points does not depend on the path taken by the test charge. A technical way o saying this is 'the electric field is **conservative**'.

UNITS

The smallest amount o negative charge available is the charge on an electron; the smallest amount o positive charge is the charge on a proton. In everyday situations this unit is ar too small so we use the **coulomb**, **C**. One coulomb o negative charge is the charge carried by a total o 6.25×10^{18} electrons.

From its definition, the unit o potential di erence (pd) is $J\,C^{-1}$. This is given a new name, the volt, V. Thus:

1 volt = 1 J C⁻¹

Voltage and potential di erence are di erent words or the same thing. Potential di erence is probably the better name to use as it reminds you that it is measuring the di erence between two points.

When working at the atomic scale, the joule is ar too big to use or a unit or energy. The everyday unit used by physicists or this situation is the electronvolt. As could be guessed rom its name, the electronvolt is simply the energy that would be gained by an electron moving through a potential di erence o 1 volt.

$$1 \text{ electronvolt} = 1 \text{ volt} \times 1.6 \times 10^{-19}\ \text{C}$$

$$= 1.6 \times 10^{-19}\ \text{J}$$

The normal SI prefixes also apply so one can measure energies in kiloelectronvolts (keV) or megaelectronvolts (MeV). The latter unit is very common in particle physics.

Example

Calculate the speed o an electron accelerated in a vacuum by a pd o 1000 V (energy = 1 KeV).

$$\text{KE o electron} = V \times e = 1000 \times 1.6 \times 10^{-19}$$

$$= 1.6 \times 10^{-16}\ \text{J}$$

$$\frac{1}{2}mv^2 = 1.6 \times 10^{-16}\ \text{J}$$

$$v = 1.87 \times 10^{7}\ \text{m s}^{-1}$$

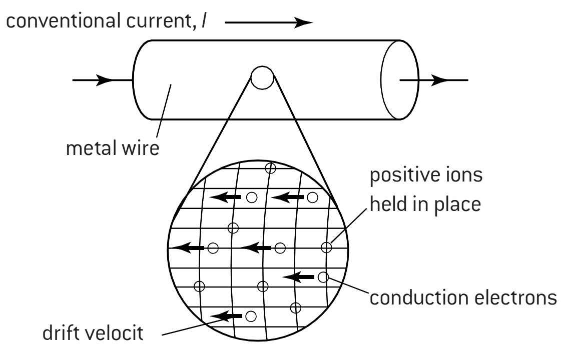
Electric current

ELECTRICAL CONDUCTION IN A METAL

Whenever charges move we say that a **current** is flowing. A current is the name or moving charges and the path that they ollow is called the **circuit**. Without a complete circuit, a current cannot be maintained or any length o time.

Current flows THROUGH an object when there is a potential di erence ACROSS the object. A battery (or power supply) is the device that creates the potential di erence.

By convention, currents are always represented as the flow o positive charge. Thus **conventional current**, as it is known, flows rom positive to negative. Although currents can flow in solids, liquids and gases, in most everyday electrical circuits the currents flow through wires. In this case the things that actually move are the negative electrons – the **conduction electrons**. The direction in which they move is opposite to the direction o the representation o conventional current. As they move the interactions between the conduction electrons and the lattice ions means that work needs to be done. There ore, when a current flows, the metal heats up. The speed o the electrons due to the current is called their **drift velocity**.



Electrical conduction in a metal

It is possible to estimate the dri t velocity o electrons using the generalized **drift speed equation**. All currents are comprised o the movement o charge-carriers and these could be positive or negative; not all currents involve just the movement o electrons. Suppose that the number density o the charge-carriers (the number per unit volume that are available to move) is n, the charge on each carrier is q and their average speed is .

In a time Δt,

the average distance moved by a charge-carrier = $\times \Delta t$

so volume o charge moved past a point = $A \times \; \Delta t$

so number o charge-carriers moved past a point = $n \times A \; \Delta t$

so charge moved past a point, $\Delta Q = nA \; \Delta t \times q$

$$\text{current } I = \frac{\Delta Q}{\Delta t}$$

$$I = nA \; q$$

It is interesting to compare:

- A typical dri t speed o an electron: 10^{-4} m s^{-1}
  (5A current in metal conductor o cross section 1 mm^2)
- The speeds o the electrons due to their random motion: 10^6 m s^{-1}
- The speed o an electrical signal down a conductor: approx. 3×10^8 m s^{-1}

CURRENT

Current is defined as the **rate of flow of electrical charge**. It is always given the symbol, I. Mathematically the definition or current is expressed as ollows:

$$\text{Current} = \frac{\text{charge flowed}}{\text{time taken}}$$

$$I = \frac{\Delta Q}{\Delta t} \text{ or (in calculus notation) } I = \frac{dQ}{dt}$$

$$1 \text{ ampere} = \frac{\textbf{1 coulomb}}{\textbf{1 second}}$$

$$1 \text{ A} = 1 \text{ C s}^{-1}$$

I a current flows in just one direction it is known as a **direct current**.
A current that constantly changes direction (first one way then the other) is known as an **alternating current** or ac.

In SI units, the ampere is the base unit and the coulomb is a derived unit

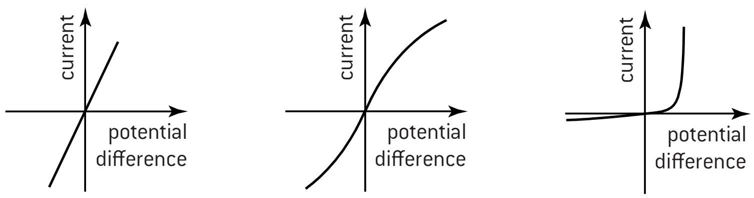
1 C = 1 A s

Electric circuits

OHM'S LAW – OHMIC AND NON-OHMIC BEHAVIOUR

The graphs below show how the current varies with potential di erence or some typical devices.

(a) metal at constant temperature **(b)** lament lamp **(c)** diode



I current and potential di erence are proportional (like the metal at constant temperature) the device is said to be **ohmic**. Devices where current and potential di erence are not proportional (like the filament lamp or the diode) are said to be **non-ohmic**.

Ohm's law states that the current flowing through a piece o metal is proportional to the potential di erence across it providing the temperature remains constant.

In symbols,

$V \quad I$ [i temperature is constant]

A device with constant resistance (in other words an ohmic device) is called a **resistor**.

RESISTANCE

Resistance is the mathematical ratio between potential di erence and current. I something has a high resistance, it means that you would need a large potential di erence across it in order to get a current to flow.

$$\text{Resistance} = \frac{\text{potential di erence}}{\text{current}}$$

In symbols, $R = \frac{V}{I}$

We define a new unit, the ohm, Ω, to be equal to one volt per amp.

1 ohm = 1 V A⁻¹

POWER DISSIPATION

$$\text{Since potential di erence} = \frac{\text{energy di erence}}{\text{charge flowed}}$$

$$\text{And current} = \frac{\text{charge flowed}}{\text{time taken}}$$

This means that potential di erence × current

$$= \frac{\text{(energy di erence)}}{\text{(charge flowed)}} \times \frac{\text{(charge flowed)}}{\text{(time taken)}} = \frac{\text{energy di erence}}{\text{time}}$$

This energy di erence per time is the power dissipated by the resistor. All this energy is going into heating up the resistor. In symbols:

$P = V \times I$

Sometimes it is more use ul to use this equation in a slightly di erent orm, e.g.

$P = V \times I$ but $V = I \times R$ so

$P = (I \times R) \times I$

$P = I^2 R$

Similarly $P = \frac{V^2}{R}$

CIRCUITS – KIRCHOFF'S CIRCUIT LAWS

An electric circuit can contain many di erent devices or **components**. The mathematical relationship $V = IR$ can be applied to any component or groups o components in a circuit.

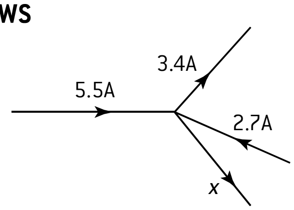
When analysing a circuit it is important to look at the circuit as a whole. The power supply is the device that is providing the energy, but it is the whole circuit that determines what current flows through the circuit.

Two undamental conservation laws apply when analysing circuits: the conservation o electric charge and the conservation o energy. These laws are collectively known as Kircho 's circuit laws and can be stated mathematically as:

First law: $\sum I = 0$ (junction)

Second law: $\sum V = 0$ (loop)

The first law states that the algebraic sum o the currents at any junction in the circuit is zero. The current flowing into a junction must be equal to the current flowing out o a junction. In the example (right) the unknown current $x = 5.5 + 2.7 - 3.4 = 4.8$ A



The second law states that around any loop, the total energy per unit charge must sum to zero. Any source o potential di erence within the loop must be completely dissipated across the components in the loop (potential drop across the component). Care needs to be taken to get the sign o any pd correct.

- I the chosen loop direction is rom the negative side o a battery to its positive side, this is an increase in potential and the value is positive when calculating the sum.
- I the direction around the loop is in the same direction as the current flowing through the component, this is a potential drop and the value is negative when calculating the sum.

EXAMPLE

A 1.2 kW electric kettle is plugged into the 250 V mains supply. Calculate

(i) the current drawn

(ii) its resistance

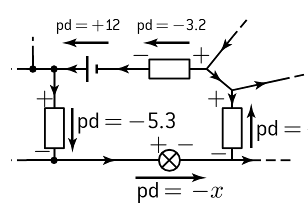
(i) $I = \frac{1200}{250} = 4.8$ A

(ii) $R = \frac{250}{4.8} = 52\ \Omega$

The example below shows one loop in a larger circuit. Anti-clockwise consideration o the loop means that:

$12.0 - 5.3 - x + 2.7 - 3.2 = 0.$

The potential di erence across the bulb, $x = 6.2$ V

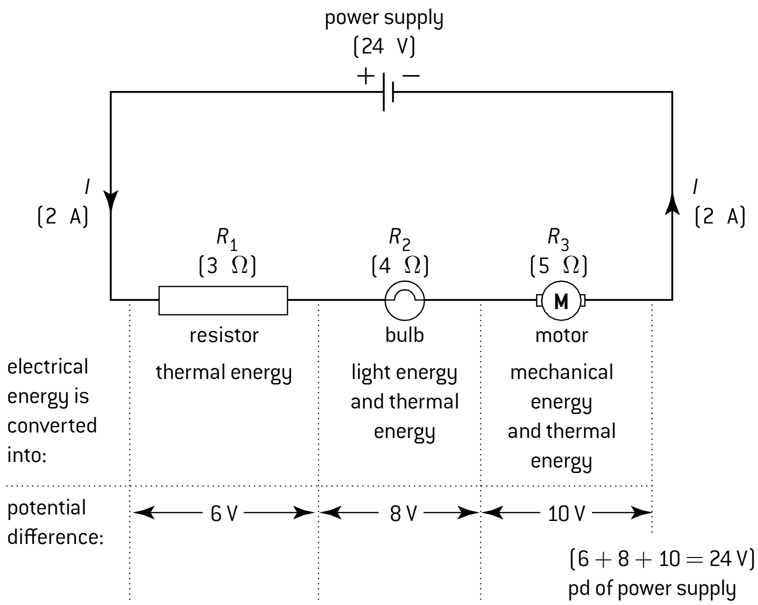


An example o the use o Kircho 's circuit laws is shown on page 59.

Resis ors in series and parallel

RESISTORS IN SERIES

A **series circuit** has components connected one a ter another in a continuous chain. The current must be the same everywhere in the circuit since charge is conserved. The total potential di erence is shared among the components.



Total resistance = 3Ω + 4 Ω + 5 Ω = 12 Ω

Example o a series circuit

We can work out what share they take by looking at each component in turn, e.g.

The potential di erence across the resistor $= I \times R_1$

The potential di erence across the bulb $= I \times R_2$

$\boldsymbol{R_{total} = R_1 + R_2 + R_3}$

This always applies to a series circuit. Note that $V = IR$ correctly calculates the potential di erence across each individual component as well as calculating it across the total.

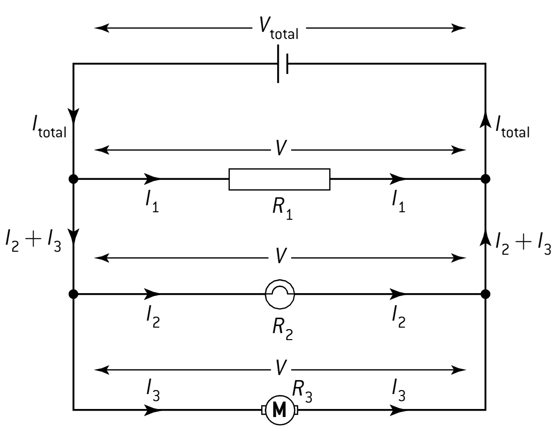
ELECTRICAL METERS

A current-measuring meter is called an **ammeter**. It should be connected in series at the point where the current needs to be measured. A per ect ammeter would have zero resistance.

A meter that measures potential di erence is called a **voltmeter**. It should be placed in parallel with the component or components being considered. A per ect voltmeter has infinite resistance.

RESISTORS IN PARALLEL

A **parallel circuit** branches and allows the charges more than one possible route around the circuit.



Example o a parallel circuit

Since the power supply fixes the potential di erence, each component has the same potential di erence across it. The total current is just the addition o the currents in each branch.

$$I_{total} = I_1 + I_2 + I_3$$

$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{total}} = \frac{1}{3} + \frac{1}{4} + \frac{1}{5}\ \Omega^{-1}$$

$$= \frac{20 + 15 + 12}{60}\ \Omega^{-1}$$

$$= \frac{47}{60}\ \Omega^{-1}$$

$$R_{total} = \frac{60}{47}\ \Omega$$

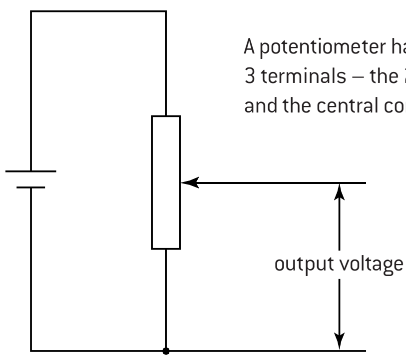
$$= 1.28\ \Omega$$

Potential divider circuits and sensors

POTENTIAL DIVIDER CIRCUIT

The example on the right is an example o a circuit involving a **potential divider**. It is so called because the two resistors 'divide up' the potential di erence o the battery. You can calculate the 'share' taken by one resistor rom the ratio o the resistances but this approach does not work unless the voltmeter's resistance is also considered. An ammeter's internal resistance also needs to be considered. One o the most common mistakes when solving problems involving electrical circuits is to assume the current or potential di erence remains constant a ter a change to the circuit. A ter a change, the only way to ensure your calculations are correct is to start again.

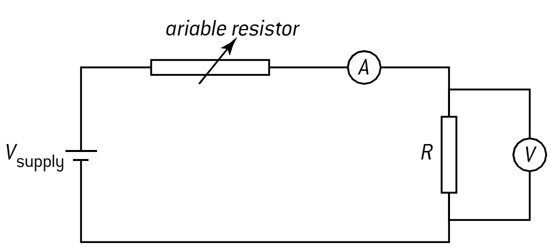
A variable potential divider (a **potentiometer**) is o ten the best way to produce a variable power supply. When designing the potential divider, the smallest resistor that is going to be connected needs to be taken into account: the potentiometer's resistance should be significantly smaller.



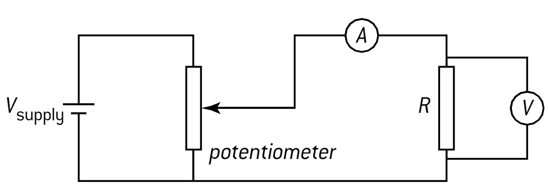
In order to measure the *V–I* characteristics o an unknown resistor *R*, the two circuits (*A* and *B*) below are constructed. Both will both provide a range o readings or the potential di erence, *V*, across and current, *I*, through *R*. Providing that *R* >> the resistance o the potentiometer, this circuit (circuit B) is pre erred because the range o readings is greater.

- Circuit *B* allows the potential di erence across *R* (and hence the current through *R*) to be reduced down to zero. Circuit *A* will not go below the minimum value achieved when the variable resistor is at its maximum value.
- Circuit *B* allows the potential di erence across *R* (and hence the current through *R*) to be increased up to the maximum value V_{supply} that can be supplied by the power supply in regular intervals. The range o values obtainable by Circuit *A* depends on a maximum o resistance o the variable resistor.

Circuit *A* – variable resistor



Circuit *B* – potentiometer

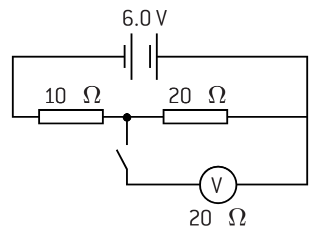


EXAMPLE

In the circuit below the voltmeter has a resistance o 20 kΩ. Calculate:

(a) the pd across the 20 kΩ resistor with the switch open

(b) the reading on the voltmeter with the switch closed.



(a) $\text{pd} = \frac{20}{(20 + 10)} \times 6.0 = 4.0\text{ V}$

(b) resistance o 20 kΩ resistor and voltmeter combination, *R*, given by:

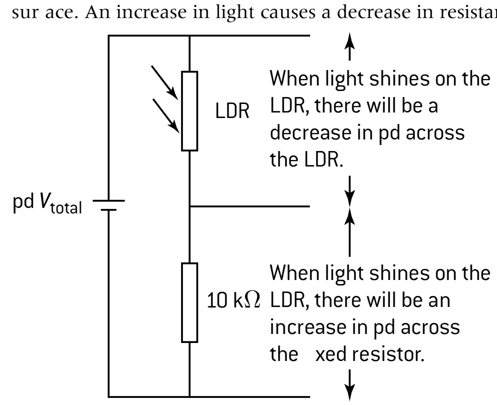
$$\frac{1}{R} = \frac{1}{20} + \frac{1}{20}\ \text{k}\Omega^{-1}$$

$$R = 10\text{ k}\Omega$$

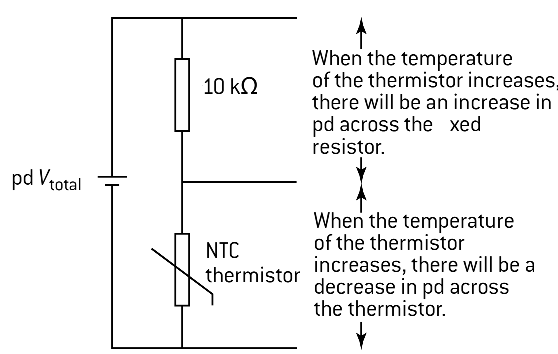
$$\text{pd} = \frac{10}{(10 + 10)} \times 6.0 = 3.0\text{ V}$$

SENSORS

A **light-dependent resistor** (**LDR**), is a device whose resistance depends on the amount o light shining on its sur ace. An increase in light causes a decrease in resistance.



A **thermistor** is a resistor whose value o resistance depends on its temperature. Most are semi-conducting devices that have a **negative temperature coe ficient** (**NTC**). This means that an increase in temperature causes a decrease in resistance. Both o these devices can be used in potential divider circuits to create sensor circuits. The output potential di erence o a **sensor circuit** depends on an external actor.



Resistivit

RESISTIVITY

The resistivity, ρ, o a material is defined in terms o its resistance, R, its length l and its cross-sectional area A.

$$R = \rho \frac{l}{A}$$

The units o resistivity must be ohm metres (Ω m). Note that this is the ohm multiplied by the metre, not 'ohms per metre'.

Example

The resistivity o copper is 3.3×10^{-7} Ω m; the resistance o a 100 m length o wire o cross-sectional area 1.0 mm^2 is:

$$R = 3.3 \times 10^{-7} \times \frac{100}{10^{-4}} = 0.3\ \Omega$$

INVESTIGATING RESISTANCE

The resistivity equation predicts that the resistance R o a substance will be:

a) Proportional to the length l o the substance

b) Inversely proportional to the cross-sectional area A o the substance.

These relationships can be predicted by considering resistors in series and in parallel:

a) Increasing l is like putting another resistor in series. Doubling l is the same as putting an identical resistor in series. R in series with R has an overall resistance o 2R. Doubling l means doubling R. So R l. A graph o R vs I will be a straight line going through the origin.

b) Increasing A is like putting another resistor in parallel. Doubling A is the same as putting an identical resistor in parallel. R in parallel with R has an overall resistance o $\frac{R}{2}$. Doubling A means halving R. So R $\frac{1}{A}$. A graph o R vs $\frac{1}{A}$ will be a straight line going through the origin.

To practically investigate these relationships, we have:

Independent variable:	Either l or A
Control variables:	A or l (depending on above choice); Temperature; Substance.
Data collection:	For each value o independent variable: • a range o values or V and I should be recorded • R can be calculated rom the gradient o a V vs I graph.
Data analysis	Values o R and the independent variable analysed graphically.

Possible sources o error/uncertainty include:

- Temperature variation o the substance (particularly i currents are high). Circuits should not be le t connected.
- The cross-sectional area o the wire is calculated by measuring the wire's diameter, d, and using $A = \pi\ ^2 = \frac{\pi d^2}{4}$. Several sets o measurements should be taken along the length o the wire and the readings in a set should be mutually perpendicular.
- The small value o the wire's diameter will mean that the uncertainties generated using a ruler will be large. This will be improved using a **vernier calliper** or a **micrometer**.

E ample of use of Kirchoff's laws

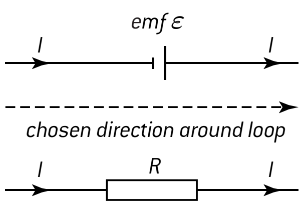
KIRCHOFF CIRCUIT LAWS EXAMPLE

Great care needs to be taken when applying Kirchoff's laws to ensure that every term in the equation is correctly identified as positive or negative. The concept of emf (see page 60) as sources of electrical energy can be used along with $V = IR$ to provide an alternative statement of the second law which may help avoid confusion: 'Round any closed circuit, the sum of the emfs is equal to the sum of the products of current and resistance'.

$$\sum(\text{emf}) = \sum(IR)$$

Process to follow

- Draw a full circuit diagram.
- It helps to set up the equations in symbols before substituting numbers and units.
- It helps to be as precise as possible. Potential difference V is a difference between two points in the circuit so specify which two points are being considered (use labels).
- Give the unknown currents symbols and mark their directions on the diagram. If you make a mistake and choose the wrong direction for a current, the solution to the equations will be negative.
- Use Kirchoff's first law to identify appropriate relationships between currents.
- Identify a loop to apply Kirchoff's second law. Go all around the loop in one direction (clockwise or anticlockwise) adding the emfs and $I \times R$ in senses shown below:



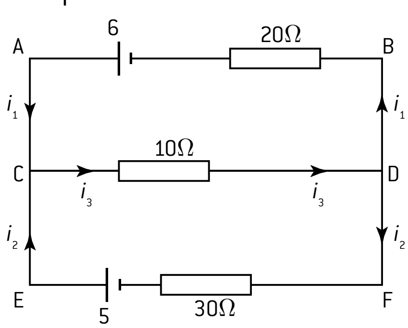
With chosen direction around loop in the direction shown, ε and IR are both ***positive*** *in the Kirchoff equation:*

$$\sum(\text{emf}) = \sum(IR)$$

(If chosen direction opposite to that shown, values are negative)

- The total number of different equations generated by Kirchoff's laws needs to be the same as the number of unknowns for the problem to be able to be solved.
- Use simultaneous equations to substitute and solve for the unknown values.
- A new loop can be identified to check that calculated values are correct.

Example



Kirchoff 1st law junction C(or D)

$$i_1 + i_2 = i_3 \qquad (1)$$

Kirchoff 2nd law and ACDB

$$10i_3 + 20i_1 = 6 \qquad (2)$$

Sub (1) into (2)

$$10\,(i_1 + i_2) + 20i_1 = 6$$

$$30i_1 + 10i_2 = 6 \qquad (3)$$

Kirchoff 2nd law and CEFD

$$-30i_2 - 10i_3 = -5 \qquad (4)$$

Sub (1) into (4)

$$30i_2 + 10(i_1 + i_2) = 5$$

$$10i_1 + 40i_2 = 5 \qquad (5)$$

$$(3) \times 4 \qquad 120i_1 + 40i_2 = 24 \qquad (6)$$

$$(6) - (5) \qquad 110i_1 = 19$$

$$i_1 = 0.1727\text{ A}$$

$$= \mathbf{172.7}\quad\textbf{A}$$

$$(3) \Rightarrow 10i_2 = 6 - 30i_1$$

$$= 0.8182$$

$$i_2 = 0.08182\text{ A}$$

$$= \mathbf{81.8}\quad\textbf{A}$$

$$i_3 = 172.7 + 81.8\text{ mA}$$

$$= \mathbf{254.5}\quad\textbf{A}$$

In ernal resis ance and cells

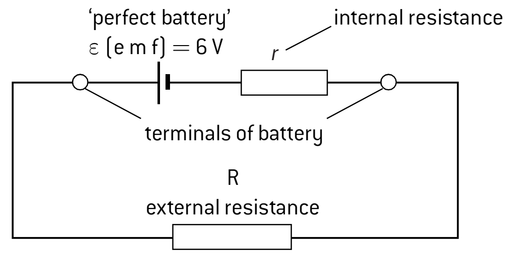
ELECTROMOTIVE FORCE AND INTERNAL RESISTANCE

When a 6V battery is connected in a circuit some energy will be used up inside the battery itself. In other words, the battery has some **internal resistance**. The TOTAL energy difference per unit charge around the circuit is still 6 volts, but some of this energy is used up inside the battery. The energy difference per unit charge from one terminal of the battery to the other is less than the total made available by the chemical reaction in the battery.

For historical reasons, the TOTAL energy difference per unit charge around a circuit is called the **electromotive force** (**emf**). However, remember that it is not a force (measured in newtons) but an energy difference per charge (measured in volts).

In practical terms, emf is exactly the same as potential difference if no current flows.

$\varepsilon = I\,(R + r)$



$$\text{e m f} = I \times R_{\text{total}}$$
$$= I(r + R)$$
$$= Ir + IR$$
$$IR = \text{emf} - Ir$$

terminal p d, V 'lost' volts

$$V = \varepsilon - Ir$$

CELLS AND BATTERIES

An electric **battery** is a device consisting of one or more cells joined together. In a cell, a chemical reaction takes place, which converts stored chemical energy into electrical energy. There are two different types of cell: primary and secondary.

A **primary** cell cannot be recharged. During the lifetime of the cell, the chemicals in the cell get used in a non-reversible reaction. Once a primary cell is no longer able to provide electrical energy, it is thrown away. Common examples include zinc–carbon batteries and alkaline batteries.

A **secondary** cell is designed to be recharged. The chemical reaction that produces the electrical energy is reversible. A reverse electrical current charges the cell allowing it to be reused many times. Common examples include a lead–acid car battery, nickel–cadmium and lithium-ion batteries.

The **charge capacity** of a cell is how much charge can flow before the cells stops working. Typical batteries have charge capacities that are measured in Amp-hours (A h). 1 A h is the charge that flows when a current of 1 A flows for one hour i.e. 1 A h = 3600 C.

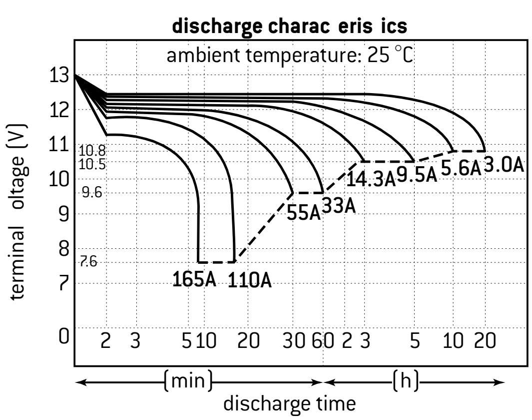
DISCHARGE CHARACTERISTICS

When current (and thus electrical energy) is drawn from a cell, the terminal potential difference varies with time. A perfect cell would maintain its terminal pd throughout its lifetime; real cells, however, do not. The terminal potential difference of a typical cell:

- loses its initial value quickly,
- has a stable and reasonably constant value for most of its lifetime.

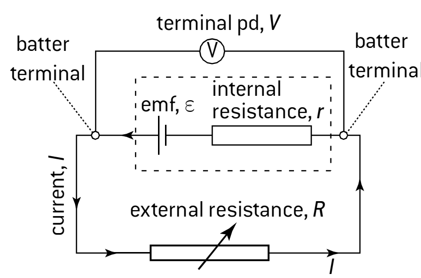
This is followed by a rapid decrease to zero (cell discharges).

The graph below shows the discharge characteristics for one particular type of lead–acid car battery.



DETERMINING INTERNAL RESISTANCE EXPERIMENTALLY

To experimentally determine the internal resistance r of a cell (and its emf ε), the circuit below can be used:



Procedure:

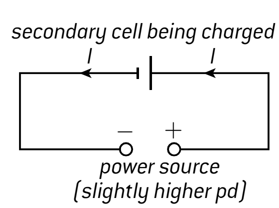
- Vary external resistance R to get a number (ideally 10 or more) of matching readings of V and I over as wide a range as possible.
- Repeat readings.
- Do not leave current running for too long (especially at high values of I).
- Take care that nothing overheats.

Data analysis:

- The relevant equation, $V = \varepsilon - Ir$ was introduced above.
- A plot of V on the -axis and I on the x-axis gives a straight line graph with
  - gradient $= -r$
  - -intercept $= \varepsilon$

RECHARGING SECONDARY CELLS

In order to recharge a secondary cell, it is connected to an external DC power source. The negative terminal of the secondary cell is connected to the negative terminal of the power source and the positive terminal of the power source with the positive terminal of the secondary cell. In order for a charging current, I, to flow, the voltage output of the power source must be slightly higher than that of the battery. A large difference between the power source and the cell's terminal potential difference means that the charging process will take less time but risks damaging the cell.



Magne ic force and fields

MAGNETIC FIELD LINES

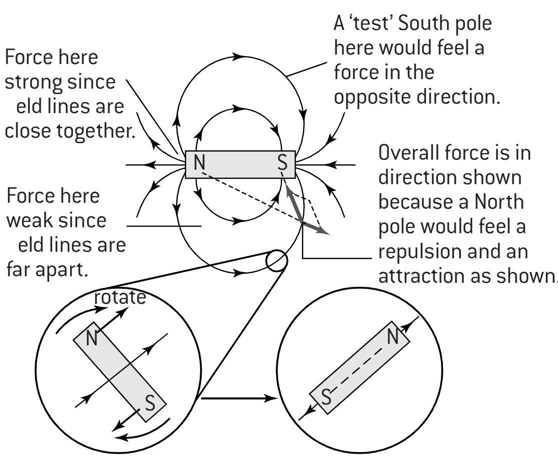
There are many similarities between the magnetic force and the electrostatic force. In fact, both forces have been shown to be two aspects of one force – the electromagnetic interaction (see page 78). It is, however, much easier to consider them as completely separate forces to avoid confusion.

Page 52 introduced the idea of electric fields. A similar concept is used for magnetic fields. A table of the comparisons between these two fields is shown below.

	Electric field	Magnetic field
Symbol	*E*	*B*
Caused by …	Charges	Magnets (or electric currents)
A ects …	Charges	Magnets (or electric currents)
Two types o …	Charge: positive and negative	Pole: North and South
Simple orce rule:	Like charges repel, unlike charges attract	Like poles repel, unlike poles attract

In order to help visualize a magnetic field we, once again, use the concept of field lines. This time the field lines are lines of magnetic field – also called **flux** lines. If a 'test' magnetic North pole is placed in a magnetic field, it will feel a force.

- The direction of the force is shown by the direction of the field lines.
- The strength of the force is shown by how close the lines are to one another.

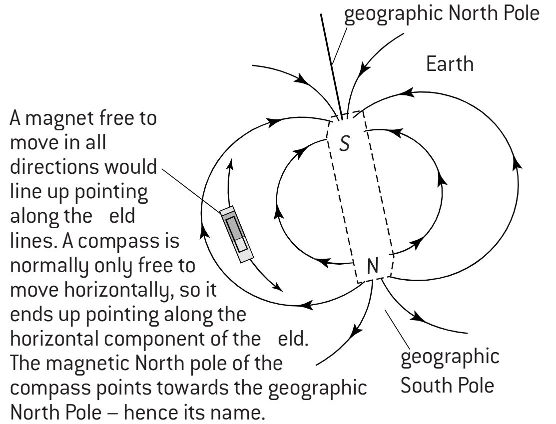


A small magnet placed in the field would rotate until lined up with the field lines. This is how a compass works. Small pieces of iron (iron filings) will also line up with the field lines – they willbe induced to become little magnets.

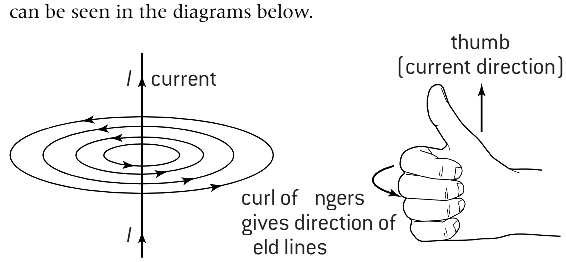
Field pattern of an isolated bar magnet

Despite all the similarities between electric fields and magnetic fields, it should be remembered that they are very different. For example:

- A magnet does not feel a force when placed in an electric field.
- A positive charge does not feel a force when placed stationary in a magnetic field.
- Isolated charges exist whereas isolated poles do not.
- The Earth itself has a magnetic field. It turns out to be similar to that of a bar magnet with a magnetic South pole near the geographic North Pole as shown below.



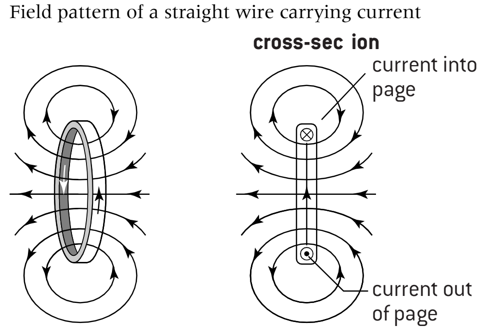
An electric current can also cause a magnetic field. The mathematical value of the magnetic fields produced in this way is given on page 63. The field patterns due to different currents can be seen in the diagrams below.



The field lines are circular around the current.

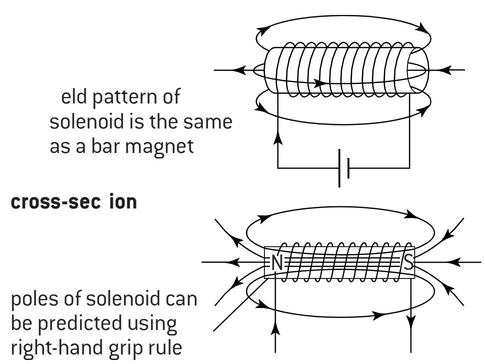
The direction of the field lines can be remembered with the right-hand grip rule. If the thumb of the right hand is arranged to point along the direction of a current, the way the fingers of the right hand naturally curl will give the direction of the field lines.

Field pattern of a straight wire carrying current



Field pattern of a flat circular coil

A long current-carrying coil is called a solenoid.

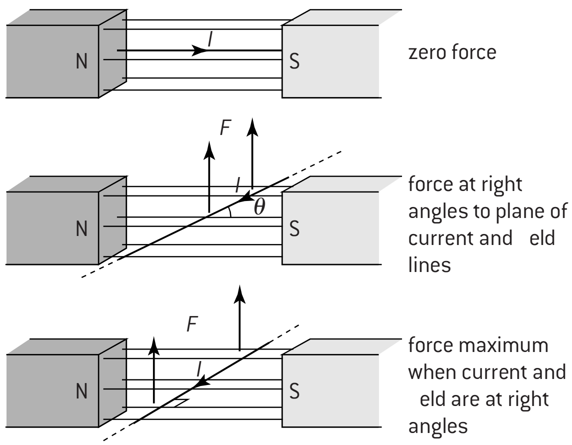


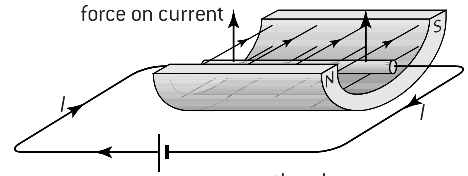
Field pattern for a solenoid

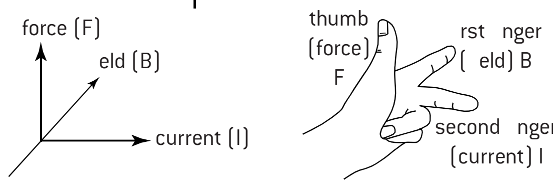
Magne ic forces

MAGNETIC FORCE ON A CURRENT

When a current-carrying wire is placed in a magnetic field the magnetic interaction between the two results in a orce. This is known as the **motor effect**. The direction o this orce is at right angles to the plane that contains the field and the current as shown below.







Fleming's le t-hand rule

Experiments show that the orce is proportional to:

- the magnitude o the magnetic field, B
- the magnitude o the current, I
- the length o the current, L, that is in the magnetic field
- the sine o the angle, θ, between the field and current.

The magnetic field strength, B is defined as ollows:

$$F = BIL \sin\theta \quad \text{or}$$

$$B = \frac{F}{IL \sin\theta}$$

A new unit, the tesla, is introduced. 1 T is defined to be equal to 1 N A^{-1} m^{-1}. Another possible unit or magnetic field strength is Wb m^{-2}. Another possible term is magnetic flux density.

MAGNETIC FORCE ON A MOVING CHARGE

A single charge moving through a magnetic field also eels a orce in exactly the same way that a current eels a orce.

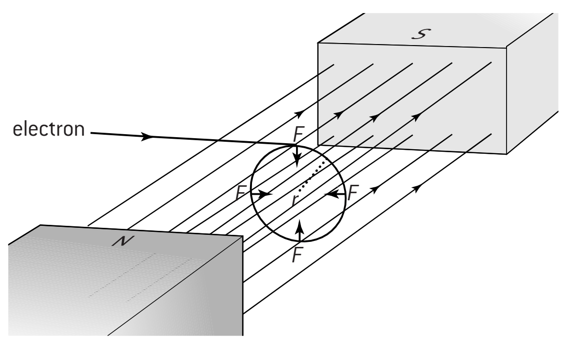
In this case the orce on a moving charge is proportional to:

- the magnitude o the magnetic field, B
- the magnitude o the charge, q
- the velocity o the charge, v
- the sine o the angle, θ, between the velocity o the charge and the field.

We can use these relationships to give an alternative definition o the magnetic field strength, B. This definition is exactly equivalent to the previous definition.

$$F = Bqv \sin\theta \quad \text{or} \quad B = \frac{F}{qv \sin\theta}$$

Since the orce on a moving charge is always at right angles to the velocity o the charge the resultant motion can be circular. An example o this would be when an electron enters a region where the magnetic field is at right angles to its velocity as shown below.



An electron moving at right angles to a magnetic field

Examples of he magne ic field due o curren s

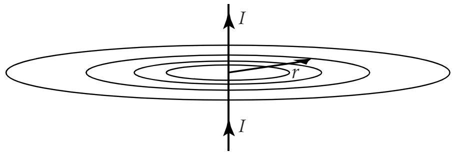
The formulae used on this page do not need to be remembered.

STRAIGHT WIRE

The field pattern around a long straight wire shows that as one moves away rom the wire, the strength o the field gets weaker. Experimentally the field is proportional to:

- the value o the current, I
- the inverse o the distance away rom the wire, r. I the distance away is doubled, the magnetic field will halve.
- The field also depends on the medium around the wire. These actors are summarized in the equation:

$$B = \frac{\mu I}{2\pi r}$$



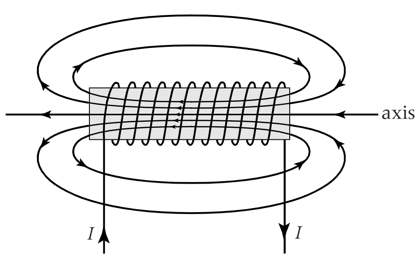
Magnetic field o a straight current

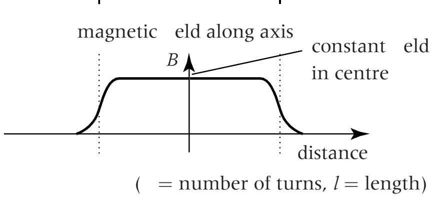
The constant μ is called the permeability and changes i the medium around the wire changes. Most o the time we consider the field around a wire when there is nothing there – so we use the value or the permeability o a vacuum, μ_0. There is almost no di erence between the permeability o air and the permeability o a vacuum. There are many possible units or this constant, but it is common to use N A^{-2} or T m A^{-1}.

Permeability and permittivity are related constants. In other words, i you know one constant you can calculate the other. In the SI system o units, the permeability o a vacuum is defined to have a value o exactly $4\pi \times 10^{-7}$ N A^{-2}. See the definition o the ampere (right) or more detail.

MAGNETIC FIELD IN A SOLENOID

The magnetic field o a solenoid is very similar to the magnetic field o a bar magnet. As shown by the parallel field lines, the magnetic field inside the solenoid is constant. It might seem surprising that the field does not vary at all inside the solenoid, but this can be experimentally verified near the centre o a long solenoid. It does tend to decrease near the ends o the solenoid as shown in the graph below.



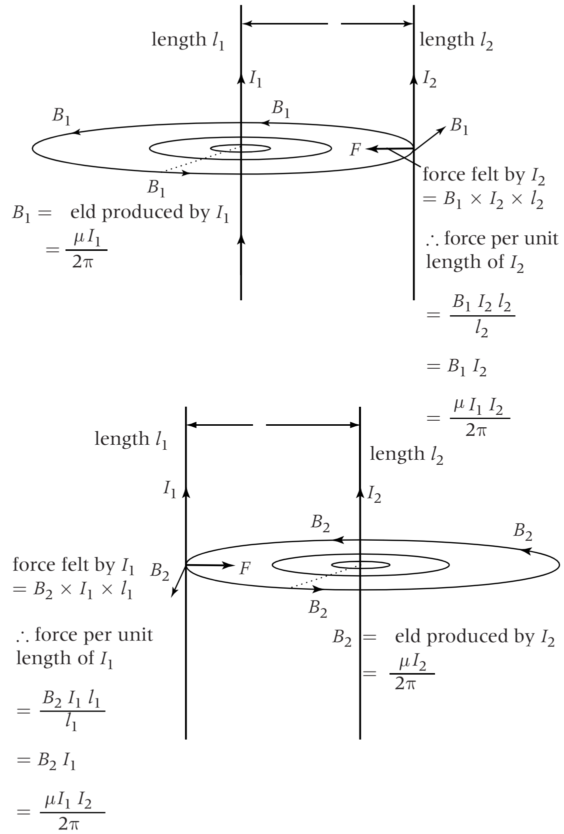


(= number of turns, l = length)

Variation o magnetic field in a solenoid

TWO PARALLEL WIRES – DEFINITION OF THE AMPERE

Two parallel current-carrying wires provide a good example o the concepts o magnetic field and magnetic orce. Because there is a current flowing down the wire, each wire is producing a magnetic field. The other wire is in this field so it eels a orce. The orces on the wires are an example o a Newton's third law pair o orces.



Magnitude o orce per unit length on either wire $= \frac{\mu I_1 I_2}{2\pi r}$

This equation is experimentally used to define the ampere. The coulomb is then defined to be one ampere second. I we imagine two infinitely long wires carrying a current o one amp separated by a distance o one metre, the equation would predict the orce per unit length to be 2×10^{-7} N. Although it is not possible to have infinitely long wires, an experimental set-up can be arranged with very long wires indeed. This allows the orces to be measured and ammeters to be properly calibrated.

The mathematical equation or this constant field at the centre o a long solenoid is

$$B = \mu \left(\frac{n}{l}\right) I$$

Thus the field only depends on:

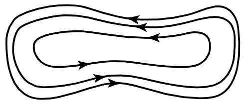
- the current, I
- the number o turns per unit length, $\frac{n}{l}$
- the nature o the solenoid core, μ

It is independent o the cross-sectional area o the solenoid.

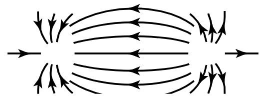
IB Quest ons – electr c ty and magnet sm

1. Which **one** of the field patterns below could be produced by two point charges?

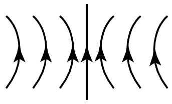
A.



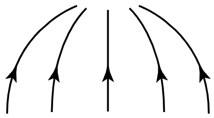
C.



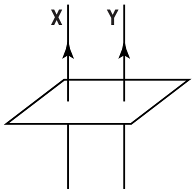
B.



D.

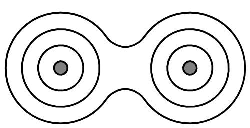


2. Two long, vertical wires **X** and **Y** carry currents in the same direction and pass through a horizontal sheet of card.

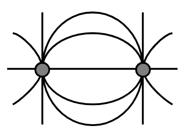


Iron filings are scattered on the card. Which **one** of the following diagrams best shows the pattern formed by the iron filings? (The dots show where the wires **X** and **Y** enter the card.)

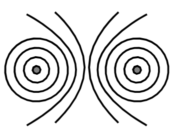
A.



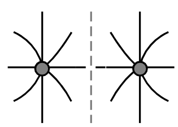
C.



B.

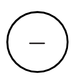


D.



3. This question is about the electric field due to a charged sphere and the motion of electrons in that field.

The diagram below shows an isolated metal sphere in a vacuum that carries a negative electric charge of 9.0 nC.



a) On the diagram draw arrows to represent the electric field pattern due to the charged sphere. [3]

b) The electric field strength at the surface of the sphere and at points outside the sphere can be determined by assuming that the sphere acts as though a point charge of magnitude 9.0 nC is situated at its centre. The radius of the sphere is 4.5×10^{-2} m. Deduce that the magnitude of the field strength at the surface of the sphere is 4.0×10^{4} V m^{-1}. [1]

An electron is initially at rest on the surface of the sphere.

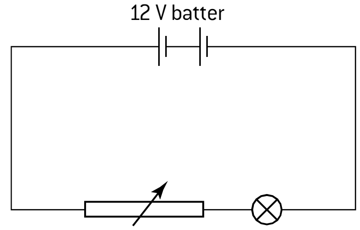
c) (i) Describe the path followed by the electron as it leaves the surface of the sphere. [1]

(ii) Calculate the initial acceleration of the electron. [3]

(iii) State and explain whether the acceleration of the electron remains constant, increases or decreases as it moves away from the sphere. [2]

(iv) At a certain point P, the speed of the electron is 6.0×10^{6} m s^{-1}. Determine the potential difference between the point P and the surface of the sphere. [2]

4. In order to measure the voltage-current (*V-I*) characteristics of a lamp, a student sets up the following electrical circuit.



a) On the circuit above, add circuit symbols showing the correct positions of an ideal ammeter **and** an ideal voltmeter that would allow the *V-I* characteristics of this lamp to be measured. [2]

The voltmeter and the ammeter are connected correctly in the circuit above.

b) Explain why the potential difference across the lamp

(i) cannot be increased to 12 V. [2]

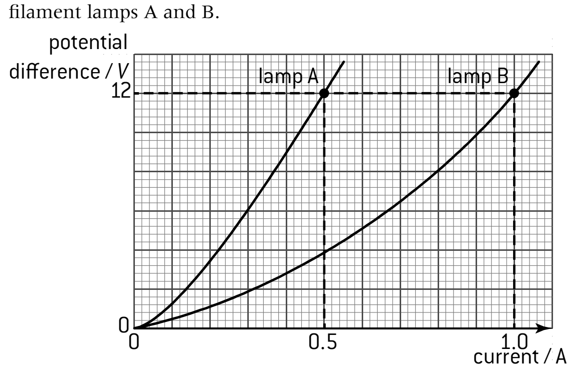
(ii) cannot be reduced to zero. [2]

An alternative circuit for measuring the *V-I* characteristic uses a *potential di ider*. [3]

c) (i) Draw a circuit that uses a potential divider to enable the *V-I* characteristics of the filament to be found. [3]

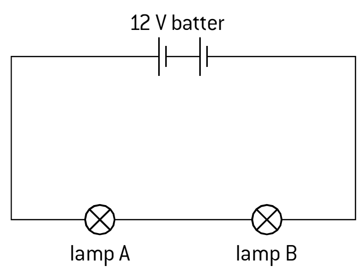
(ii) Explain why this circuit enables the potential difference across the lamp to be reduced to zero volts. [2]

The graph below shows the *V-I* characteristic for two 12 V filament lamps A and B.



d) State and explain which lamp has the greater power dissipation for a potential difference of 12 V. [3]

The two lamps are now connected in series with a 12 V battery as shown below.



e) (i) State how the current in lamp A compares with that in lamp B. [1]

(ii) Use the *V-I* characteristics of the lamps to deduce the total current from the battery. [4]

(iii) Compare the power dissipated by the two lamps. [2]