11 ELECTROMAGNETIC INDUCTION (AHL)

Introduction

The physics of electromagnetic induction has profound implications for the way we generate electrical energy and therefore for the way we live. Every year throughout the world about 10^{20} J of energy are converted into an electrical form using electromagnetism. This vast conversion of energy enables us to feed our ever-growing appetite for technologies that require electricity. It also raises considerable issues about the sustainability of the energy sources used for the conversion.

11.1 Electromagnetic induction

Understandings

- Electromotive force (emf)
- Magnetic flux and magnetic flux linkage
- Faraday's law of induction
- Lenz's law

Nature of science

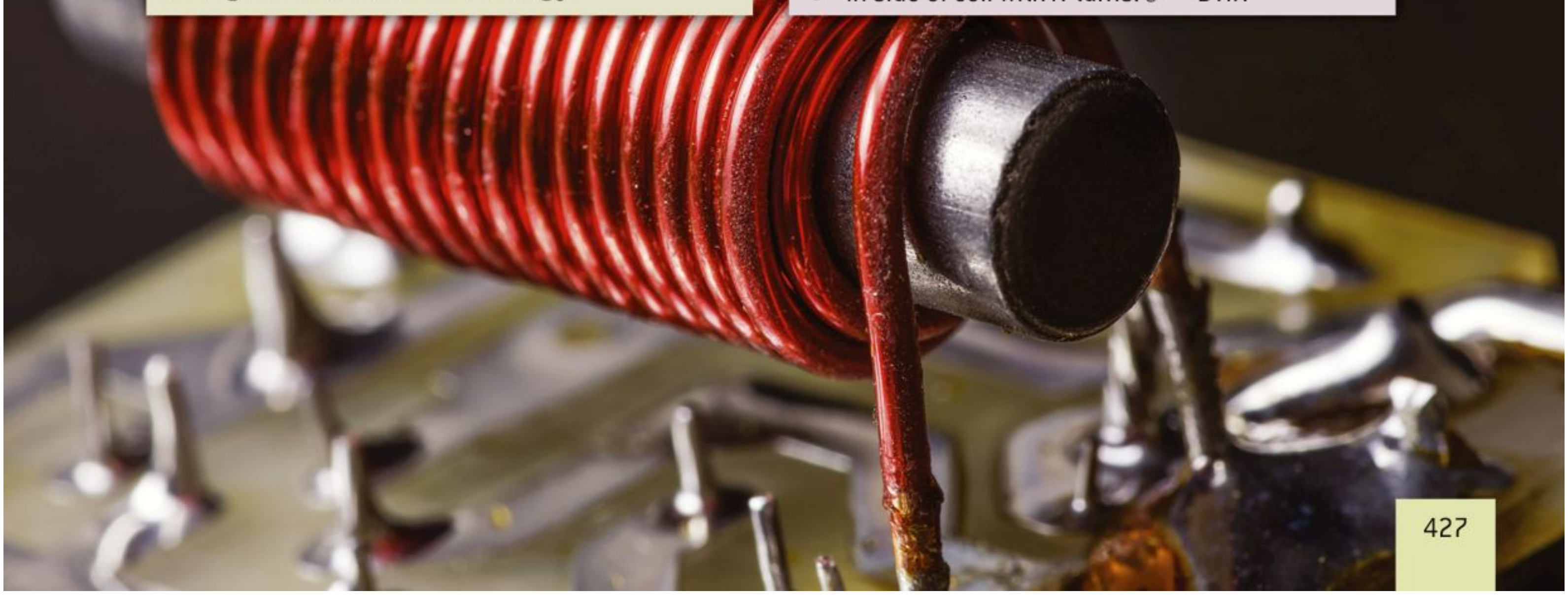
Much of the physics in this sub-topic was discovered through painstaking experimentation by a handful of scientists. Pre-eminent among them was Michael Faraday who observed currents induced in a coil when magnetic fields were varied nearby. These observations led him to the laws of electromagnetic induction that we use in our large-scale generation of electrical energy.

Applications and skills

- Describing the production of an induced emf by a changing magnetic flux and within a uniform magnetic field
- Solving problems involving magnetic flux, magnetic flux linkage, and Faraday's law
- Explaining Lenz's law through the conservation of energy

Equations

- Flux: $\Phi = BA\cos\theta$
- Faraday's / Neumann's equation: $\varepsilon = -N\dfrac{\Delta\Phi}{\Delta t}$
- emf induced in moving rod: $\varepsilon = Bvl$
- in side of coil with N turns: $\varepsilon = BvlN$



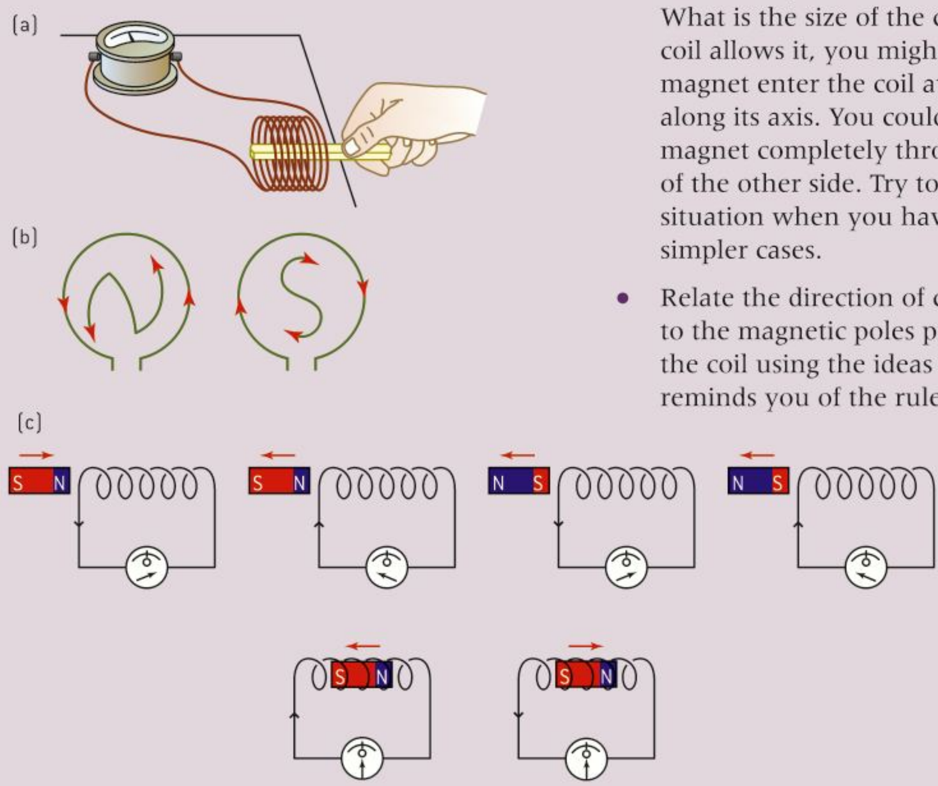
Inducing an emf

In Topic 5 we saw that when an electric charge moves in a magnetic field, then a force acts on the charge (and therefore on the conductor in which it is moving). In a reverse sense, a movement or change in a magnetic field relative to a stationary charge gives rise to an electric current. This phenomenon is called **electromagnetic induction**.

Investigate!

Making a current

- Begin with a magnet and a coil or a solenoid of wire. Arrange the coil horizontally and connect it to a galvanometer (a form of sensitive ammeter, with the zero in the middle of the scale). You can use a laboratory coil, or you can wind your own from suitable metal wire using a cylinder as a former.
- Move the bar magnet so that its north-seeking pole approaches one end of the coil and observe the effect on the meter. Record the direction of the current as indicated by the meter and the peak value shown.
- Repeat the movement, moving the bar magnet with its south-seeking pole towards the coil.
- Move the bar magnet away from the coil.
- Change the speed with which you move the magnet.
- Compare the current directions for each case and also the size of the current produced.
- Now try moving the coil and keeping the magnet still. Does this change your observation?
- Now try moving the coil and the magnet at the same speed and in the same direction. What is the size of the current now? If your coil allows it, you might also try making the magnet enter the coil at an angle rather than along its axis. You could also try moving the magnet completely through the coil and out of the other side. Try to interpret this complex situation when you have understood the simpler cases.
- Relate the direction of current flow in the coil to the magnetic poles produced at the ends of the coil using the ideas in Topic 5. (Figure 1(b) reminds you of the rule.)



▲ Figure 1 (a) the experiment, (b) the N and S rule, and (c) typical results.

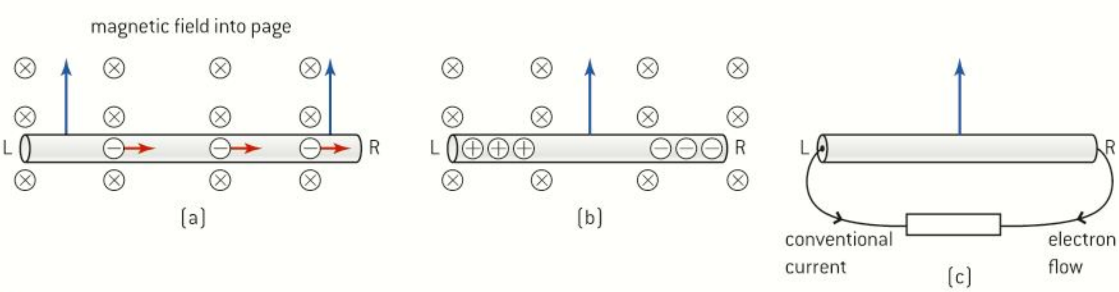
The results for this simple experiment are shown in figure 1(c). You should focus both on the direction of the conventional current flow and on the magnitude of the current. You are observing results similar to those made by Faraday in the 19th century.

A number of conclusions emerge from these simple experiments:

- The current only appears when there is relative motion between the coil and the magnet; either or both can move to produce the effect. However, if both coil and magnet move with no relative motion between them then no current occurs. Only movement of the wire in the coil relative to the field is important.
- When the north-seeking pole of the magnet is inserted into the coil, the current in the coil tends to reduce the magnet's motion by producing another north-seeking pole at the magnet end of the coil. Push a south-seeking pole in and another south pole appears at the magnet end of the coil. It is as though the system acts to repel the bar magnet and to reduce its movement. The system appears to oppose any change in the magnetic flux; the greater the rate of change, the greater is this opposition. We will look at this again later.
- In the same way, when a magnetic pole is moved away from the coil, the opposite pole is formed by the current in the coil in an attempt to attract the magnet and reduce its speed of motion.
- Moving the coil at greater speeds relative to the magnet increases the sizes of the currents. The effect is at a maximum when the axis of the magnet between its poles is perpendicular to the area of cross-section of the coil.

The keys to understanding these effects lie in what we said earlier in Topic 5 about the motion of charges in a magnetic field and what we know about the internal structure of the conducting wire that makes up the coil.

Figure 2(a) shows electrons in a metal rod that is moving through a uniform, unchanging magnetic field. Free electrons in the wire are moving upwards with the rod in an external magnetic field that acts into the page. A force acts on each electron to the right. This force is equivalent in direction to that which would act on a positive charge moving down the page (a conventional current downwards). This direction is perpendicular to both the field and the direction of motion of the rod and is determined using Fleming's left-hand rule.



▲ Figure 2 Electrons forced to move in a magnetic field experience a force.

In figure 2(b), with no external connection between L and R the electrons accumulate at the right-hand end (R) of the rod making it negatively charged, and a lack of electrons at the left-hand end (L) makes it positive. A potential difference exists between L and R, L being at the higher potential. When there is no external connection between L and R, no current will circulate. Charges will accumulate at the end of the rod, that is electrons will move to one end leaving the other end positive. Without a current in a closed circuit no work is required, no transformation of energy takes place.

If the circuit is closed externally between L and R (figure 2(c)), a flow of electrons will occur as shown. Inside the rod the conventional current flows from R to L. The electrons flow out of the right-hand end of the rod and this is a conventional current in the external circuit from L to R. A current has been generated, or induced, in the circuit.

The system is acting to move the electrons through the resistor and, because this is a transformation of energy into an electric form, the source of the energy is termed an electromotive force (emf). As usual, we can identify the amount of energy transferred for each coulomb of charge that moves around the circuit and we use the term **induced emf** to show that the emf arises from an induction effect. (The word "induction" is another term used in the early days of the study of electromagnetism.)

Nature of science

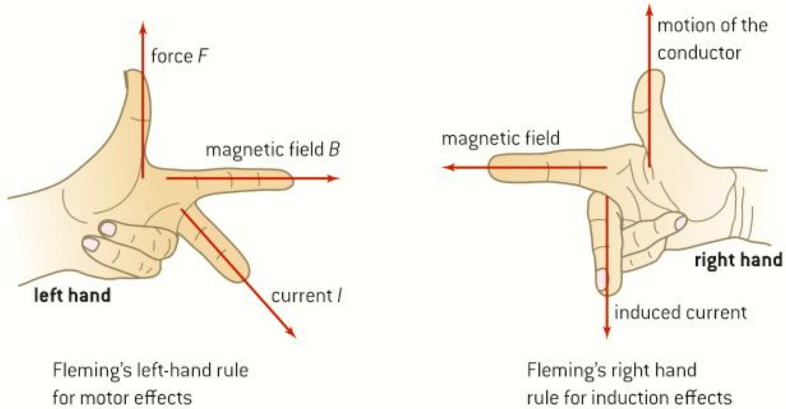
Electromagnetic force

Perhaps you can now see why the term electromagnetic force arose in the early days of electromagnetic induction – and why it still persists. Some physicists object that no force acts when the term emf is applied to electric cells, batteries, piezoelectric devices, and so on – therefore, they say, emf is not a good expression. It is true that it is difficult to see how the word "force" can apply in the case of a cell. But in the case of electromagnetic induction, it is clear that a force is acting on the electrons in the conductor that is being moved and so the term emf continues to be used in physics. The fact that we often use the abbreviation emf rather than the full expression is a reminder that we should not focus on the term "force" but rather on the units of emf: J C^{-1}.

Lenz's law

An important aspect of electromagnetic induction is that the induced emf exists *whether the charge flows in a complete circuit or not*. In the case shown in figure 2(b) the circuit is incomplete. Electrons flow along the rod until an excess of them sets up an electric field which repels further electrons, stopping further flow. It is only when the circuit is complete that charge flows. An induced emf is *always* generated by the system and the induced current will exist *only* if there is a complete circuit.

We can look at the system in terms of a possible direction rule. Fleming's left-hand rule was able to predict the force on, and therefore a flow of, the electrons. This flow of electrons is equivalent to a conventional current



▲ Figure 3 Fleming's rules.

acting in the opposite direction. You have two choices: either use Fleming's left-hand rule to work out the force direction from first principles and let this lead you to the conventional current direction (the argument is given above), or you can use another rule (Fleming's right-hand rule), which uses the symmetry between our left and right hands to give the relationship between the motion of the conductor, the direction of the field and the direction of the induced conventional current. It's your choice!

The observations you made in the *Investigate!* can be interpreted by looking closely at the directions of current in the coil relative to the movement of field and coil. A rule that describes this was summed up by the German scientist Heinrich Lenz in 1833. He stated that

> **the direction of the induced current is such as to oppose the change that created the current.**

Check your experimental notes (or the diagrams in figure 1 that sum them up) and see if your results confirm this law.

In fact, Lenz's law is little more than the conservation of energy. Suppose that, rather than opposing the induced effect, the change were to enhance it. This would imply an attraction instead of a repulsion between magnet and coil; the magnet would be pulled into the coil, accelerating as it goes. This would increase the speed and lead to an even greater acceleration. The magnet would move faster and faster into the coil, gaining kinetic energy from nowhere. Conservation of energy tells us this cannot happen.

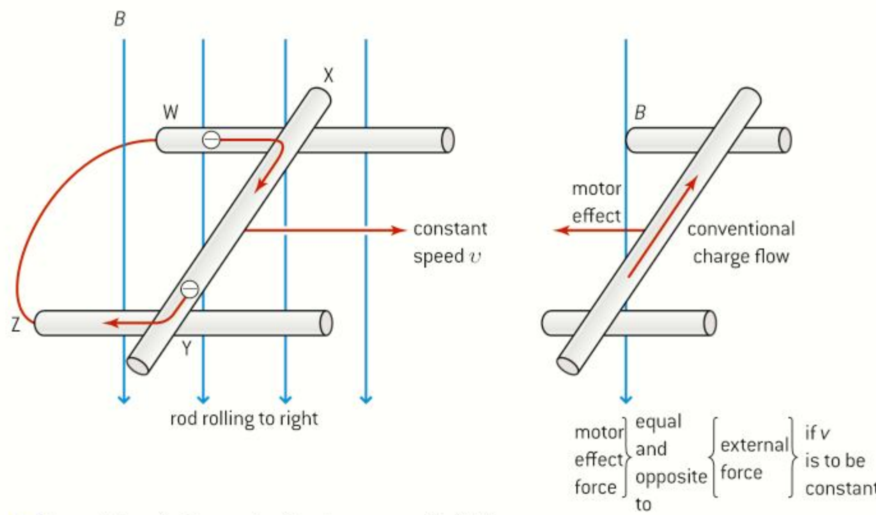
Another way to look at the consequences of Lenz's law is to realize that you cannot do work without having some opposition. The induced current in the coil is such that the induced field produced by this current opposes the motion of the magnet you are holding. If the circuit is open, there is no current, no opposition, and no electric energy produced. If you move the magnet very fast you will clearly feel the opposite force acting on you!

Electrons had not been discovered in Lenz's time and we can see how his law arises from first principles. In Topic 5 we saw that when a current flows within a magnetic field the current produces a magnetic field which distorts the original field pattern. The result was that a force acted on (and could accelerate) the current-carrying conductor. This was the effect that led us to the basis of an electric motor.

Nature of science

Another rule

Another possibility is to use the right hand with its thumb in the direction of the current and fingers in the direction of the magnetic field. By pushing you get the direction of the force on a positive charge or a conventional current. If an electron is the moving charge, point the thumb in the opposite direction to get the force in opposite direction This convention can also be used to get the direction of the magnetic field due a conventional current e.g. by curling the fingers around a wire with current.



▲ Figure 4 Conducting rod rolling in a magnetic field.

In electromagnetic induction, there is a current in the conductor as a result of the motion of the conductor. Figure 4 extends the example of the moving rod that is now rolling to the right at a constant speed through the magnetic field on a pair of rails. The rails conduct and form part of a complete electrical circuit WXYZ. (Rolling means that we do not have to worry about friction between the rod and the rails.) Charges, driven by the induced emf, flow around the circuit giving rise to an induced current in the direction shown. This induced current interacts with the uniform magnetic field to give rise to a force - the motor effect force that we discussed in Topic 5 (page 234). If you now use Fleming's left-hand rule you will see that the induced current leads to a motor effect (a force) acting to the left in figure 4, that is, opposite to the direction in which the conductor is moving. If the rod is to move at a constant speed then (by Newton's first law of motion) an external force must be exerted on it. This is where the energy conversion comes in. The work done by the external force to keep the conductor moving at a constant speed appears as electrical energy in the conductor.

Magnetic flux and flux density

We can use the physics from Topic 5 to extend these qualitative ideas. The magnitude of this magnetic force is BIl where B is the magnetic field strength, I is the induced current in the rod, and l is the length of the rod. Fleming's left-hand rule shows that the magnetic force arising from the induced current opposes the original force. In other words, the opposing magnetic force is to the left if the original applied force is to the right. The net force is zero and the rod moves at constant speed. It is possible for work to be done because of the opposition to the motion presented by the external magnetic field that produces a magnetic force on the induced current. No opposition, no work possible (or needed!).

From Newton's first law, to keep the rod in figure 4 moving at constant velocity, a constant force equal to BIl must act on the rod to the right.

The energy we have to supply therefore in a time Δt is

$$\textit{force} \times \textit{distance moved}$$

which is

$$BIl \times \Delta x$$

where Δx is the distance moved to the right by the rod.

The induced emf is equal to the energy per coulomb supplied to the system. In other words

$$\frac{\textit{energy supplied}}{\textit{charge moved}} = \varepsilon = \frac{BIl \times \Delta x}{Q} = \frac{BIl\Delta x}{I\Delta t}$$

and therefore

$$\varepsilon = Blv$$

where v is the speed of the rod.

Cancelling and rearranging gives

$$\varepsilon = \frac{B\,\Delta A}{\Delta t} = B \times \textit{rate of change of area}$$

This is because the area that is swept through by the rod in a time t is $l\Delta x = \Delta A$ (because it is the amount by which the area *changes* in time Δt)

So, in words,

$$\textit{induced emf} = \textit{magnetic flux density} \times \textit{rate of change of area}$$

This introduces you to an alternative term in magnetism for what was earlier called the magnetic field strength – **magnetic flux density**.

The magnetic field strength is numerically equivalent to magnetic flux density.

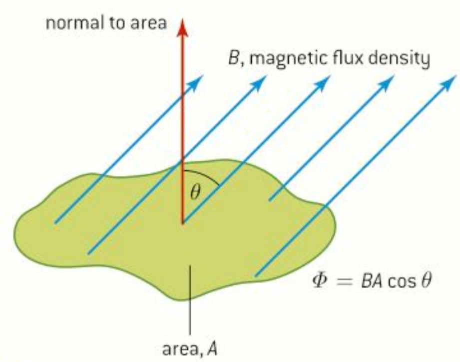
In Topic 5 we used the term "magnetic field strength" because in that topic we were concerned with basic ideas of field. In both electrostatics and gravity, the term field strength has a meaning of $\frac{\textit{force}}{\textit{mass}}$ or $\frac{\textit{force}}{\textit{charge}}$ depending on the context. We defined magnetic field strength in Topic 5 as

$$\frac{\textit{force}}{\textit{current} \times \textit{length}}$$

However, this definition does not take account of the old but helpful view of magnetic field lines as lines directed from a north-seeking to a south-seeking pole with a line density that is a measure of the strength of the field. It is this visualization of a field in terms of lines (close together when the field is strong, and well-separated when the field is weak) that links magnetic field strength to magnetic flux density.

When lines of force (field lines) are close together, then the magnetic field strength is large. There will be many lines through a given area. We say also that the magnetic flux density is large. The total number of lines per square metre is a measure of the magnetic flux density and therefore the total number of lines in a given area is a measure of the magnetic flux.

Flux is an old English word that has the meaning of "flow" and one way to think about flux is to imagine a windsock used to show the direction and speed of wind at an airfield. If the wind is strong then the flux



▲ Figure 5 Flux and flux density.

density is high. The flux is the number of streamlines going through the sock. If the wind has the same speed for two windsocks of different sizes then the windsock with the larger opening will have a larger flux even though the flux density is the same.

If magnetic flux density is defined as the number of flux lines per unit area, then flux itself must be equivalent to

flux density × *area*

so that, in symbols,

$$\Phi = B \times A$$

This equation assumes that B and A are at right angles. Figure 5 shows the relationship between area and flux density when this is not the case. In this case, we need to consider the component of the field normal to the plane. A normal to the area is constructed and this normal makes an angle θ with the lines of flux. So,

$\Phi = BA \cos\theta$ so that when $\theta = 0$ (area normal to field lines) $\Phi = BA$, and when $\theta = 90°$, $\Phi = 0$.

To sum up:

- Magnetic flux density B is related to the number of field lines per unit area. It is a vector quantity.
- Magnetic flux is equal to BA (also written as Φ). It is a scalar quantity.
- The equation $\Phi = BA \cos\theta$ is used if the area is not normal to the lines.

The unit of flux is the weber (Wb) and is defined in terms of the emf induced when a magnetic field changes. The equation $\varepsilon = B\frac{\Delta A}{\Delta t}$ can be re-written as $\varepsilon = \frac{\Delta(BA)}{\Delta t}$ or $\frac{\textit{change in flux}}{\textit{time taken for change}}$ and so

a rate of change of flux of one weber per second induces an emf of one volt across a conductor

For a particular conductor, knowledge of a magnetic field in terms of the magnetic flux and the rate at which it changes allows a direct calculation of the magnitude of the induced emf that will appear.

We can now make a direct link between flux density and field strength.

The magnetic flux density $\left(\frac{\text{flux}}{\text{area over which it acts}}\right.$, measured in weber metre$^{-2}\Big)$ is numerically equal to the magnetic field strength $\left(\frac{\text{force}}{\text{current} \times \text{length}}\right)$.

One tesla (T) ≡ one weber per square metre (Wb m^{-2})

We therefore also have a link between changes in the magnetic field strength and the induced emf.

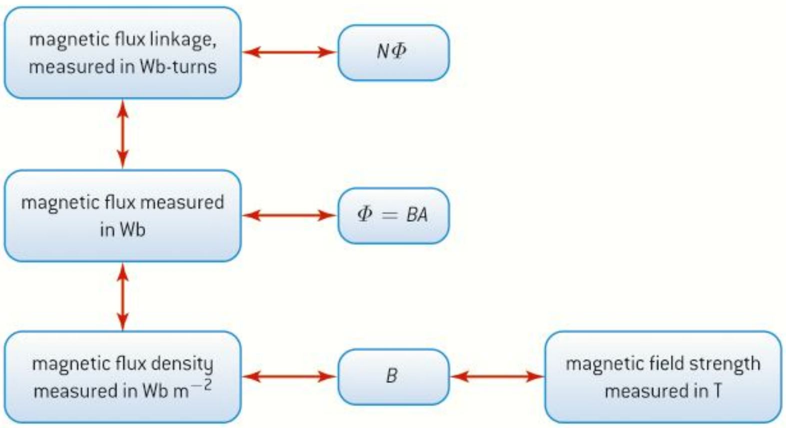
Magnetic flux linkage

Finally, one more quantity appears. Our derivation of $\varepsilon = B\frac{\Delta A}{\Delta t}$ above used a single rod rolling along two rails. This is equivalent to a single rectangular coil of wire that is gradually increasing in area. Imagine this single turn of coil gradually increasing its area to include more and more field lines. As before the emf across the ends of the coil will be

equal to the rate of change of area multiplied by the magnetic flux density. If there are N turns of wire in the coil then the induced emf will be N times greater so that $= NB\frac{\Delta A}{\Delta t} = \frac{\Delta(N\Phi)}{\Delta t}$. $N\Phi$ is known as the **magnetic flux linkage**.

The unit of flux linkage is often written as **weber turns,** although this is entirely equivalent to weber because the number of turns is simply a number.

The relationships between these interlinked quantities can be shown as:



Magnetic induction is summed up in a law devised by Faraday himself that is known as Faraday's law, it states that

> **the induced emf in a circuit is equal to the rate of change of magnetic flux linkage through the circuit.**

In our usual notation this is written algebraically as

$$\varepsilon = -\frac{N\Delta\Phi}{\Delta t}$$

The negative sign is added to include Lenz's law. In its full mathematical form, the equation is also known as Neumann's equation. This equation combines Faraday's and Lenz's ideas. It reminds us that, (as we shall see in the next section) magnetic flux can be changed in three different ways: by changing the area of cross-section with time $\left(\frac{\Delta A}{\Delta t}\right)$, by changing θ with time $\left[\frac{\Delta(\cos\theta)}{\Delta t}\right]$ or by changing magnetic flux density with time $\left(\frac{\Delta B}{\Delta t}\right)$.

Nature of science

Cutting lines of force

Faraday first introduced the field-line model though his model is not quite the same as our modern interpretation of a magnetic field. He considered the lines of force to be at the edges of "tubes of force", like elongated elastic bands. At the time, an invisible "aether", thought to have elastic properties, was considered to fill space. Later on, Faraday and others took the concept of the field line further by suggesting that it was the action of the conductor "cutting" the tubes of force that led to an induced emf. This is a helpful way to think of the process, although it conceals the link between a charge being moved in a magnetic field and the magnetic force that acts on the charge as a result. But we need always to remember that Faraday and the others did not know of the existence of the electron, and that they were very familiar with the ideas of field lines. In the nineteenth century, Maxwell refined these models by including both electrostatic and electromagnetic forces in one set of equations.

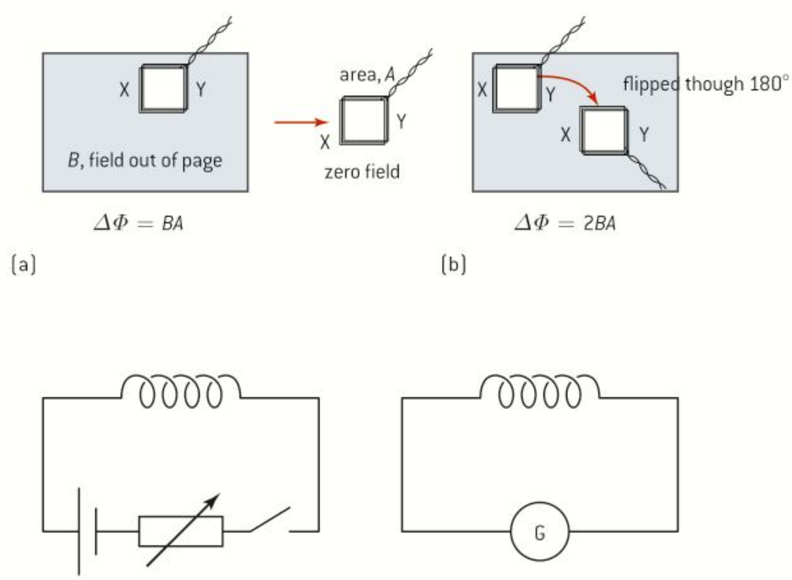
This illustrates two things about the nature of science: the way in which scientists allow a discovery to illuminate prior knowledge in a different way, and the power of the visual image to help us to comprehend a phenomenon.

Changing fields and moving coils

An emf can be electromagnetically induced in a number of ways that, on the face of it, appear different from each other:

- A wire or coil can move in an unchanging magnetic field (the example of the rolling rod above).
- The magnetic field can change in strength but the conductor does not move or change.
- A coil can change its size or orientation in an unchanging magnetic field.
- Combinations of these changes can occur.

We will look at these cases in the context of a rectangular coil interacting with a uniform (but possibly changing) magnetic field.



▲ Figure 6 Moving coils and changing fields.

Case 1: Straight wire moving in a uniform field

This is the case of the rolling rod above. The change in area per second is lv, the length l of the rod multiplied by v, the speed of the rod.

The induced emf is therefore $\varepsilon = Bvl$ when the wire moves at 90° to the field lines. If the wire motion is not at 90° to the field then, as usual, the component of field at 90° to the direction should be used.

Case 2: Coil moving

The coil can move as shown in figure 6(a) from one position in a magnetic field to another position where the field may be different. If the coil begins and ends in positions where the field is identical, then there is no change in the flux linkage and there is no induced emf. Although the coil is cutting lines, the same number is being cut on opposite sides of the coil. Two emfs are induced but in opposite senses and they therefore cancel out.

If the coil moves from a position where the flux is Φ to a position where the flux is zero, the change in flux linkage is $N\Phi$ and the induced emf ε is

$$\varepsilon = \frac{N\Phi}{\textit{time taken for change}}$$

An interesting variant of this occurs where a coil in a field is flipped through 180° (figure 6(b)). To visualize this, look at things from the point of view of the coil. The field lines appear to reverse their direction through the coil, and so the change in flux is $\Phi - (-\Phi)$, in other words, 2Φ. So the emf induced will be equal to $\frac{2N\Phi}{\textit{time taken to rotate coil}}$.

When a coil rotates in a field the emf produced instantaneously depends on the rate of change of the flux linkage, and this in turn depends on the angle the coil makes instantaneously with the field. If the coil rotates at a constant angular speed, then the emf output varies in a sinusoidal way. This is the basis of an alternating current (ac) generator as we shall see later.

Case 3: Magnetic field changes

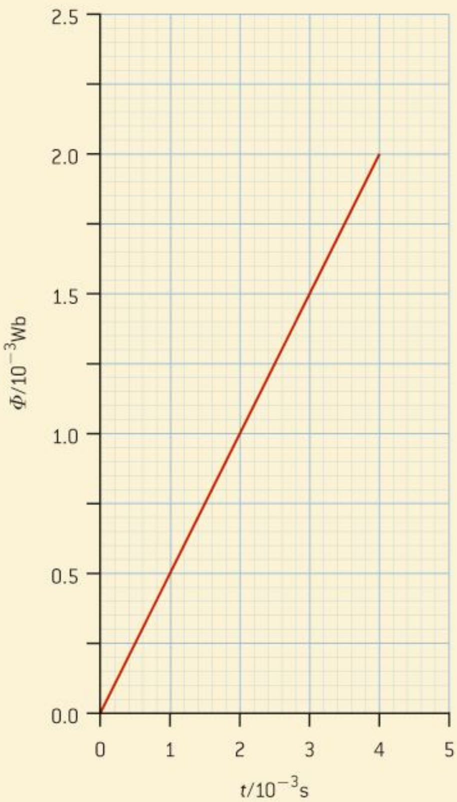
Sometimes the field changes from one value to another – it gets stronger or weaker – but the coil does not move. The act of cutting field lines is not so obvious here.

Suppose the field is being turned on from zero. Before the field begins to change there are no field lines inside the coil. You can think of the lines as moving from the outside into the area bounded by the coil. The change stops when the flux density is at its final value. In so doing the lines must have cut through the stationary coil.

Again, the solution is not difficult. Now $\varepsilon = -\frac{N\Delta\Phi}{\Delta t}$ becomes $\varepsilon = -NA\frac{\Delta B}{\Delta t}$ because only B is changing. You need to know the rate at which the field is changing with time (or the total change and the total time over which it happens). Another example is the case of two coils face-to-face as in figure 6(c). One coil is connected to a galvanometer alone, the other coil is connected to a circuit with a cell, a variable resistor and a switch. In what way will you expect the galvanometer reading to change when the switch is closed and remains closed? When the switch is opened? When the switch is closed and the resistance in the circuit is varied?

Worked examples

1 The graph shows the variation of magnetic flux with time through a coil of 500 turns.



Calculate the magnitude of the emf induced in the coil.

Solution

The change in flux is 2×10^{-3} Wb and this occurs in a time of 4.0 ms. the rate of change of flux is the gradient of this graph (as always) – as the flux is proportional to the time we can use any corresponding values of ϕ and t.

$$\frac{\Delta\Phi}{\Delta t} = \frac{2\ \text{mWb}}{4\ \text{ms}} = 0.5\text{V}$$

thus the induced emf $= 500 \times 0.5\ \text{V} = 250\ \text{V}$

2 A small cylindrical magnet and an aluminum cylinder (which is non-magnetic) of similar shape and mass are dropped from rest down a vertical copper tube of length 1.5 m.

a) Show that the aluminum cylinder will take about 0.5 s to reach the bottom of the tube.

b) The magnet takes 5 s to reach the bottom of the tube. Explain why the objects take different times to reach the bottom.

Solution

a) Use a kinematic equation, e.g. $s = ut + \frac{1}{2}at^2$. Then $1.5 = \frac{1}{2} \times 9.8 \times t^2$

$t = 0.55$ s.

b) As the magnet falls the copper tube experiences a changing magnetic flux, and as a result an emf is induced in the walls of the tube. This emf results in a current in the tube. The current leads to another magnetic field that opposes the motion of the magnet by Lenz's law. There is an upward force on the magnet so that its acceleration is less than the value for free fall. In the case of the aluminium cylinder no current arises and it falls with the usual acceleration.

Nature of science

Applications of electromagnetic induction

There are many applications of electromagnetic induction over and above the generation of electrical energy. They include electromagnetic braking, which is used in large commercial road vehicles; the use of an induction coil to generate the large pds required to provide the spark that ignites the petrol–air mixture in a car engine; and the generation of the signal in geophones and metal detectors. In each of these examples, a changing magnetic field leads to the generation of an emf and demonstrates the physics developed in this sub-topic.

11.2 Power generation and transmission

Understanding

- Alternating current (ac) generators
- Average power and root mean square (rms) values of current and voltage
- Transformers
- Diode bridges
- Half-wave and full-wave rectification

Nature of science

The provision of abundant, cheap electrical energy has been one of the ways in which an abstract science and its technological development have directly affected the lives of many people on the planet. Who could have imagined the enormous impact that Faraday's discoveries would make? ...certainly not Faraday himself! This is an example of research in pure science leading to great practical applications.

Applications and skills

- Explaining the operation of a basic ac generator, including the effect of changing the generator frequency
- Solving problems involving the average power in an ac circuit
- Solving problems involving step-up and step-down transformers
- Describing the use of transformers in ac electrical power distribution
- Investigating a diode bridge rectification circuit experimentally
- Qualitatively describing the effect of adding a capacitor to a diode bridge rectification circuit

Equations

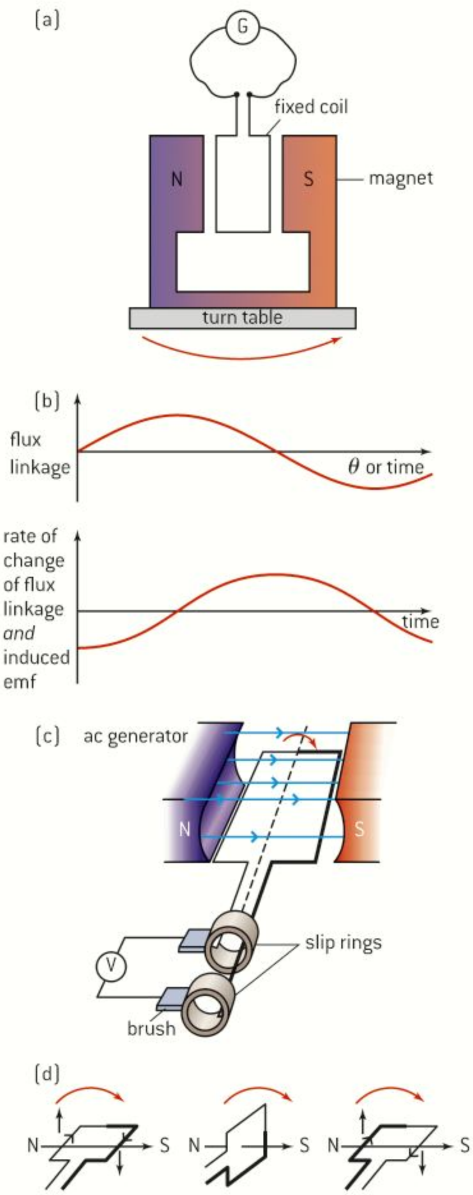
- rms and peak values: $I_{\text{r.m.s.}} = \frac{I_0}{\sqrt{2}}$
- potential difference: $V_{\text{r.m.s.}} = \frac{V_0}{\sqrt{2}}$
- resistance: $R = \frac{V_0}{I_0} = \frac{V_{\text{r.m.s.}}}{I_{\text{r.m.s.}}}$
- maximum power: $P_{\text{max}} = I_0 V_0$
- average power: $P_{\text{average}} = \frac{1}{2} I_0 V_0$
- transformer equation: $\frac{\varepsilon_p}{\varepsilon_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p}$

Introduction

We have seen that a rod rolling along two parallel rails generates induced emf and induced current; this is hardly a sensible way to generate electrical energy on a large scale. The practicalities of generation were solved by scientists from the 1830s onwards, first for direct current, and later for alternating current.

Alternating current (ac) generators

In this IB course we focus on the ac generator because it is commonly used for the generation of energy. Such a generator consists of a coil with a large number of turns; the coil rotates relative to a magnetic field.



▲ Figure 1 Basic ac generator.

For the moment we will imagine a fixed coil placed between the poles of a U-shaped magnet that stands at the centre of a rotating turntable (figure 1(a)). The turntable can turn at different angular speeds and the coil can have different numbers of turns and cross-sectional areas. The coil is connected to a galvanometer or to a data logger that registers the emf across the coil. When the magnetic flux in the coil is maximum, the emf induced (current) is minimum (0) and vice-versa. Changing the speed of the turntable changes the frequency of the emf as well as its amplitude. Increasing the number of turns or increasing the area of the coil will increase the amplitude of the emf but leave the frequency unchanged if the turntable speed does not change.

Magnetic field lines cut the coil as the turntable rotates and an induced emf is generated. However the emf varies as the turntable rotates. Figure 1(b) shows how the flux linkage varies with θ, the angle between the normal to the coil and field lines. When θ is equal to 90°, the field lines lie in the plane of the coil and so the flux through the coil is zero ($\cos 90° = 0$ in equation $\phi = BA\cos\theta$). When θ is equal to 0° the field lines are at 90° to the plane of the coil and the flux through the coil is a maximum. When the turntable rotation speed is constant, this graph also shows how the flux varies with time. The graph is a sine curve.

The emf induced in the coil is equal to $-\frac{\Delta\phi}{\Delta t}$, in other words the negative of the gradient of the flux-time graph. Figure 1(b) also shows how the induced emf varies with time; this graph can be obtained either by differentiation or by a consideration of the gradient of the flux-time graph. When the normal to the coil and the field lines are parallel (θ is 90°) then the emf is zero because the coil does not – instantaneously – cut the field lines at all.

However, while some ac generators have a rotating magnetic field, others have fixed magnets and a rotating coil (Figure 1(c)). The principle is however the same. The direction of charge flow in the coils varies with the position of the coil. The use of a direction rule shows this.

When the left-hand wire is moving upwards as shown in Figure 1(d), the conventional current direction in this wire is to the back of the coil. The right-hand wire is moving downwards at the same instant and the current in this wire is towards the front of the coil. Charge flows clockwise (looking from above) in the coil and out into the external circuit.

Half a cycle later the sides of the coil will have exchanged position. Charge is again flowing clockwise, but because the coil has rotated, the current (so far as the meter is concerned) is in the opposite direction. Figure 1(d) shows this too.

If there were wires permanently connecting the coil to the meter they would quickly become twisted. Energy needs to be extracted from the generator without this happening. Slip rings are used for this. The ends of the coil terminate in two rings of metal that rotate with the coil about the same axis. Two stationary brushes, connected to the external part of

the circuit, press onto the rotating rings and charge flows out into the circuit through these connections (Figure 1(c)).

The essential requirements for an ac generator are therefore:

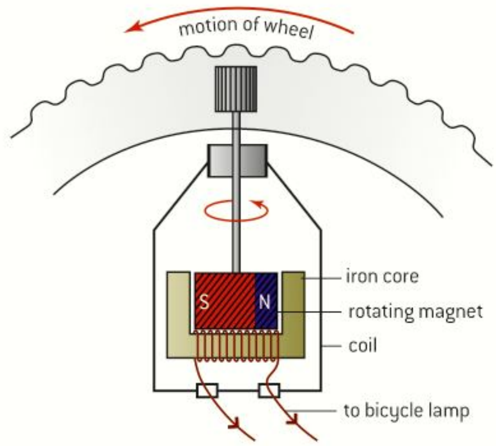
- a rotating coil
- a magnetic field
- relative movement between the coil and the magnetic field
- a suitable connection to the outside world.

Real-life generators are more sophisticated than our simple models and an Internet search will allow you to see many different types of ac generator.

For real generators, there is another issue that arises because an induced current is generated in the coils. As we saw in Sub-topic 11.1, any moving conductor carrying an induced current in a magnetic field has two forces acting on it: the force that moves it, and an opposite force that arises because of the induced current. This also applies to the rotating current-carrying coil in the ac generator. Fleming's left-hand rule and Lenz's law show that this force opposes whatever is turning the coil.

If a generator coil is being rotated clockwise by an external agent, then the magnetic forces that arise from the induced current interacting with the magnetic field will exert a turning force anticlockwise on the coil. Some of the energy provided by the external agent turning the coil has to be used to overcome this anticlockwise magnetic effect. This reduces the induced current that can be made available to the external circuit. However, remember that with no opposition, no work is done and no energy will be transferred from the agent (doing the turning) to the coil (and its associated electrical circuit).

This is easily demonstrated using a bicycle dynamo (a device similar to our first rotating-magnet generator) (see figure 2). In this type of dynamo a permanent magnet is rotated in the gap inside a coil. When the lamp is switched off so that no induced current is produced, the dynamo is relatively easy to turn (remember, there will still be an induced emf across the terminals). When the dynamo supplies current and lights the lamp, more effort is required to rotate the dynamo at the same speed since an opposing magnetic force will act on the current (coil). This is Lenz's law in action.



▲ Figure 2 Bicycle dynamo.

Nature of science

Modelling an ac generator

Although the following derivation will not be required in the examination, it shows you how Faraday's law can be used to model the behaviour of a simple ac generator.

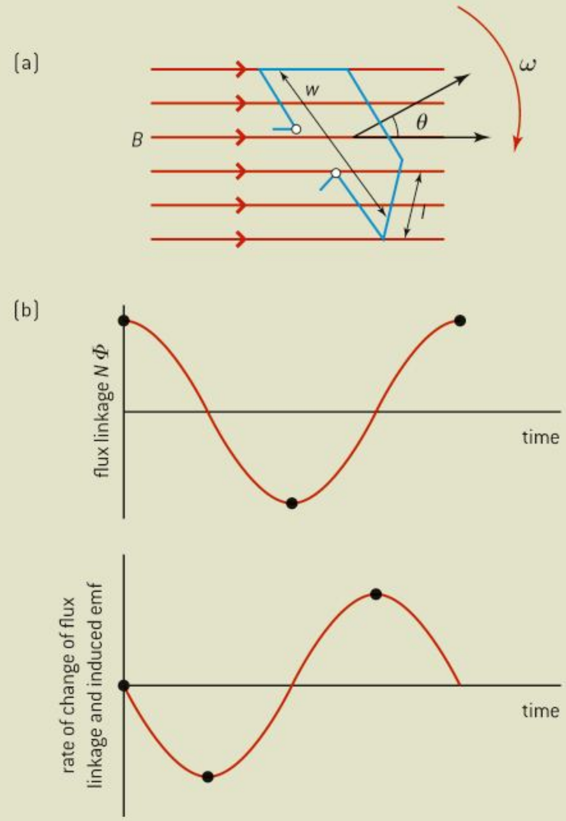
The coil has an average length l and an average width of w with N turns (these dimensions are shown in figure 3(a)). The area of the coil A is therefore $l \times w$.

At the instant when the normal to the plane of the coil is at an angle θ to the magnetic field, the flux linkage through the coil is $N \times \Phi$, which is $N \times BA\cos\theta$. The coil spins at a constant angular speed ω In time t the angle swept is θ $(= \omega t)$.

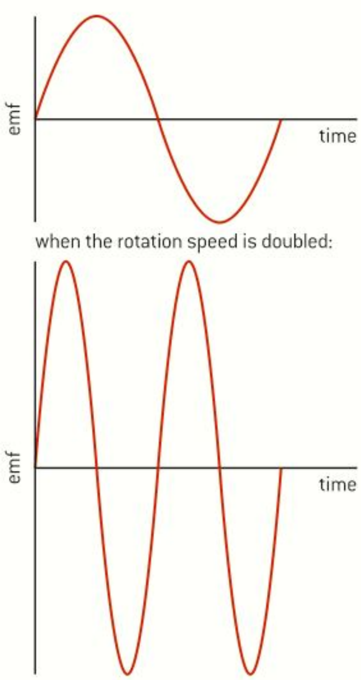
Therefore the flux linkage varies with t as $NB \cos \omega t$ and a graph showing how flux linkage varies with time has a cosine shape with maximum and minimum values of NBA at $t = 0$ and $-NBA$ when the coil is halfway through one cycle.

The value of the induced emf at any instant is equal to the $(-)$ rate of change of the flux linkage $(N\Phi)$ and this is the the $(-)$ gradient of the flux linkage–time graph. So the graph of induced emf is a (negative) sine curve with an equation of which has maximum and minimum values for the emf of $\varepsilon_0 = \pm BAN\omega$.

A supply in which the current and voltage vary as a sine wave is an **alternating supply.** Although ω is used for convenience in the mathematics of the emf induced in the generator, for everyday purposes we use the frequency. This has the same meaning as elsewhere in physics: the number of cycles that occur each second. A generator that rotates 50 times in one second has an alternating current output at a frequency of 50 Hz.



▲ Figure 3



▲ Figure 4 Increasing the rotation speed of the coil.

The model above shows that the output emf of a generator is sinusoidal and that one complete rotation of the coil through 360° gives one cycle of the alternating current.

Figure 4 shows the effect on the emf of changing the angular speed of the coil (without changing any other feature of the coil or field). If the angular speed of the coil is increased then:

- the coil will take a shorter time to complete one cycle and so there will be more cycles every second hence the frequency increases.
- the time between maximum and minimum flux linkage will decrease and therefore (as the flux linkage is constant) the rate of change increases and hence the peak emf increases.

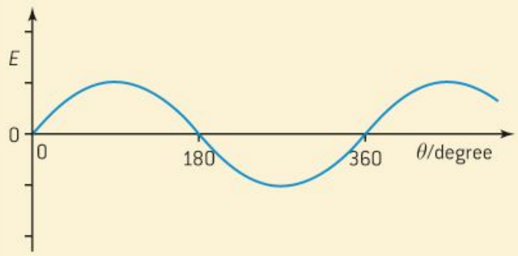
Other ways to increase the output of the emf, but without changing frequency, include increasing:

- the magnetic field strength (B)
- the number of turns on the rotating coil (N)
- the larger coil area (A).

Worked example

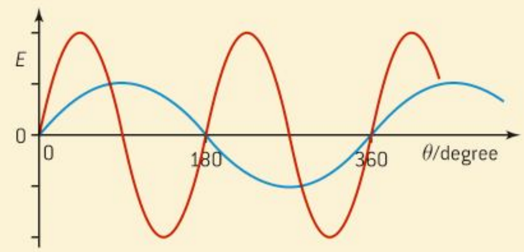
A coil rotates at a constant rate in a uniform magnetic field. The variation of the emf E with angle θ between the coil and the field direction is shown.

Copy the graph below and, on the same axes, add the emf that will be produced when the rate of rotation of the coil is doubled. Explain your answer.



Solution

The rate of change of the flux linkage will double, so the magnitude of the peak emf will also double. This is because Faraday's law states that the induced emf is proportional to the rate of change of flux linkage. However, the coil now rotates in half the time, so the time for one cycle will be halved. On the same scales, the new graph is:



Measuring alternating currents and voltages

The current and voltage of an alternating supply change constantly throughout one cycle. Measuring these quantities is not straightforward because, with positive and negative half-cycles, the average value for current or voltage over one cycle is zero.

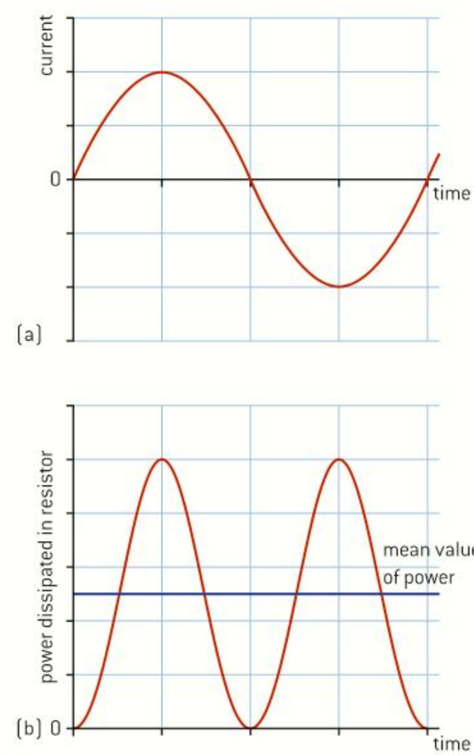
One way around this problem is to consider the power supplied to a resistance R connected to the generator. The instantaneous power dissipated in the resistance is I^2R where I is the instantaneous current.

Figure 5 shows both the current–time graph and the power dissipated–time graph with the same time axes. Notice the difference between the two:

- The power–time graph is always positive (which we would expect because the power is I^2R and a number squared is always positive).
- The power graph cycles at twice the frequency of the current.

To see this, suppose that the time period of the ac generator is very large taking 10 s to turn once through one cycle. If you watch a filament lamp supplied with an ac supply of such a low frequency you will see the lamp flash on and off *twice* in each cycle. The lamp is on when the emf is near its maximum (positive) and minimum (negative) values.When we look at a filament lamp powered by the AC mains, persistence of vision prevents us seeing its flashing like this because it is switched on and off at 100 or 120 times per second (twice the normal mains frequency of 50 or 60 Hz).

Alternating values are measured using the equivalent direct current that delivers the same power as the ac over one cycle. A lamp supplied from a dc supply would have the same brightness as the average brightness of our ac lamp flashing on and off twice a cycle. This equivalent dc current is that which gives the average value of the power in the power–time graph. Because the average value of a $\sin^2$ graph is halfway between the peak and zero values and the curve is symmetrical about this line (shown on figure 5(b)), the areas above and below the average line are the same.



▲ Figure 5 Current–time and power–time graphs for ac.

So the mean power that an ac circuit supplies is $\frac{1}{2} I_{max}^2 R$ where I_{max} is the peak value of the current. The dc current required to give this power is $\sqrt{\frac{1}{2} I_{max}^2}$ which is $\frac{I_{max}}{\sqrt{2}}$.

This value is known as the **root mean square (rms) current** $\boldsymbol{I}_{rms} = \frac{\boldsymbol{I}_{max}}{\sqrt{\mathbf{2}}}$. In a similar way $V_{rms} = \frac{V_{max}}{\sqrt{2}}$. The power P dissipated in a resistance $= I_{rms} V_{rms} = \frac{I_{max}}{\sqrt{2}} \frac{V_{max}}{\sqrt{2}} = \frac{I_{max} V_{max}}{2}$

with the usual equivalents:

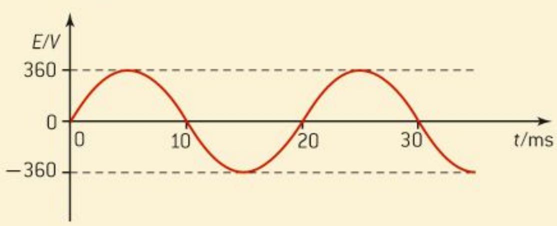
$$P = I_{rms} V_{rms} = \frac{1}{2} I_{rms}^2 R = \frac{1}{2} \frac{V_{rms}^2}{R}$$

You will not need to know how to prove the relationship between peak values and rms values for the examination.

Many countries use alternating current for their electrical supply to homes and industry. We will look at some of the reasons for this later, but different countries have made differing decisions about the potential differences and frequencies at which they transmit and use electrical energy. Thus, in some parts of the world the supply voltage is about 100 V; in others it is roughly 250 V. Likewise, frequencies are usually either 50 Hz or 60 Hz.

Worked examples

1 The diagram below shows the variation with time t of the emf E generated in a rotating coil.



Calculate:

a) the rms value of the emf

b) the frequency of rotation of the coil.

Solution

a) the peak value of the emf is 360 V, so the rms value is $\frac{360}{\sqrt{2}} = 255$ V

b) $f = \frac{1}{T} = \frac{1}{0.02} = 50$ Hz

2 A resistor is connected in series with an alternating current supply of negligible internal resistance. The peak value of the supply voltage is 140 V and the peak value of the current in the resistor is 9.5 A. Calculate the average power dissipation in the resistor.

Solution

$$\text{The average power} = \frac{\textit{peak current} \times \textit{peak pd}}{2}$$

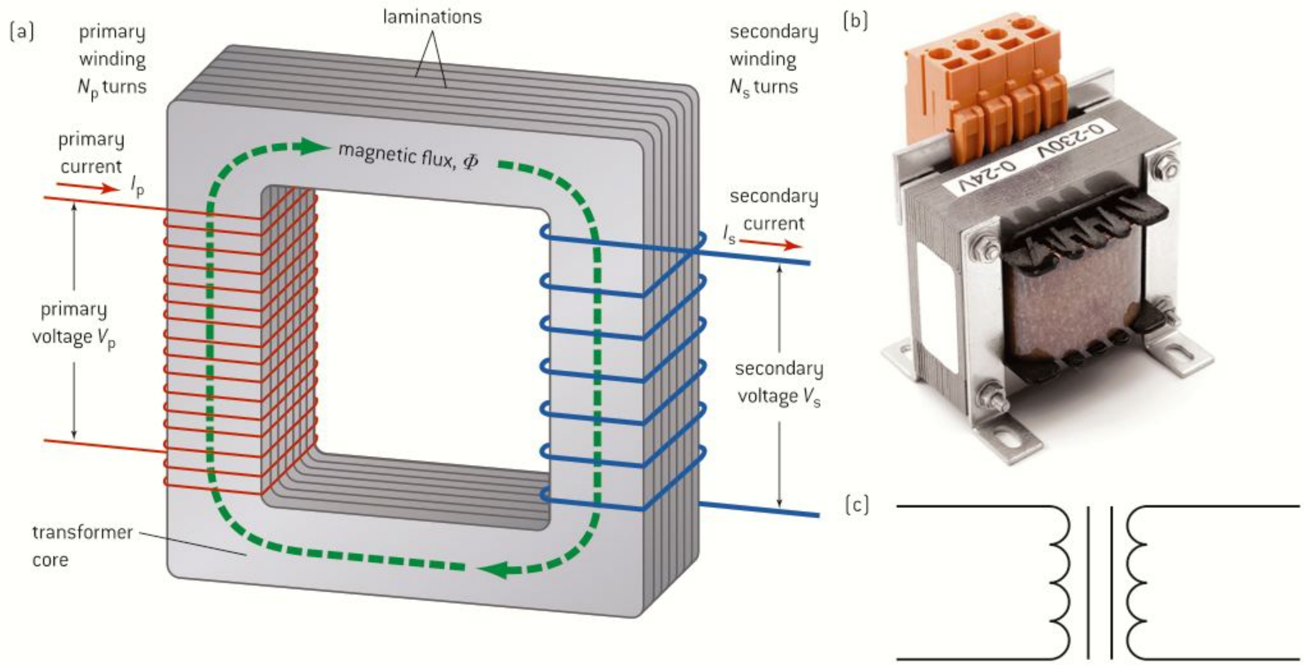
$$= \frac{1}{2} \times 140 \times 9.5 = 670 \text{ W}$$

Transformers

One of the reasons why ac supplies are so common is because a device called a transformer can be used to change alternating supplies from one pd to another. Transformers come in many shapes and sizes ranging from devices that convert voltages at powers of many megawatts down to the small devices used to power domestic devices that need a low-voltage supply.

A transformer consists of three parts:

- an input (or primary) coil
- an output (or secondary) coil
- an iron core on which both coils are wound.



▲ Figure 6 Transformers in theory and practice.

Figure 6 shows a schematic diagram of a transformer (a), a real-life transformer (b), and also the transformer symbol used in IB examinations (c).

The operation of a transformer, like the ac generator, relies on electromagnetic induction.

- Alternating current is supplied to the primary coil.
- A magnetic field is produced by the current in the primary coil and this field links around a core made from a magnetic material, usually soft iron. (The basic ideas behind the production of this field were covered in Topic 5.)
- Because the primary current is alternating, the magnetic field in the core alternates at the same frequency also. The field goes first in one direction around the core and then reverses its direction.
- The transformer is designed so that the changing flux also links the secondary coil.
- The secondary coil has a changing field inside it and an induced alternating emf appears at its terminals. When the coil is connected to an external load, charge will flow in the circuit of the secondary coil and its load.
- Energy has been transferred from the primary to the secondary circuit through the core.

Suppose that an alternating pd with a peak value of V_p is applied to the primary coil and that this results in a flux of Φ in the core. The flux linked to the secondary coil of N_s turns is therefore $N_s\Phi$ and the induced emf in the secondary coil is $V_s = N_s \frac{\Delta\Phi}{\Delta t}$. There is also a flux linkage to the primary coil even though the primary current was originally responsible for setting up the field in the first place. This gives rise to $\varepsilon_p = N_p \frac{\Delta\Phi}{\Delta t}$

where ε_p and N_p are the induced emf and the number of turns in the primary coil. This induced primary emf will oppose the applied pd.

If we assume that the resistance of the primary coil is negligible, then ε_p will be equal in magnitude to V_p. Thus, because $\frac{\Delta\Phi}{\Delta t}$ is the same for both coils $\frac{\varepsilon_p}{N_p} = \frac{V_p}{N_p} = \frac{V_s}{N_s}$ and

$$\frac{\varepsilon_p}{\varepsilon_s} = \frac{V_p}{V_s} = \frac{N_p}{N_s}$$

This is known as the **transformer rule**. It relates the number of turns on the coils to the input and output voltages:

- When $N_s > N_p$ the output voltage is greater than the input voltage. This is known as a **step-up** transformer.
- When $N_s < N_p$ the output voltage is less than the input voltage. This is known as a **step-down** transformer.
- The terms "step-up" and "step-down" refer to changes in the alternating voltages *not* to the currents.

You may wonder how zero current in the primary coil can lead to any energy transfer to the secondary. The answer is: it cannot. The equation applies to the case where there is no current in the secondary (in other words it has not yet been connected to a load). Once again Lenz's law has a part to play. When the secondary coil supplies current (because a resistor is now connected across its terminals), then because charge flows through this secondary coil, another magnetic field is set up in the coil and through the core. This magnetic field tries to oppose the changes occurring in the system and so tends to reduce the flux in the core. This in turn reduces the opposing emf in the primary (often called a "back emf" for this reason) and so now there will be an overall current in the primary that allows energy to be transferred.

Electrical engineers use a more complex theory of the transformer than the one presented here to take account of this and other factors. But for our simple theory, which assumes that the transformer loses no energy, the energy entering the primary is equal to the energy leaving the secondary so

$$I_p V_p = I_s V_s$$

where I_p and I_s are the currents in the primary and secondary circuits respectively. This is a second transformer equation that you should know be able to use.

In fact, many transformers have an efficiency that is close to 100% as energy losses can be reduced by good design. The **efficiency of a transformer** is

$$\frac{\text{energy supplied by secondary coil}}{\text{energy supplied to primary coil}} \times 100\%$$

Ways to improve the efficiency include:

Laminating the core

- Iron is a good conductor and the changing flux in the transformer produces currents flowing inside the iron core itself. These are known as **eddy currents**. To prevent these, transformer designers use thin

layers of insulating material that separate layers (sheets) of iron in the core (shown in Figure 6(a)). This has the result that although the magnetic properties of the iron are largely unaltered, the electrical resistance of the core is significantly increased. The currents are forced to travel along longer paths within the layers hence increasing electrical resistance and reducing the current. Laminations reduce the energy losses that result from a reduction in the amount of flux and from a rise in temperature of the iron that would occur if the eddy currents were large.

Choosing the core material

- The magnetic material of the core is a "soft" magnetic material. It can be magnetized and demagnetized very readily and also maintain high fluxes too. These are all desirable properties for the core.

Choosing the wire in the coils

- Low-resistance wires are used in the primary and secondary coils as high resistances would lead to heating losses (called **joule heating**) in the coils.

Core design

- It is important that flux is not allowed to leak out of the core. As much flux as possible should link both coils so that the maximum rate of change of flux linkage occurs.

Note

Hard and soft magnetic materials

We sometimes talk about magnetic materials being "soft" or "hard". A soft material, such as iron, can be easily magnetised by another magnet or a current-carrying coil. When the magnetic influence is removed, however, the iron loses all or most of its magnetism easily. Such materials are excellent for the cores of transformers because they respond well to the variations in magnetic field. Hard materials such as some iron alloys (a steel) do not magnetise easily but are good at retaining the magnetism. These materials are best for the manufacture of permanent magnets such as the ones you use in the laboratory.

Worked example

A transformer steps down a mains voltage of 120 V to 5 V for a tablet computer. The computer requires 1.0 W of power to operate correctly. There are 2300 turns on the primary coil of the transformer.

a) Calculate the number of turns on the secondary coil.

b) Calculate the current in the secondary coil when it is operating correctly.

c) The input current to the primary is 0.0090 A. Calculate the efficiency of the transformer.

Solution

a) $\frac{V_p}{V_s} = \frac{N_p}{N_s}$ so $N_s = \frac{V_s \times N_p}{V_p} = \frac{5 \times 2300}{120} = 96$ turns

b) Current in the secondary $I_s = \frac{\text{tablet power}}{V_s} = \frac{1.0}{5} = 0.20\text{A}$

c) Efficiency $= \frac{\text{power supplied by secondary coil}}{\text{power supplied to primary coil}}$

$= \frac{5 \times 0.2}{120 \times 0.0090} = 0.93$ (or 93%)

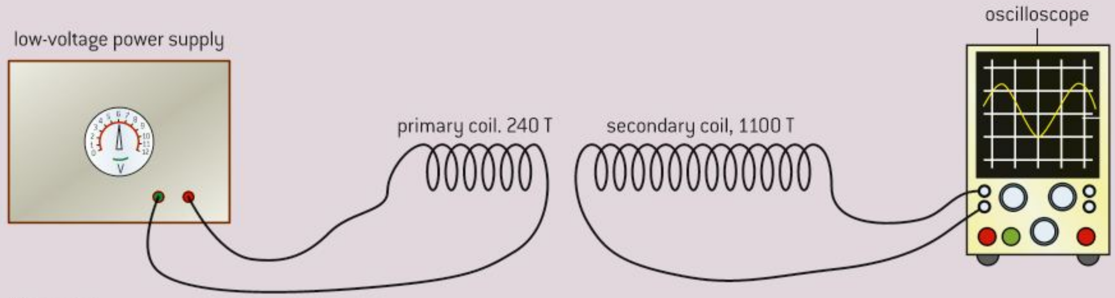
Investigate!

Transformer action

- In this experiment you are asked to investigate the basic ideas behind the transformer. You will require an ac low-voltage power supply to provide an input, an oscilloscope to view the output of your coils, and the coils themselves. One coil should have significantly less turns than the other, typical values might be 240 turns on one coil and 1100 turns on the other. You will also need additional apparatus for part of the experiment. (Some coil kits have special laminated iron cores that clip together to give a continuous magnetic circuit,)

- Connect the coil with fewer turns to the supply (this is the primary circuit) and the other coil to the oscilloscope (this is the secondary). Set the coils close so that the secondary coil is linked to the flux produced by the primary. Look closely at the output from the secondary coil and compare the relative sizes of the input and output and also their phases.
- Try the following changes to see how they affect the output:
  - Alter the separation and orientation of the coils (reverse one coil relative to the other and observe the difference). Alter the frequency of the primary current. Link the coils by inserting a piece of iron or several iron nails through the centre of both coils.
  - With the coils linked with iron compare the output voltages when the 240 turn coil is the primary and, later, when the 110 turn coil is the primary. If you have access to other coils with different numbers of turns collect data to complete the table.

Number of primary turns N_p	Primary pd, V_p/V	Number of secondary turms N_s	Secondary pd, V_s/V	$\frac{V_p}{V_s}$	$\frac{N_p}{N_s}$



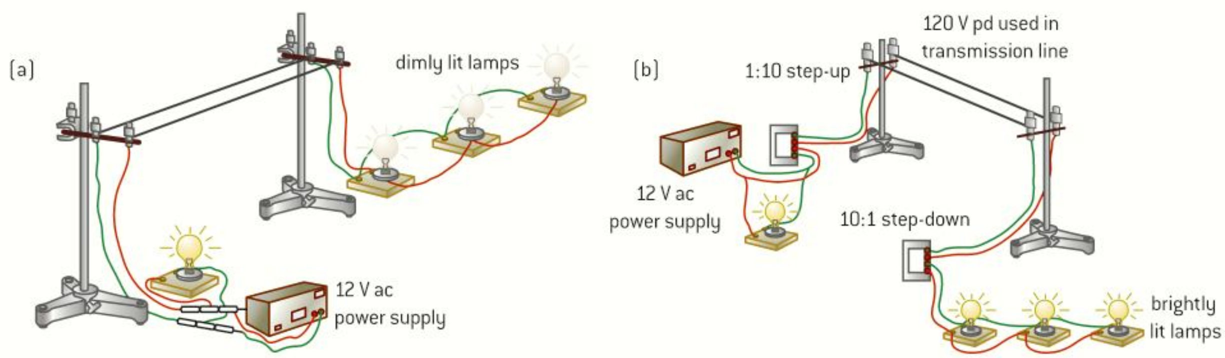
▲ Figure 7

Transformers in action

Many countries have an electrical grid system so that each separate community within the country does not have to provide its own energy. In the event of power failures in one part of the grid system, energy can be diverted where needed. When energy has to be sent over long distances it is advantageous to send it at very high voltages. This is because transmitting at high voltages and low currents helps to reduce the energy lost in the power transmission lines that are used in the grid.

A laboratory example will help here:

Figure 8 shows two alternative ways to transmit electrical energy from one point in a school laboratory to another. In both circuits a power supply acts as the "generator" providing energy at an alternating potential difference of 12 V. This energy is transmitted through two transmission lines (leads strung between two retort stands). In one case (Figure 8(a)), the energy is tramsitted at 12 V, but in the other (Figure 8(b)) a step-up transformer with a turns ratio of 1:10 is used to transmit the energy at 120 V. At the other end of this transmission line, a step-down transformer returns the pd supply to the voltage level required by the lamps in the circuit.



▲ Figure 8 Transmission at high voltages.

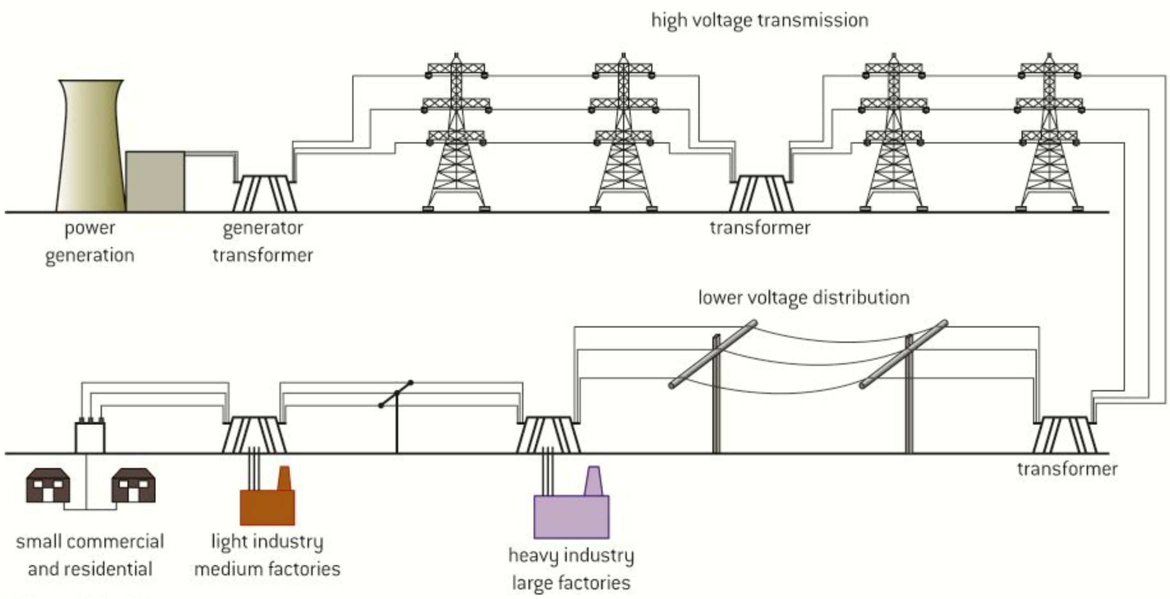
A numerical calculation will help you to compare the two cases: In each case, there are three lamps each rated at 0.3 A, 12V (a total power requirement of 3.6 W) and a transmission line of total resistance 1.5 ohms.

Without the transformers, the total current required by lamps will be 0.9 A and power loss in transmission line $= I^2R = 0.81 \times 1.5 = 1.2$ W.

With the voltage stepped up to 120 V along the transmission line, the total current will be stepped down to 0.09 A during transmission. This means the power loss in the transmission line $= I^2R = 0.081 \times 1.5 = 0.012$ W.

So, stepping *up* the voltage by a factor of 10 reduces the current by a factor of 10 and therefore reduces the power lost during transmission by a factor of 100 (i.e. 10^2). This is an important saving for electricity supply companies (and their customers!). Transformers play a crucial role in increasing the efficiency of transmission.

In practice, grid systems are usually like the one shown in figure 9. Numbers are not given on this diagram as they vary from country to country. Find out the details of the grid voltages used where you live.



▲ Figure 9 A grid system.