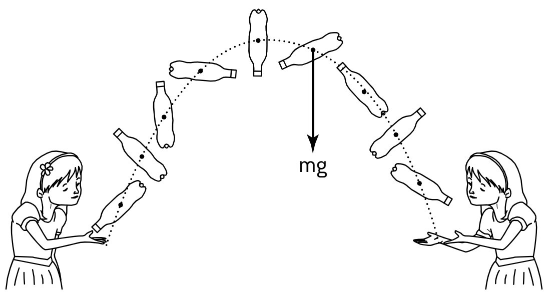
Transla ional and ro a ional mo ion

CONCEPTS

The complex motion o a rigid body can be analysed as a **combination** o two types o motion: *translation* and *rotation*. Both these types o motion are studied separately in this study guide (pages 9 and 65).



A bottle thrown through the air – the centre o mass o the bottle ollows a path as predicted by projectile motion. In addition the bottle rotates about one (or more) axes.

Translational motion	Rotational motion
Every particle in the object has the same instantaneous velocity	Every particle in the object moves in a circle around the same axis o rotation
Displacement, s, measured in m	Angular displacement, θ, measured in radians [rad]
Velocity, v, is the rate o change o displacement measured in m s^{-1} $v = \frac{ds}{dt}$	Angular velocity, ω, is the rate o change o angle measured in rad s^{-1} $\omega = \frac{d\theta}{dt}$
Acceleration, a, is the rate o change o velocity measured in m s^{-2} $a = \frac{dv}{dt}$	Angular acceleration, α, is the rate o change o angular velocity measured in rad s^{-2} $\alpha = \frac{d\omega}{dt}$

Comparison o linear and rotational motion

Translational motion is described using displacements, velocities and linear accelerations; all these quantities apply to the **centre of mass** o the object. Rotational motion is described using angles (angular displacement), angular velocities and angular accelerations; all these quantities apply to circular motion about a given axis o rotation.

The concept o angular velocity, ω, has already been introduced with the mechanics o circular motion (see page 66) and is linked to the requency o rotation by the ollowing ormula:

$$\omega = 2\pi f$$

angular velocity requency

EQUATIONS OF UNIFORM ANGULAR ACCELERATION

The definitions o average linear velocity and average linear acceleration can be rearranged to derive the constant acceleration equations (page 11). An equivalent rearrangement derives the equations o constant angular acceleration.

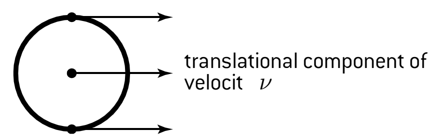
Translational motion		Rotational motion	
Displacement	s	Angular displacement	θ
Initial velocity	u	Initial angular velocity	ω_i
Final velocity	v	Final angular velocity	ω_f
Time taken	t	Time taken	t
Acceleration	a [constant]	Angular acceleration	α [constant]
$v = u + at$		$\omega_f = \omega_i + \alpha t$	
$s = ut + \frac{1}{2}at^2$		$\theta = \omega_i t + \frac{1}{2}\alpha t^2$	
$v^2 = u^2 + 2as$		$\omega_f^2 = \omega_i^2 + 2\alpha\theta$	
$s = \frac{(v+u)t}{2}$		$\theta = \frac{(\omega_f + \omega_i)t}{2}$	

EXAMPLE: BICYCLE WHEEL

When a bicycle is moving orward at constant velocity v, the di erent points on the wheel each have di erent velocities. The motion o the wheel can be analysed as the addition o the translational and the rotational motion.

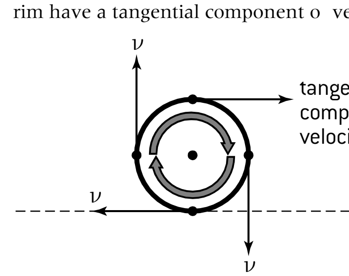
a) Translational motion

The bicycle is moving orward at velocity v so the wheel's centre o mass has orward translational motion o velocity v. All points on the wheel's rim have a translational component orward at velocity v.



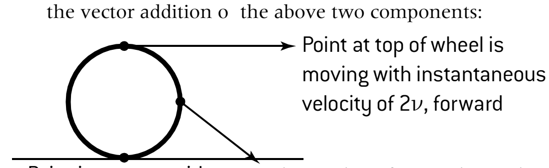
b) Rotational motion

The wheel is rotating around the central axis o rotation at a constant angular velocity ω. All points on the wheel's rim have a tangential component o velocity v ($= r\omega$)



c) Combined motion

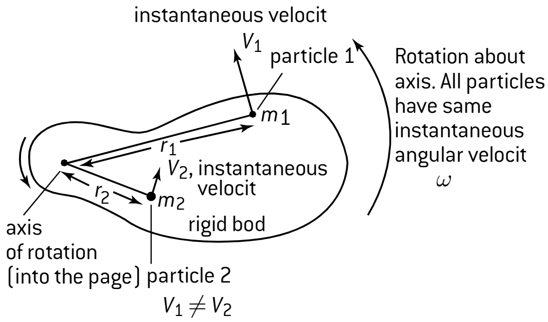
The motion o the di erent points on the wheel's rim is the vector addition o the above two components:



Transla ional and ro a ional rela ionships

RELATIONSHIP BETWEEN LINEAR AND ROTATIONAL QUANTITIES

When an object is just rotating about a fixed axis, and there is no additional translational motion o the object, all the individual particles that make up that object have di erent instantaneous values o linear displacement, linear velocity and linear acceleration. They do, however, all share the same instantaneous values o angular displacement, angular velocity and angular acceleration. The link between these values involves the distance rom the axis o rotation to the particle.



a) Displacements

Distance travelled on circular path

Angular displacement

$$s = r\theta$$

Distance rom axis o rotation to particle

b) Instantaneous velocities

Linear instantaneous velocity (along the tangent)

Angular velocity

$$v = r\omega$$

Distance rom axis o rotation to particle

c) Accelerations

The total linear acceleration o any particle is made up o two components:

a) The **centripetal acceleration**, a_r, (towards the axis o rotation – see page 65), also known as the **radial acceleration**.

Tangential velocity

Angular velocity

$$a_r = \frac{v^2}{r} = r\omega^2$$

Centripetal acceleration (along the radius)

Distance rom axis o rotation to particle

b) An additional **tangential acceleration**, a_t, which results rom an angular acceleration taking place. I $\alpha = 0$, then $a_t = 0$.

Instantaneous acceleration (along the tangent)

Angular acceleration

$$a_t = r\alpha$$

Distance rom axis o rotation to particle

The total acceleration o the particle can be ound by vector addition o these two components: $a = r\sqrt{\omega^4 + \alpha^2}$

Translational and rotational equilibrium

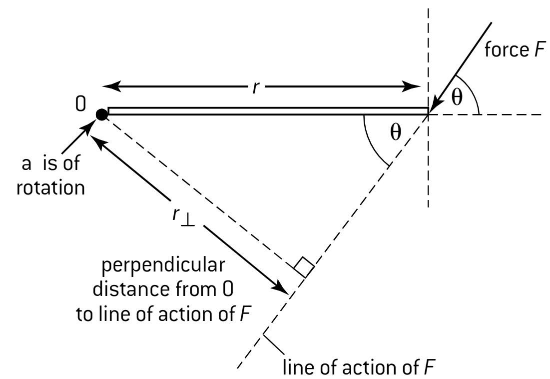
THE MOMENT OF A FORCE: THE TORQUE Γ

A particle is in equilibrium i its acceleration is zero. This occurs when the vector sum o all the external orces acting on the particle is zero (see page 16). In this situation, all the orces pass through a single point and sum to zero. The orces on real objects do not always pass through the same point and can create a turning e ect about a given axis. The turning e ect is called the **moment of the force** or the **torque**. The symbol or torque is the Greek uppercase letter gamma, Γ.

The moment or torque Γ o a orce, F about an axis is defined as the product o the orce and the perpendicular distance rom the axis o rotation to the line o action o the orce.

$$\underbrace{\Gamma}_{\text{moment or torque}} = \underbrace{F}_{\text{orce}}\underbrace{r_\perp}_{\text{perpendicular distance}}$$

$$\Gamma = Fr\sin\theta$$

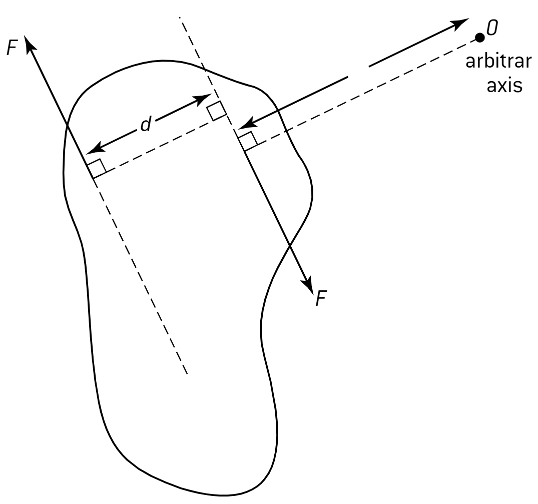


Note:

- The torque and energy are both measured in N m, but only energy can also be expressed as joules.
- The direction o any torque is clockwise or anticlockwise about the axis o rotation that is being considered. For the purposes o calculations, this can be treated as a vector quantity with the direction o the torque vector considered to be along the axis o rotation. In the example above, the torque vector is directed into the paper. I the orce F was applied in the opposite direction, the torque vector would be directed out o the paper.

COUPLES

A **couple** is a system o   orces that has no resultant orce but which does produce a turning e ect. A common example is a pair o equal but anti-parallel orces acting with di erent points o application. In this situation, the resultant torque is the same about all axes drawn perpendicular to the plane defined by the orces.



Torque of forces $= F(\ + d) - F$
$= F_d$ clockwise
This result is independent of position of axis, O

ROTATIONAL AND TRANSLATIONAL EQUILIBRIUM

I a resultant orce acts on an object then it must accelerate (page 17). When there is no resultant orce acting on an object then we know it to be in translational equilibrium (page 16) as this means its acceleration must be zero.

Similarly, i there is a resultant torque acting on an object then it must have an angular acceleration, α. Thus an object will be in **rotational equilibrium** only i the vector sum o all the external torques acting on the object is zero.

I an object is not moving and not rotating then it is said to be in **static equilibrium**. This must mean that the object is in both rotational and translational equilibrium.

For rotational equilibrium:

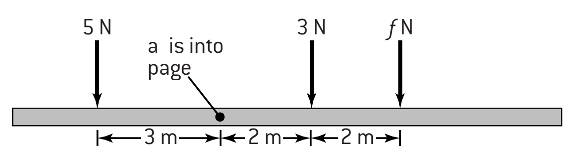
$$\alpha = 0 \therefore \sum \Gamma = 0$$

In 2D problems (in the x-y plane), it is su ficient to show that there is no torque about any **one** axis perpendicular to the plane being considered (parallel to the z-axis). In 3D problems, three axis directions (x, y and z) would need to be considered.

For translational equilibrium:

$$a = 0 \therefore \sum F = 0$$

In 2D problems, it is su ficient to show that there is no resultant orce in **t o** di erent directions. In 3D problems three axis directions (x, y and z) would need to be considered.



In the example above, or rotational equilibrium:

$f = 2.25$ N

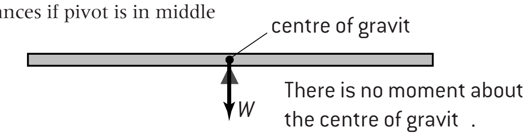
Equilibrium e amples

CENTRE OF GRAVITY

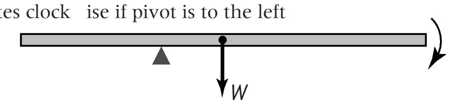
The effect of gravity on all the different parts of the object can be treated as a single force acting at the object's **centre of gravity**.

If an object is of uniform shape and density, the centre of gravity will be in the middle of the object. If the object is not uniform, then finding its position is not trivial – it is possible for an object's centre of gravity to be outside the object. Experimentally, if you suspend an object from a point and it is free to move, then the centre of gravity will always end up below the point of suspension.

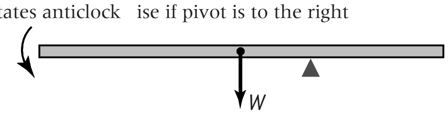
(a) plank balances if pivot is in middle



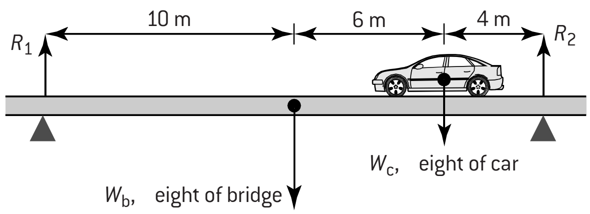
(b) plank rotates clock   ise if pivot is to the left



(c) plank rotates anticlock   ise if pivot is to the right



EXAMPLE 1



When a car goes across a bridge, the forces (on the bridge) are as sho   n.

Taking moments about right-hand support:

clock   ise moment = anticlock   ise moment

$$(R_1 \times 20 \text{ m}) = (W_b \times 10 \text{ m}) + (W_c \times 4 \text{ m})$$

Taking moments about left-hand support:

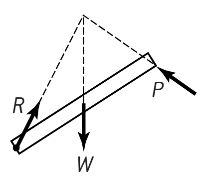
$$(R_2 \times 20 \text{ m}) = (W_b \times 10 \text{ m}) + (W_c \times 16 \text{ m})$$

Also, since bridge is not accelerating:

$$R_1 + R_2 = W_b + W_c$$

When solving problems to do with rotational equilibrium remember:

- All forces at an axis have zero moment about that axis.
- You do not have to choose the pivot as the axis about which you calculate torques, but it is often the simplest thing to do (for the reason above).
- You need to remember the sense (clockwise or anticlockwise).
- When solving two-dimensional problems it is sufficient to show that an object is in rotational equilibrium about any ONE axis.
- Newton's laws still apply. Often an object is in rotational AND in translational equilibrium. This can provide a simple way of finding an unknown force.
- The weight of an object can be considered to be concentrated at its centre of gravity.
- If the problem only involves three non-parallel forces, the lines of action of all the forces must meet at a single point in order to be in rotational equilibrium.



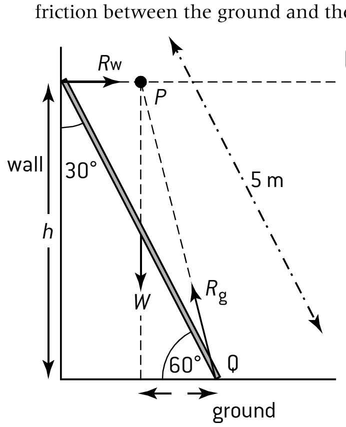
3 forces must meet at a point if in equilibrium

EXAMPLE 2

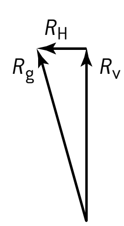
A ladder of length 5.0 m leans against a smooth wall (no friction) at an angle of 30° to the vertical.

a) Explain why the ladder can only stay in place if there is friction between the ground and the ladder.

b) What is the minimum coefficient of static fraction between the ladder and the ground for the ladder to stay in place?



(a) The reaction from the wall, R_w and the ladder's weight meet at point P. For equilibrium the force from the ground, R_g must also pass through this point (for zero torque about P). $\therefore$ R_g is as shown and has a horizontal component (i.e. friction must be acting)



(b) Equilibrium conditions:-

(↑) $W = R_v$ ①

(→) $R_H = R_w$ ②

moments about Q $R_w h = W$ ③

$$F_f \leq \mu_s R$$

$$\therefore R_H \leq \mu_s R_v$$

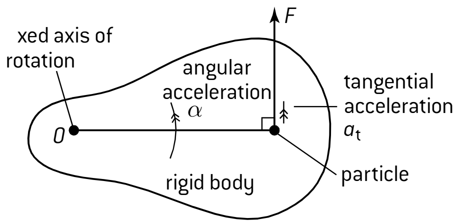
using ① & ② $\Rightarrow \mu_s \geq \frac{R_w}{W}$

③ $\Rightarrow \mu_s \geq \frac{}{h} = \frac{2.5 \cos 60}{5.0 \sin 60}$

$$\therefore \mu_s \geq 0.29$$

Ne ton's second la — moment of inertia

NEWTON'S SECOND LAW – DEFINITION OF MOMENT OF INERTIA



Newton's second law as applied to one particle in a rigid body

Newton's second law applies to every particle that makes up a large object and must also apply i the object is undergoing rotational motion. In the diagram above, the object is made up o lots o small particles each with a mass *m*. *F* is the *tangential component* o the resultant orce that acts *on one particle*. The other component, the radial component, cannot produce angular acceleration so it is not included. For this particle we can apply Newton's second law:

$F = m\,a_t = mr\alpha$

so torque $\Gamma = (mr\alpha)r = mr^2\,\alpha$

Similar equations can be created or all the particles that make up the object and summed together:

$\Gamma = \quad mr^2\,\alpha$

or $\quad \Gamma_{ext} = \alpha \quad mr^2 \qquad (1)$

Note that:

- Newton's third law applies and, when summing up all the torques, the internal torques (which result rom the internal orces between particles) must sum to zero. Only the external torques are le t.
- Every particle in the object has the same angular acceleration, α.

The moment o inertial, *I*, o an object about a particular axis is defined by the summation below:

moment o inertia — the distance o the particle rom the axis or rotation

$$I = \quad mr^2$$

mass o an individual particle in the object

Note that moment o inertia, *I*, is

- A scalar quantity
- Measured in kg m² (not kg m⁻²)
- Dependent on:
  - ◊ The mass o the object
  - ◊ The way this mass is distributed
  - ◊ The axis o rotation being considered.

Using this definition, equation 1 becomes:

resultant external torque in N m — angular acceleration in rad s⁻²

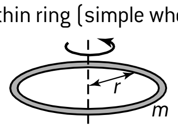
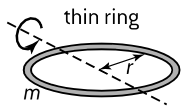
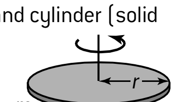
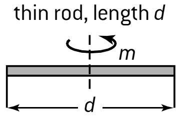
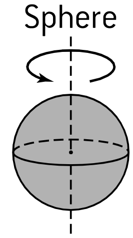
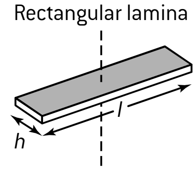
$$\Gamma = I\alpha$$

moment o inertia in kg m²

This is Newton's second law or rotational motion and can be compared to $F = ma$

MOMENTS OF INERTIA FOR DIFFERENT OBJECTS

Equations or moments o inertia in di erent situations do not need to be memorized.

Object	Axis of rotation	moment of inertia
thin ring (simple wheel) 	through centre, perpendicular to plane	mr^2
thin ring 	through a diameter	$\frac{1}{2}\,mr^2$
disc and cylinder (solid ywheel) 	through centre, perpendicular to plane	$\frac{1}{2}\,mr^2$
thin rod, length *d* 	through centre, perpendicular to rod	$\frac{1}{12}\,md^2$
Sphere 	through centre	$\frac{2}{5}\,mr^2$
Rectangular lamina 	*Through the centre of mass, perpendicular to the plane of the lamina*	$m\left(\frac{l^2 + h^2}{12}\right)$

EXAMPLE

A torque o 30 N m acts on a wheel with moment o inertia 600 kg m². The wheel starts o at rest.

a) What angular acceleration is produced?

b) The wheel has a radius o 40 cm. A ter 1.5 minutes:

i. what is the angular velocity o the wheel?

ii. how ast is a point on the rim moving?

a) $\Gamma = I\,\alpha \Rightarrow \alpha = \frac{\Gamma}{I} = \frac{30}{600} = 5.0 \times 10^{-2}$ rad s⁻²

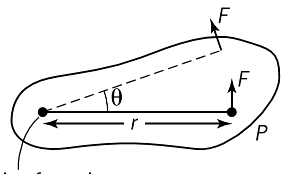
b) i. $\omega = \alpha t = 5.0 \times 10^{-2} \times 90 = 4.5$ rad s⁻¹

ii. $v = r\,\omega = 0.4 \times 4.5 = 1.8$ m s⁻¹

Rotational d namics

ENERGY OF ROTATIONAL MOTION

Energy considerations o ten provide simple solutions to complicated problems. When a torque acts on an object, work is done. In the absence o any resistive torque, the work done on the object will be stored as rotational kinetic energy.



Calculation o work done by a torque

In the situation above, a orce F is applied and the object rotates. As a result, an angular displacement o θ occurs. The work done, W, is calculated as shown below:

$W = F \times$ (distance along arc) $= F \times r\theta = \Gamma\theta$

Using $\Gamma = I\alpha$ we know that $W = I\alpha\theta$

We can apply the constant angular acceleration equation to substitute or $\alpha\theta$:

$$\ _f^2 = \ _i^2 + 2\alpha\theta$$

$$W = I\left(\frac{\ _f^2}{2} - \frac{\ _i^2}{2}\right) = \frac{1}{2} I \ _f^2 - \frac{1}{2} I \ _i^2$$

This means that we have an equation or rotational KE:

$$E_{K_{rot}} = \frac{1}{2} I \ ^2$$

Work done by the torque acting on object = change in rotational KE o object

The total KE is equal to the sum o translational KE and the rotational KE:

Total KE = translational KE + rotational KE

Total KE $= \frac{1}{2} M \ ^2 + \frac{1}{2} I \ ^2$

ANGULAR MOMENTUM

For a single particle

The linear momentum, p, o a particle o mass m which has a tangential speed is m .

The angular momentum, L, is defined as the *moment* o the linear momentum about the axis o rotation

Angular momentum, $L = (m \ )r = (mr \ )r = (mr^2 \ )$

For a larger object

The angular momentum L o an object about an axis o rotation is defined as

Angular momentum, $L = \sum(mr^2 \ )$

$L = I$

Note that total angular momentum, L, is:

- a vector (in the same way that a torque is considered to be a vector or calculations)
- measured in kg m² s⁻¹ or N m s
- dependent on all rotations taking place. For example, the total angular momentum o a planet orbiting a star would involve:
  - ◊ the spinning o the planet about an axis through the planet's centre o mass and
  - ◊ the orbital angular momentum about an axis through the star.

CONSERVATION OF ANGULAR MOMENTUM

In exactly the same way that Newton's laws can be applied to linear motion to derive:

- the concept o the impulse o a orce
- the relationship between impulse and change in momentum
- the law o conservation o linear momentum,

then Newton's laws can be applied to angular situations to derive:

- The concept o the **angular impulse**:

  Angular impulse is the product o torque and the time or which the torque acts:

  angular impulse $= \Gamma\Delta t$

  I the torque varies with time then the total angular impulse given to an object can be estimated rom the area under the graph showing the variation o torque with time. This is analogous to estimating the total impulse given to an object as a result o a varying orce (see page 23).

- The relationship between angular impulse and change in angular momentum:

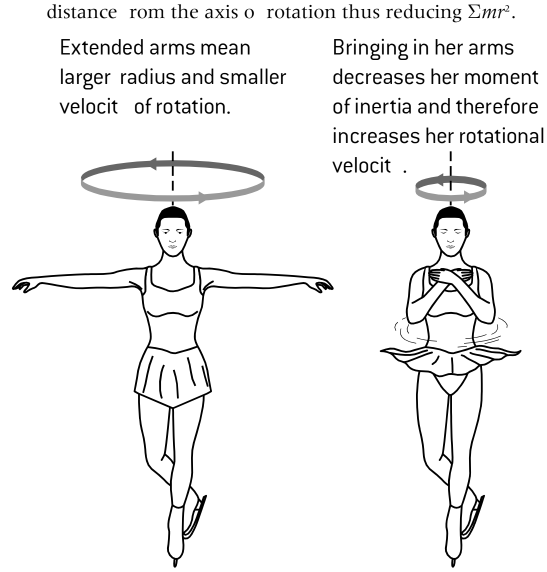
  angular impulse applied to an object = change o angular momentum experienced by the object

- The law o conservation o angular momentum.

The total angular momentum o a system remains constant provided no resultant external torque acts.

Examples:

a) A skater who is spinning on a vertical axis down their body can reduce their moment o inertia by drawing in their arms. This allows their mass to be redistributed so that the mass o the arms is no longer at a significant distance rom the axis o rotation thus reducing Σmr^2.



b) The Earth–Moon system produces tides in the oceans. As a result o the relative movement o water, riction exists between the oceans and Earth. This provides a torque that acts to reduce the Earth's spin on its own axis and thus reduces the Earth's angular momentum. The conservation o angular momentum means that there must be a corresponding increase in the orbital angular momentum o the Earth–Moon system. As a result, the Earth–Moon separation is slowly increasing.

Solving rotational problems

SUMMARY COMPARISON OF EQUATIONS OF LINEAR AND ROTATIONAL MOTION

Every equation for linear motion has a corresponding angular equivalent:

	Linear motion	**Rotational motion**
Physics principles	A resultant external force on a point object causes acceleration. The value of the acceleration is determined by the mass and the resultant force.	A resultant external torque on an extended object causes rotational acceleration. The value of the angular acceleration is determined by the moment of inertia and the resultant torque.
Newton s second law	$F = m\,a$	$\Gamma = I\,\alpha$
Work done	$W = F\,s$	$W = \Gamma\,\theta$
Kinetic energy	$E_K = \frac{1}{2}\,m\ \ ^2$	$E_{K_{rot}} = \frac{1}{2} I\ \ ^2$
Power	$P = F$	$P = \Gamma$
Momentum	$p = m$	$L = I$
Conservation of momentum	The total linear momentum of a system remains constant provided no resultant external force acts.	The total angular momentum of a system remains constant provided no resultant external torque acts.
Symbols used	Resultant force F Mass m Acceleration a Displacement s Velocity Linear momentum p	Resultant torque Γ Moment of inertia I Angular acceleration α Angular displacement θ Angular velocity Angular momentum L

PROBLEM SOLVING AND GRAPHICAL WORK

When analysing any rotational situation, the simplest approach is to imagine the equivalent linear situation and use the appropriate equivalent relationships.

a) Graph of angular displacement vs time

This graph is equivalent to a graph of linear displacement vs time. In the linear situation, the area under the graph does not represent any useful quantity and the gradient of the line at any instant is equal to the instantaneous velocity (see page 10). **Thus the gradient of an angular displacement vs time graph gives the instantaneous angular velocity.**

b) Graph of angular velocity vs time

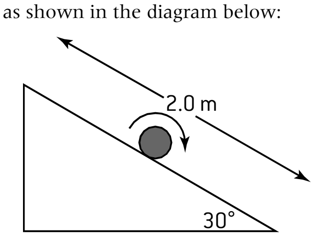
This graph is equivalent to a graph of linear velocity vs time. In the linear situation, the area under the graph represents the distance gone and the gradient of the line at any instant is equal to the instantaneous acceleration (see page 10). **Thus the area under an angular velocity vs time graph gives the total angular displacement and the gradient of an angular velocity vs time graph gives the instantaneous angular acceleration.**

c) Graph of torque vs time

This graph is equivalent to a graph of force vs time. In the linear situation, the area under the graph represents the total impulse given to the object which is equal to the change of momentum of the object (see page 23). **Thus the area under the torque vs time graph represents the total angular impulse given to the object which is equal to the change of angular momentum.**

EXAMPLE

A solid cylinder, initially at rest, rolls down a 2.0 m long slope of angle 30° as shown in the diagram below:



The mass of the cylinder is m and the radius of the cylinder is R. Calculate the velocity of the cylinder at the bottom of the slope.

Answer:

Vertical height fallen by cylinder = 2.0 sin30 = 1.0 m

$$PE \text{ lost} = mgh$$

$$KE \text{ gained} = \frac{1}{2} m\ \ ^2 + \frac{1}{2} I\ \ ^2$$

$$\text{but} \quad I = \frac{1}{2} mR^2 \qquad \text{(cylinder) see page 156}$$

$$\text{and} \quad = \frac{}{R}$$

$$\Rightarrow \quad KE \text{ gained} = \frac{1}{2} m\ \ ^2 + \frac{1}{2}\ \frac{mR^2}{2} \cdot \frac{^2}{R^2}$$

$$= \frac{1}{2} m\ \ ^2 + \frac{1}{4} m\ \ ^2$$

$$= \frac{3}{4} m\ \ ^2$$

Conservation of energy

$$\Rightarrow \quad mgh = \frac{3}{4} m\ \ ^2$$

$$= \sqrt{4\frac{gh}{3}}$$

$$= \sqrt{\frac{4 \times 9.8 \times 1.0}{3}}$$

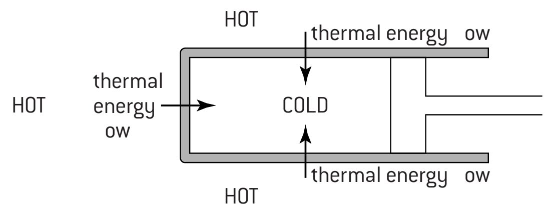
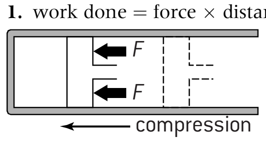
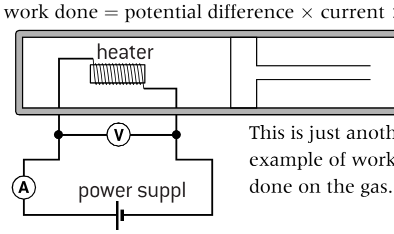
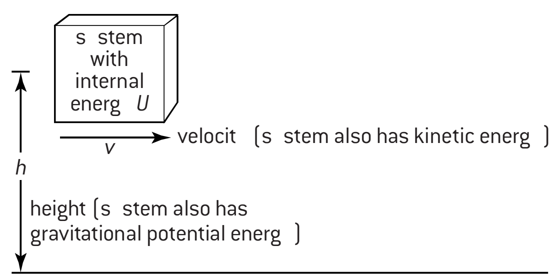
$$= 3.61 \text{ m s}^{-1}$$

Thermodynamic sys ems and concep s

DEFINITIONS

Historically, the study of the behaviour of ideal gases led to some very fundamental concepts that are applicable to many other situations. These laws, otherwise known as the laws of **thermodynamics,** provide the modern physicist with a set of very powerful intellectual tools.

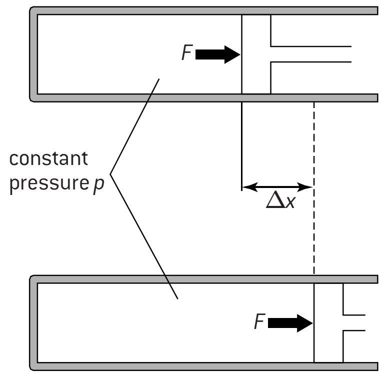
The terms used need to be explained.

Thermodynamic system	Most of the time when studying the behaviour of an ideal gas in particular situations, we focus on the macroscopic behaviour of the gas as a whole. In terms of work and energy, the gas can gain or lose thermal energy and it can do work or work can be done on it. In this context, the gas can be seen as a **thermodynamic system**.
The surroundings	If we are focusing our study on the behaviour of an ideal gas, then everything else can be called its **surroundings**. For example the expansion of a gas means that work is done by the gas on the surroundings (see below).
Heat Q	In this context heat refers to the transfer of a quantity of thermal energy between the system and its surroundings. This transfer must be as a result of a temperature difference. 
Work W	In this context, work refers to the macroscopic transfer of energy. For example **1.** work done = force × distance  *When a gas is compressed, ork is done on the gas* When a gas is compressed, the surroundings do work on it. When a gas expands it does work on the surroundings. **2.** work done = potential difference × current × time  This is just another example of work being done on the gas.
Internal energy U (*ΔU = change in internal energy*)	The internal energy can be thought of as the energy held within a system. It is the sum of the PE due to the intermolecular forces and the kinetic energy due to the random motion of the molecules. See page 26. This is different to the total energy of the system, which would also include the overall motion of the system and any PE due to external forces. In thermodynamics, it is the changes in internal energy that are being considered. If the internal energy of a gas is increased, then its temperature must increase. A change of phase (e.g. liquid → gas) also involves a change of internal energy.  The total energy of a system is not the same as its internal energy
Internal energy of an ideal monatomic gas	The internal energy of an ideal gas depends only on temperature. When the temperature of an ideal gas changes from T to $(T + \Delta T)$ its internal energy changes from U to $(U + \Delta U)$. The same ΔU always produces the same ΔT. Since the temperature is related to the average kinetic energy per molecule (see page 30), $\overline{E_K} = \frac{3}{2} k_B T = \frac{3}{2} \frac{R}{N_A} T$, the internal energy U, is the sum of the total random kinetic energies of the molecules: $U = nN_A \overline{E_K} = \frac{3}{2} nRT$ [n = number of moles; N_A = Avogadro s constant]

Work done by an ideal gas

WORK DONE DURING EXPANSION AT CONSTANT PRESSURE

Whenever a gas expands, it is doing work on its surroundings. If the pressure of the gas is changing all the time, then calculating the amount of work done is complex. This is because we cannot assume a constant force in the equation of work done (work done = force × distance). If the pressure changes then the force must also change. If the pressure is constant then the force is constant and we can calculate the work done.



Work done W = force × distance

$= F\Delta x$

Since pressure $= \dfrac{\text{force}}{\text{area}}$

$F = pA$

therefore

$W = pA\Delta x$

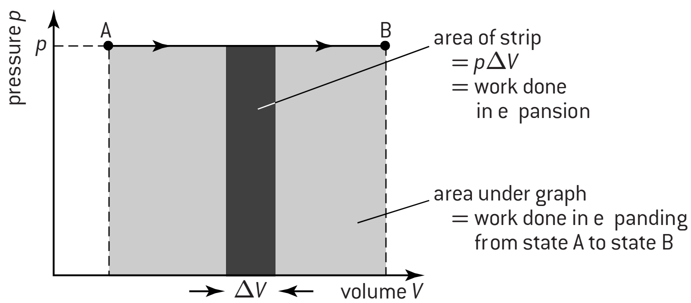
but $A\Delta x = \Delta V$

so work done $= p\Delta V$

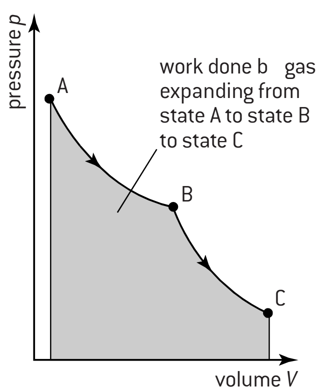
So if a gas increases its volume (ΔV is positive) then the gas does work (W is positive)

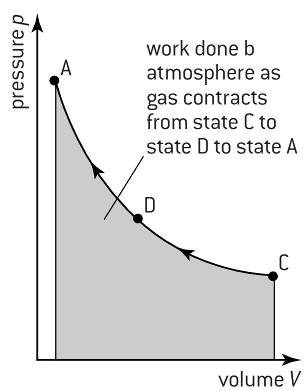
p V DIAGRAMS AND WORK DONE

It is often useful to represent the changes that happen to a gas during a thermod namic process on a pV diagram. An important reason for choosing to do this is that the area under the graph represents the work done. The reasons for this are shown below.



This turns out to be generall true for an thermod namic process.





The first law of thermod namics

FIRST LAW OF THERMODYNAMICS

There are three undamental laws o thermodynamics. The first law is simply a statement o the principle o energy conservation as applied to the system. I an amount o thermal energy Q is given to a system, then one o two things must happen (or a combination o both). The system can increase its internal energy ΔU or it can do work W.

As energy is conserved

$$Q = \Delta U + W$$

It is important to remember what the signs o these symbols mean. They are all taken rom the system's 'point o view'.

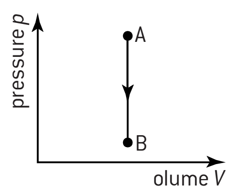
Q	I this is **positive**, then thermal energy is going into the system. I it is **negative**, then thermal energy is going out o the system.
ΔU	I this is **positive**, then the internal energy o the system is **increasing**. (The temperature o the gas is increasing.) I it is **negative**, the internal energy o the system is **decreasing**.(The temperature o the gas is decreasing.)
W	I this is **positive**, then the **system is doing work** on the surroundings.(The gas is expanding.) I it is **negative**, the **surroundings are doing work** on the system. (The gas is contracting.)

IDEAL GAS PROCESSES

A gas can undergo any number o di erent types o change or process. Four important processes are considered below. In each case the changes can be represented on a pressure–volume diagram and the first law o thermodynamics must apply. To be precise, these diagrams represent a type o process called a reversible process.

1. **Isochoric (isovolumetric)**

   In an isochoric process, also called an isovolumetric process, the gas has a constant volume. The diagram below shows an **isochoric decrease** in pressure.



   Isochoric (volumetric) change

   V = constant, or

   $\frac{p}{T}$ = constant

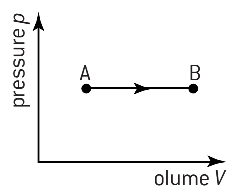
   Q negative

   ΔU negative (T↓)

   W zero

2. **Isobaric**

   In an isobaric process the gas has a constant pressure. The diagram below shows an **isobaric expansion**.



   Isobaric change

   p = constant, or

   $\frac{V}{T}$ = constant

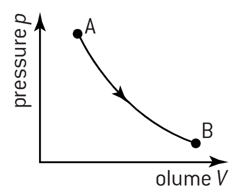
   Q positive

   ΔU positive (T↑)

   W positive

3. **Isothermal**

   In an isothermal process the gas has a constant temperature. The diagram below shows an **isothermal expansion**.



   Isothermal change

   T = constant, or

   pV = constant

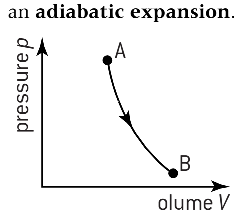
   Q positive

   ΔU zero

   W positive

4. **Adiabatic**

   In an adiabatic process there is no thermal energy trans er between the gas and the surroundings. This means that i the gas does work it must result in a decrease in internal energy. A rapid compression or expansion is approximately adiabatic. This is because done quickly there is not su ficient time or thermal energy to be exchanged with the surroundings. The diagram below shows an **adiabatic expansion**.



   Adiabatic change

   Q zero

   ΔU negative (T↓)

   W positive

   For a monatomic gas, the equation or an adiabatic process is

   $pV^{\frac{5}{3}}$ = constant

EXAMPLE

A monatomic gas doubles its volume as a result o an adiabatic expansion. What is the change in pressure?

$$p_1 V_1^{\frac{5}{3}} = p_2 V_2^{\frac{5}{3}}$$

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\frac{5}{3}}$$

$$= 0.5^{\frac{5}{3}}$$

$$= 0.31$$

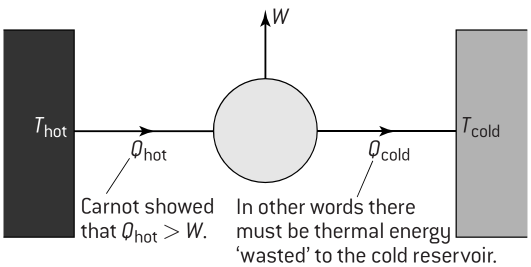
∴ final pressure = 31% o initial pressure

Second law of thermod namics and entrop

SECOND LAW OF THERMODYNAMICS

Historically the **second law o thermodynamics** has been stated in many different ways. All of these versions can be shown to be equivalent to one another.

In principle there is nothing to stop the complete conversion of thermal energy into useful work. In practice, a gas can not continue to expand forever – the apparatus sets a physical limit. Thus **the continuous conversion o thermal energy into work requires a cyclical process – a heat engine.**



This realization leads to possibly the simplest formulation of the second law of thermodynamics (the **Kelvin–Planck** formulation).

> **No heat engine, operating in a cycle, can take in heat rom its surroundings and totally convert it into work.**

Other possible formulations include the following:

> **No heat pump can trans er thermal energy rom a low-temperature reservoir to a high-temperature reservoir without work being done on it** (Clausius).

> **Heat flows rom hot objects to cold objects.**

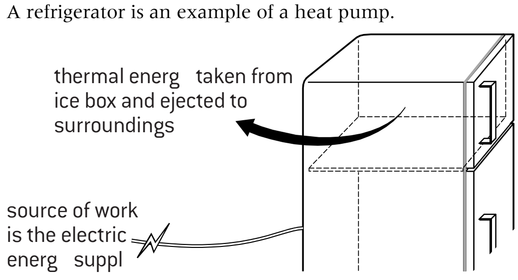
The concept of **entropy** leads to one final version of the second law.

> **The entropy o the Universe can never decrease.**

EXAMPLES

The first and second laws of thermodynamics both must apply to all situations. Local decreases of entropy are possible so long as elsewhere there is a corresponding increase.

1. A refrigerator is an example of a heat pump.



A refrigerator

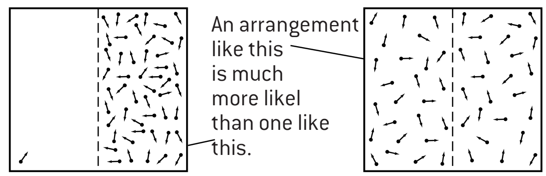
2. It should be possible to design a theoretical system for propelling a boat based around a heat engine. The atmosphere could be used as the hot reservoir and cold water from the sea could be used as the cold reservoir. The movement of the boat through the water would be the work done. This is possible BUT it cannot continue to work for ever. The sea would be warmed and the atmosphere would be cooled and eventually there would be no temperature difference.

ENTROPY AND ENERGY DEGRADATION

Entropy is a property that expresses the disorder in the system.

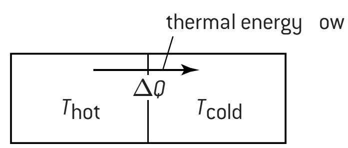
The details are not important but the entropy S of a system is linked to the number of possible arrangements W of the system. [$S = k_B \ln(W)$]

Because molecules are in random motion, one would expect roughly equal numbers of gas molecules in each side of a container.



The number of ways of arranging the molecules to get the set-up on the right is greater than the number of ways of arranging the molecules to get the set-up on the left. This means that the entropy of the system on the right is greater than the entropy of the system on the left.

In any random process the amount of disorder will tend to increase. In other words, the total entropy will always increase. The entropy change ΔS is linked to the thermal energy change ΔQ and the temperature T. ($\Delta S = \frac{\Delta Q}{T}$)

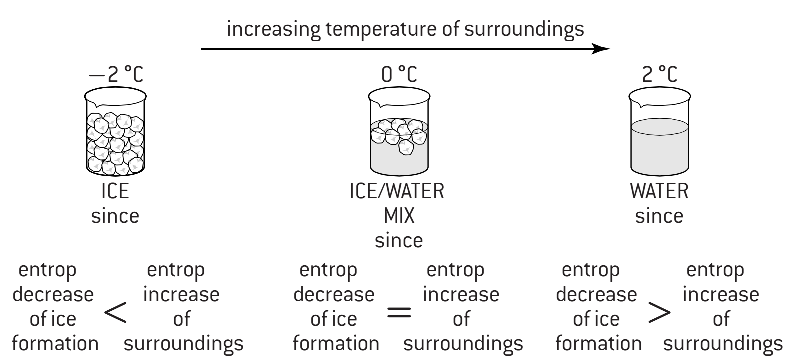


decrease of entropy $= \dfrac{\Delta Q}{T_{hot}}$ increase of entropy $= \dfrac{\Delta Q}{T_{cold}}$

When thermal energy flows from a hot object to a colder object, overall the total entropy has increased.

In many situations the idea of energy **degradation** is a useful concept. The more energy is shared out, the more degraded it becomes – it is harder to put it to use. For example, the internal energy that is 'locked' up in oil can be released when the oil is burned. In the end, all the energy released will be in the form of thermal energy – shared among many molecules. It is not feasible to get it back.

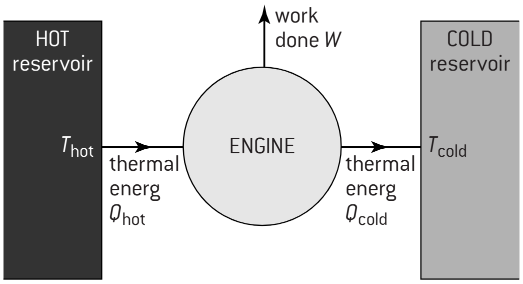
3. Water freezes at 0 °C because this is the temperature at which the entropy increase of the surroundings (when receiving the latent heat) equals the entropy decrease of the water molecules becoming more ordered. It would not freeze at a higher temperature because this would mean that the overall entropy of the system would decrease.



Heat engines and heat pumps

HEAT ENGINES

A central concept in the study o thermodynamics is the **heat engine**. A heat engine is any device that uses a source o thermal energy in order to do work. It converts heat into work. The internal combustion engine in a car and the turbines that are used to generate electrical energy in a power station are both examples o heat engines. A block diagram representing a generalized heat engine is shown below.

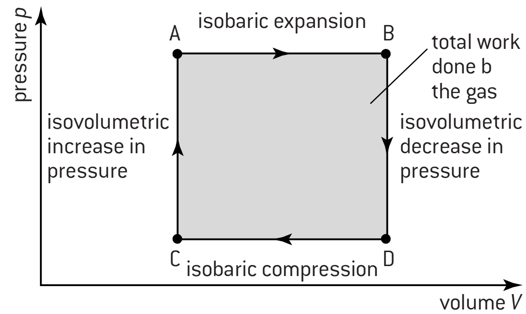


Heat engine

In this context, the word **reservoir** is used to imply a constant temperature source (or sink) o thermal energy. Thermal energy can be taken rom the hot reservoir without causing the temperature o the hot reservoir to change. Similarly thermal energy can be given to the cold reservoir without increasing its temperature.

An ideal gas can be used as a heat engine. The *pV* diagram right represents a simple example. The our-stage cycle returns the gas to its starting conditions, but the gas has done work. The area enclosed by the cycle represents the amount o work done.

In order to do this, some thermal energy must have been taken rom a hot reservoir (during the isovolumetric increase in pressure and the isobaric expansion). A di erent amount o thermal energy must have been ejected to a cold reservoir (during the isovolumetric decrease in pressure and the isobaric compression).



The thermal e ficiency o a heat engine is defined as

$$\eta = \frac{\text{work done}}{(\text{thermal energy taken rom hot reservoir})}$$

This is equivalent to

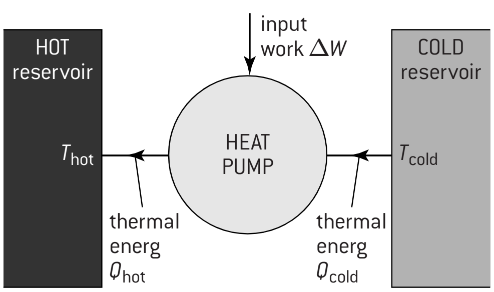
$$\eta = \frac{\text{rate o doing work}}{(\text{thermal power taken rom hot reservoir})}$$

$$\eta = \frac{\text{use ul work done}}{\text{energy input}}$$

The cycle o changes that results in a heat engine with the maximum possible e ficiency is called the **Carnot cycle**.

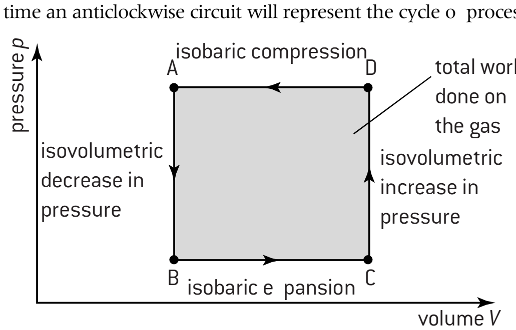
HEAT PUMPS

A **heat pump** is a heat engine being run in reverse. A heat pump causes thermal energy to be moved rom a cold reservoir to a hot reservoir. In order or this to be achieved, mechanical work must be done.



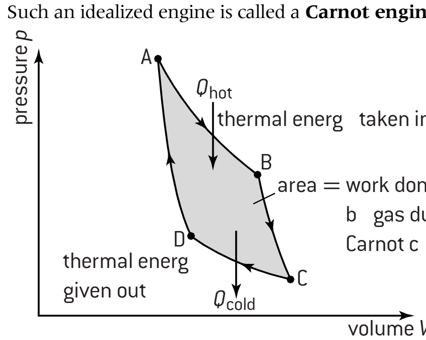
Heat pump

Once again an ideal gas can be used as a heat pump. The thermodynamic processes can be exactly the same ones as were used in the heat engine, but the processes are all opposite. This time an anticlockwise circuit will represent the cycle o processes.



CARNOT CYCLES AND CARNOT THEOREM

The Carnot cycle represents the cycle o processes or a theoretical heat engine with the maximum possible e ficiency. Such an idealized engine is called a **Carnot engine**.



Carnot cycle

It consists o an ideal gas undergoing the ollowing processes.

- Isothermal expansion (A → B)
- Adiabatic expansion (B → C)
- Isothermal compression (C → D)
- Adiabatic compression (D → A)

The temperatures o the hot and cold reservoirs fix the maximum possible e ficiency that can be achieved.

The e ficiency o a Carnot engine can be shown to be

$$\eta_{\text{Carnot}} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}} \quad \text{(where T is in kelvin)}$$

An engine operates at 300 °C and ejects heat to the surroundings at 20 °C. The maximum possible theoretical e ficiency is

$$\eta_{\text{Carnot}} = 1 - \frac{293}{573} = 0.49 = 49\%$$

Fluids at rest

DEFINITIONS OF DENSITY AND PRESSURE

The symbol representing density is the Greek letter rho, ρ. The average density o a substance is defined by the ollowing equation:

average density, mass, volume

$$\rho = \frac{m}{V}$$

- Density is a scalar quantity.
- The SI units o density are $kg\,m^{-3}$.
- Densities can also be quoted in $g\,cm^{-3}$ (see conversion actor below)
- The density o water is $1\,g\,cm^{-3} = 1{,}000\,kg\,m^{-3}$

Pressure at any point in a fluid (a gas or a liquid) is defined in terms o the orce, ΔF, that acts normally (at 90°) to a small area, ΔA, that contains the point.

pressure, normal orce, area

$$p = \frac{\Delta F}{\Delta A}$$

- Pressure is a scalar quantity – the orce has a direction but the pressure does not. Pressure acts equally in all directions.
- The SI unit o pressure is $N\,m^{-2}$ or pascals (Pa). $1\,Pa = 1\,N\,m^{-2}$
- Atmospheric pressure $\approx 10^5$ Pa
- Absolute pressure is the actual pressure at a point in a fluid. Pressure gauges o ten record the **difference** between absolute pressure and atmospheric pressure. Thus i a di erence pressure gauge gives a reading o 2×10^5 Pa or a gas, the absolute pressure o the gas is 3×10^5 Pa.

VARIATION OF FLUID PRESSURE

The pressure in a fluid increases with depth. I two points are separated by a vertical distance, d, in a fluid o constant density, ρ_f, then the pressure di erence, Δp, between these two points is:

density o fluid, gravitational field strength

$$\Delta p = \rho_f g d$$

pressure di erence due to depth, depth

The total pressure at a given depth in a liquid is the addition o the pressure acting at the sur ace (atmospheric pressure) and the additional pressure due to the depth:

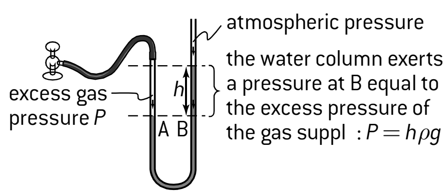
Atmospheric pressure, density o fluid, depth

$$P = P_0 + \rho_f g d$$

Total pressure, gravitational field strength

Note that:

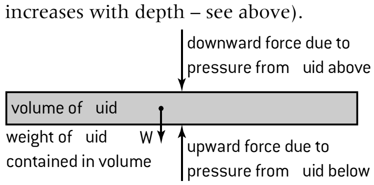
- Pressure can be expressed in terms o the equivalent depth (or head) in a known liquid. Atmospheric pressure is approximately the same as exerted by a 760 mm high column o mercury (Hg) or a 10 m column o water.
- As pressure is dependent on depth, the pressures at two points that are at the same horizontal level in the same liquid must be the same provided they are connected by that liquid and the liquid is static.



- The pressure is independent o the cross-sectional area – this means that liquids will always find their own level.

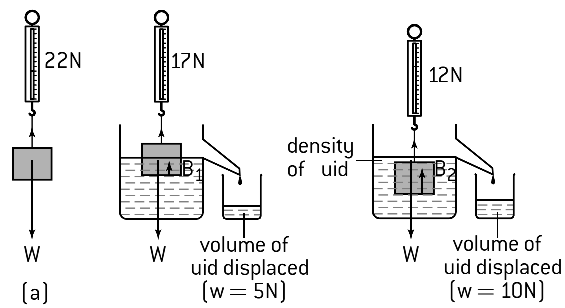
HYDROSTATIC EQUILIBRIUM

A fluid is in **hydrostatic equilibrium** when it is at rest. This happens when all the orces on a given volume o fluid are balanced. Typically external orces (e.g. gravity) are balanced by a pressure gradient across the volume o fluid (pressure increases with depth – see above).

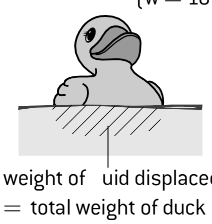


BUOYANCY AND ARCHIMEDES' PRINCIPLE

Archimedes' principle states that when a body is immersed in a fluid, it experiences a buoyancy upthrust equal in magnitude to the weight o the fluid displaced. $B = \rho_f V_f g$



A consequence o this principle is that a floating object displaces its own weight o fluid.

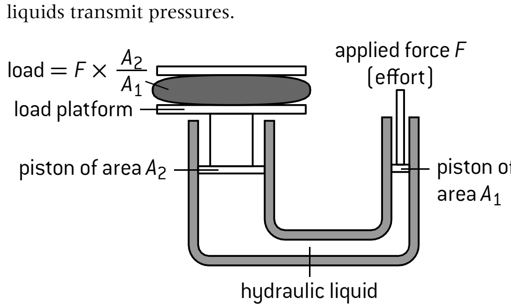


PASCAL'S PRINCIPLE

Pascal's principle states that the pressure applied to an enclosed liquid is transmitted to every part o the liquid, whatever the shape it takes. This principle is central to the design o many **hydraulic** systems and is di erent to how solids respond to orces.

When a solid object (e.g. an incompressible stick) is pushed at one end and its other end is held in place, then the same orce will be exerted on the restraining object.

Incompressible solids transmit orces whereas incompressible liquids transmit pressures.



Fluids in motion – Bernoulli e ect

THE IDEAL FLUID

In most real situations, fluid flow is extremely complicated. The ollowing properties define an ideal fluid that can be used to create a simple model. This simple model can be later refined to be more realistic.

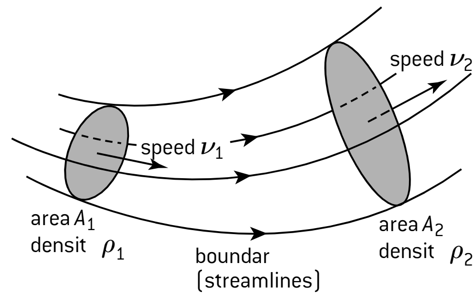
An ideal fluid:

- Is **incompressible** – thus its density will be constant.
- Is **non-viscous** – as a result o fluid flow, no energy gets converted into thermal energy. See page 167 or the definition o the viscosity o a real fluid.
- Involves a **steady flow** (as opposed to a **turbulent**, or chaotic, flow) o fluid. Under these conditions the flow is laminar (see box below). See page 167 or an analysis o turbulent flow.
- Does **not have angular momentum** – it does not rotate.

LAMINAR FLOW, STREAMLINES AND THE CONTINUITY EQUATION

When the flow o a liquid is steady or **laminar**, di erent parts o the fluid can have di erent instantaneous velocities. The flow is said to be laminar i every particle that passes through a given point has the same velocity whenever the observation is made. The opposite o **laminar flow**, turbulent flow, takes place when the particles that pass through a given point have a wide variation o velocities depending on the instant when the observation is made (see page 167).

A **streamline** is the path taken by a particle in the fluid and laminar flow means that all particles that pass through a given point in the fluid must ollow the same streamline. The direction o the tangent to a streamline gives the direction o the instantaneous velocity that the particles o the fluid have at that point. No fluid ever crosses a streamline. Thus a collection o streamlines can together define a **tube o flow**. This is tubular region o fluid where fluid only enters and leaves the tube through its ends and never through its sides.



In a time Δt, the mass, m_1, entering the cross-section A_1 is

$m_1 = \rho_1 A_1 v_1 \Delta t$

Similarly the mass, m_2, leaving the cross-section A_2 is

$m_2 = \rho_2 A_2 v_2 \Delta t$

Conservation o mass applies to this tube o flow, so

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

This is an ideal fluid and thus incompressible meaning $\rho_1 = \rho_2$, so

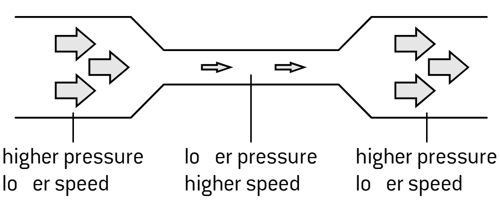
$A_1 v_1 = A_2 v_2$ or $Av = $ constant

This is the **continuity equation**.

THE BERNOULLI EFFECT

When a fluid flows into a narrow section o a pipe:

- The fluid must end up moving at a higher speed (continuity equation).
- This means the fluid must have been accelerated orwards.



- This means there must be a pressure di erence orwards with a lower pressure in the narrow section and a higher pressure in the wider section.

Thus an increase in fluid speed must be associated with a decrease in fluid pressure. This is the Bernoulli e ect – the greater the speed, the lower the pressure and vice versa.

THE BERNOULLI EQUATION

The Bernoulli equation results rom a consideration o the work done and the conservation o energy when an ideal fluid changes:

- its speed (as a result o a change in cross-sectional area)
- its vertical height as a result o work done by the fluid pressure.

The equation identifies a quantity that is always constant along any given streamline:

density o fluid　speed o fluid particles　vertical height　fluid pressure

$$\frac{1}{2}\rho v^2 + \rho g z + p = \text{constant}$$

density o fluid　gravitational field strength

Note that:

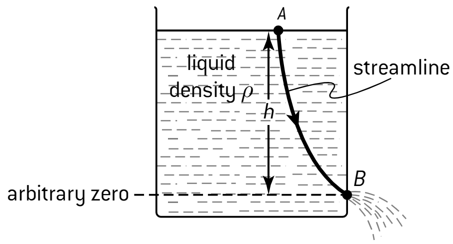
- The first term ($\frac{1}{2}\rho v^2$), can be thought o as the dynamic pressure.
- The last two terms ($\rho g z + p$), can be thought o as the static pressure.
- Each term in the equation has several possible units: N m^{-2}, Pa, J m^{-3}.
- The last o the above units leads to a new interpretation or the Bernoulli equation:

KE per unit volume + gravitational PE per unit volume + pressure = constant

Bernoulli – examples

APPLICATIONS OF THE BERNOULLI EQUATION

a) Flow out o a container



To calculate the speed o fluid flowing out o a container, we can apply Bernoulli's equation to the streamline shown above.

At A, p = atmospheric and v = zero

At B, p = atmospheric and v = ?

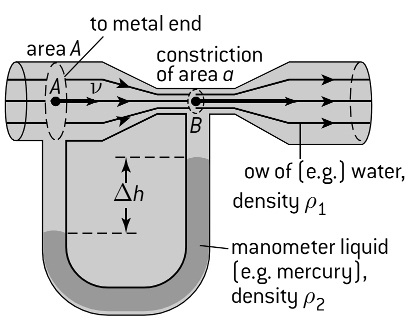
$$\frac{1}{2}\rho v^2 + \rho g z + p = \text{constant}$$

$$0 + h\rho g + p = \frac{1}{2}\rho v^2 + 0 + p$$

$$v = \sqrt{2gh}$$

b) Venturi tubes

A Venturi meter allows the rate o flow o a fluid to be calculated rom a measurement o pressure di erence between two di erent cross-sectional areas o a pipe.



- The pressure di erence between A and B can be calculated by taking readings o Δh and ρ_2 rom the attached manometer:

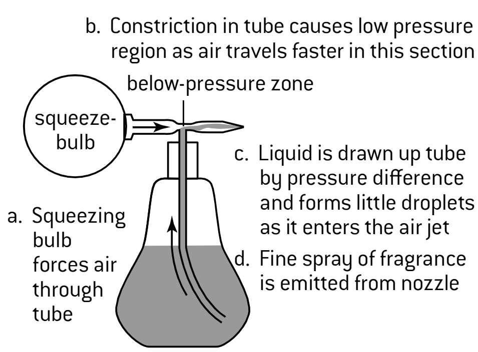
  $$P_A - P_B = \Delta h \rho_2 g$$

- This value and measurements o A, a and ρ_1 allows the fluid speed at A to be calculated by using Bernoulli's equation and the equation o continuity

  $$v = \sqrt{\frac{2\Delta h \rho_2 g}{\left[\rho_1\left(\frac{A}{a}\right)^2 - 1\right]}}$$

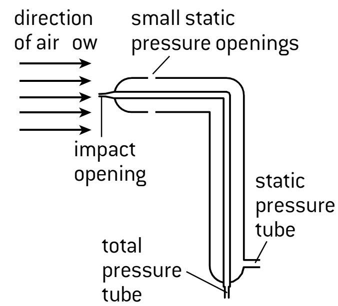
- The rate o flow o fluid through the pipe is equal to $A \times v$

c) Fragrance spray



d) Pitot tube to determine the speed o a plane

A pitot tube is attached acing orward on a plane. It has two separate tubes:

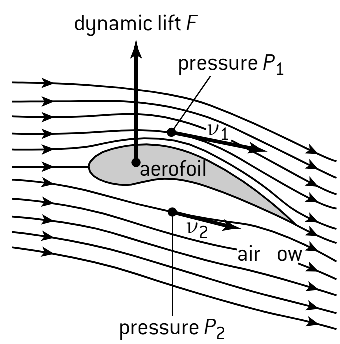


- The ront hole (impact opening) is placed in the airstream and measures the total pressure (sometimes called the stagnation pressure), P_T.
- The side hole(s) measures the static pressure, P_s.
- The di erence between P_T and P_s, is the dynamic pressure. The Bernoulli equation can be used to calculate airspeed:

  $$P_T - P_s = \frac{1}{2}\rho v^2$$

  $$v = \sqrt{\frac{2(P_T - P_s)}{\rho}}$$

e) Aero oil (aka air oil)



Note that:

- Streamlines closer together above the aero oil imply a decrease in cross-sectional area o equivalent tubes o flow above the aero oil.
- Decrease in cross-sectional area o tube o flow implies increased velocity o flow above the aero oil (equation o continuity). $v_1 > v_2$
- Since $v_1 > v_2$, $P_1 < P_2$
- Bernoulli equation can be used to calculate the pressure di erent (height di erence not relevant) which can support the weight o the aeroplane.
- When angle o attack is too great, the flow over the upper sur ace can become turbulent. This reduces the pressure di erence and leads to the plane '**stalling**'.

Viscosit

DEFINITION OF VISCOSITY

An ideal fluid does not resist the relative motion between di erent layers o fluid. As a result there is no conversion o work into thermal energy during laminar flow and no external orces are needed to maintain a steady rate o flow. Ideal fluids are non-viscous whereas real fluids are viscous. In a viscous fluid, a steady external orce is needed to maintain a steady rate o flow (no acceleration). Viscosity is an internal riction between di erent layers o a fluid which are moving with di erent velocities.

The definition o the viscosity o a fluid, η, (Greek letter Nu) is in terms o two new quantities, the **tangential stress**, τ, and the **velocity gradient**, $\frac{\Delta v}{\Delta y}$ (see RH side).

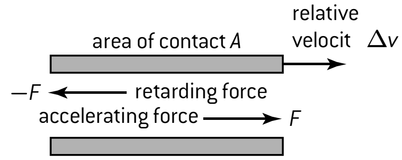
The coe ficient o viscosity η is defined as:

$$\eta = \frac{\text{tangential stress}}{\text{velocity gradient}} = \frac{F/A}{\Delta v/\Delta y}$$

- The units o η are N s m^{-2} or kg m^{-1} s^{-1} or Pa s
- Typical values at room temperature:
  - ◊ Water: 1.0×10^{-3} Pa s
  - ◊ Thick syrup: 1.0×10^{2} Pa s
- Viscosity is very sensitive to changes o temperature.

For a class o fluid, called **Newtonian fluids**, experimental measurements show that tangential stress is proportional to velocity gradient (e.g. many pure liquids). For these fluids the coe ficient o viscosity is constant provided external conditions remain constant.

A) Tangential stress

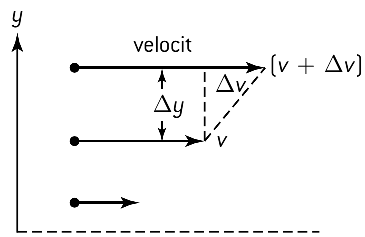


The tangential stress is defined as:

$$\tau = \frac{F}{A}$$

- Units o tangential stress are N m^{-2} or Pa

B) Velocity gradient



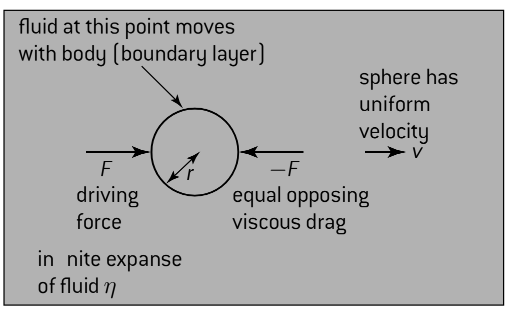
The velocity gradient is defined as:

$$\text{velocity gradient} = \frac{\Delta v}{\Delta y}$$

- Units o velocity gradient are s^{-1}

STOKES' LAW

Stokes' law predicts the viscous drag orce F_D that acts on a per ect sphere when it moves through a fluid:



Drag orce acting on sphere in N   viscosity o fluid in Pa s

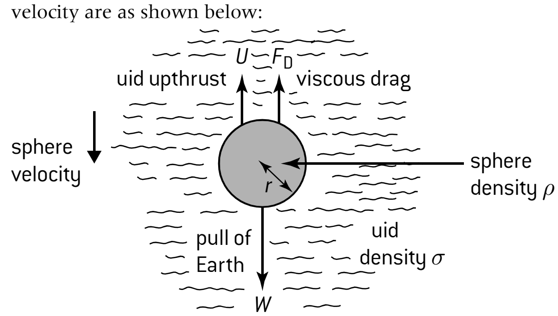
$$F_D = 6\pi\eta r v$$

radius o sphere in m   velocity o sphere in m s^{-1}

Note Stokes' law assumes that:

- The speed o the sphere is small so that:
  - ◊ the flow o fluid past the sphere is streamlined
  - ◊ there is no slipping between the fluid and the sphere
- The fluid is infinite in volume. Real spheres alling through columns o fluid can be a ected by the proximity o the walls o the container.
- The size o the particles o the fluid is very much smaller than the size o the sphere.

The orces on a sphere alling through a fluid at terminal velocity are as shown below:



At terminal velocity v_t,

$$W = U + F_D$$

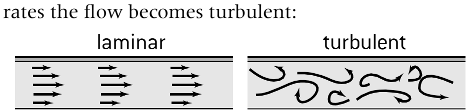
$$F_D = U - W$$

$$6\pi\eta r v_t = \frac{4}{3}\pi r^3(\rho - \sigma)g$$

$$v_t = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

TURBULENT FLOW – THE REYNOLDS NUMBER

Streamline flow only occurs at low fluid flow rates. At high flow rates the flow becomes turbulent:



It is extremely di ficult to predict the exact conditions when fluid flow becomes turbulent. When considering fluid flow down a pipe, a use ul number to consider is the Reynolds number, R, which is defined as:

Reynolds number   speed o bulk flow   radius o pipe   density o fluid

$$R = \frac{v r \rho}{\eta}$$

viscosity o fluid

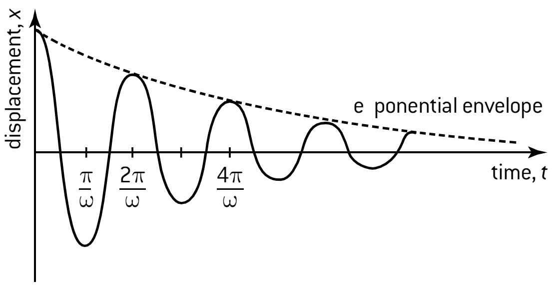
Note that:

- The Reynolds number does not have any units – it is just a ratio.
- Experimentally, fluid flow is o ten laminar when R < 1000 and turbulent when R > 2000 but precise predictions are di ficult.

Forced oscilla ions and resonance (1)

DAMPING

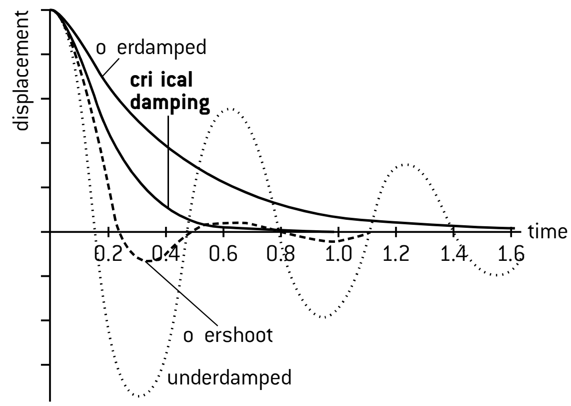
Damping involves a rictional orce that is always in the opposite direction to the direction o motion o an oscillating particle. As the particle oscillates, it does work against this resistive (or dissipative) orce and so the particle loses energy. As the total energy o the particle is proportional to the (amplitude)^2 o the SHM, the amplitude decreases exponentially with time.



The above example shows the e ect o **light damping** (the system is said to be **underdamped**) where the resistive orce is small so a small raction o the total energy is removed each cycle. The time period o the oscillations is not a ected and the oscillations continue or a significant number o cycles. The time taken or the oscillations to 'die out' can be long.

Heavy damping or **overdamping** involves large resistive orces (e.g. the SHM taking place in a viscous liquid) and can completely prevent the oscillations rom taking place. The time taken or the particle to return to zero displacement can again be long.

Critical damping involves an intermediate value or resistive orce such that the time taken or the particle to return to zero displacement is a minimum. E ectively there is no 'overshoot'. Examples o critically damped systems include electric meters with moving pointers and door closing mechanisms.

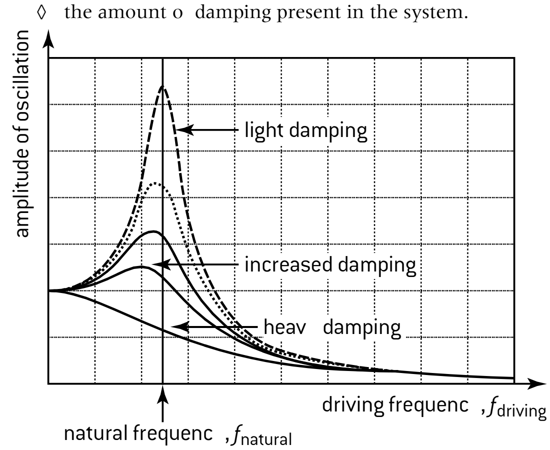


NATURAL FREQUENCY AND RESONANCE

I a system is temporarily displaced rom its equilibrium position, the system will oscillate as a result. This oscillation will be at the **natural frequency of vibration** o the system. For example, i you tap the rim o a wine glass with a kni e, it will oscillate and you can hear a note or a short while. Complex systems tend to have many possible modes o vibration each with its own natural requency.

It is also possible to orce a system to oscillate at any requency that we choose by subjecting it to a changing orce that varies with the chosen requency. This periodic driving orce must be provided rom outside the system. When this **driving frequency** is first applied, a combination o natural and orced oscillations take place which produces complex **transient** oscillations. Once the amplitude o the transient oscillations 'die down', a steady condition is achieved in which:

- The system oscillates at the driving requency.
- The amplitude o the orced oscillations is fixed. Each cycle energy is dissipated as a result o damping and the driving orce does work on the system. The overall result is that the energy o the system remains constant.
- The amplitude o the orced oscillations depends on:
  - ◊ the comparative values o the natural requency and the driving requency
  - ◊ the amount o damping present in the system.



Resonance occurs when a system is subject to an oscillating orce at exactly the same requency as the natural requency o oscillation o the system.

Q FACTOR AND DAMPING

The degree o damping is measured by a quantity called the quality actor or Q actor. It is a ratio (no units) and the definition is:

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy lost per cycle}}$$

Since the energy stored is proportional to the square o amplitude o the oscillation, measurements o decreasing amplitude with time can be used to calculate the Q actor. The Q actor is approximately equal to the number o oscillations that are completed be ore damping stops the oscillation.

Typical orders o magnitude or di erent Q- actors:

Car suspension:	1
Simple pendulum:	10^3
Guitar string:	10^3
Excited atom:	10^7

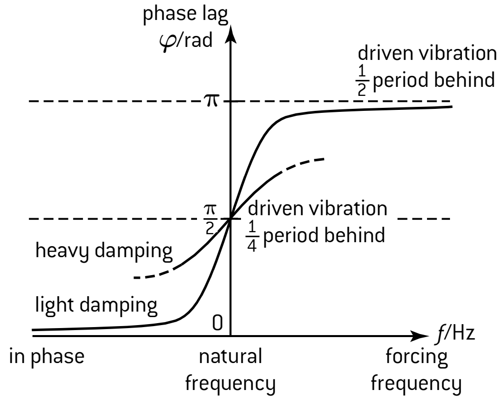
When a system is in resonance and its amplitude is constant, the energy provided by the driving requency during one cycle is all used to overcome the resistive orces that cause damping. In this situation, the Q actor can be calculated as:

$$Q = 2\pi \times \text{resonant requency} \times \frac{\text{energy stored}}{\text{power loss}}$$

Resonance (2)

PHASE OF FORCED OSCILLATIONS

After transient oscillations have died down, the frequency of the forced oscillations equals the driving frequency. The phase relationship between these two oscillations is complex and depends on how close the driven system is to resonance:



EXAMPLES OF RESONANCE

	Comment
Vibrations in machinery	When in operation, the moving parts of machinery provide regular driving forces on the other sections of the machinery. If the driving frequency is equal to the natural frequency, the amplitude of a particular vibration may get dangerously high. e.g. at a particular engine speed a truck's rear view mirror can be seen to vibrate.
Quartz oscillators	A quartz crystal feels a force if placed in an electric field. When the field is removed, the crystal will oscillate. Appropriate electronics are added to generate an oscillating voltage from the mechanical movements of the crystal and this is used to drive the crystal at its own natural frequency. These devices provide accurate clocks for microprocessor systems.
Microwave generator	Microwave ovens produce electromagnetic waves at a known frequency. The changing electric field is a driving force that causes all charges to oscillate. The driving frequency of the microwaves provides energy, which means that water molecules in particular are provided with kinetic energy – i.e. the temperature is increased.
Radio receivers	Electrical circuits can be designed (using capacitors, resistors and inductors) that have their own natural frequency of electrical oscillations. The free charges (electrons) in an aerial will feel a driving force as a result of the frequency of the radio waves that it receives. Adjusting the components of the connected circuit allows its natural frequency to be adjusted to equal the driving frequency provided by a particular radio station. When the driving frequency equals the circuit's natural frequency, the electrical oscillations will increase in amplitude and the chosen radio station's signal will dominate the other stations.
Musical instruments	Many musical instruments produce their sounds by arranging for a column of air or a string to be driven at its natural frequency which causes the amplitude of the oscillations to increase.
Greenhouse effect	The natural frequency of oscillation of the molecules of greenhouse gases is in the infra-red region. Radiation emitted from the Earth can be readily absorbed by the greenhouse gases in the atmosphere. See page 92 for more details.

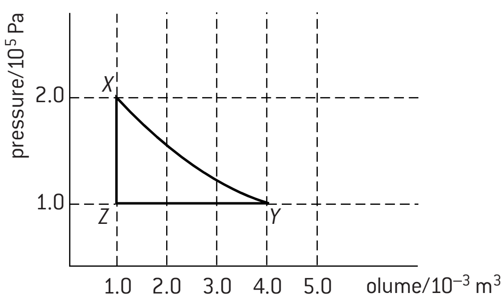
IB Quest ons – opt on B – eng neer ng phys cs

1. A sphere o mass m and radius r rolls, without slipping, rom rest down an inclined plane. When it reaches the base o the plane, it has allen a vertical distance h. Show that the speed o the sphere, v, when it arrives at the base o the incline is given by:

   $v = \sqrt{\frac{10gh}{7}}$ [4]

2. A flywheel o moment o inertia 0.75 kg m^2 is accelerated uni ormly rom rest to an angular speed o 8.2 rad s^{-1} in 6.5 s.
   a) Calculate the resultant torque acting on the flywheel during this time. [2]
   b) Calculate the rotational kinetic energy o the flywheel when it rotates at 8.2 rad s^{-1} [2]
   c) The radius o the flywheel is 15 cm. A breaking orce applied on the circum erence and brings it to rest rom an angular speed o 8.2 rad s^{-1} in exactly 2 revolutions. Calculate the value o the breaking orce. [2]

3. A fixed mass o a gas undergoes various changes o temperature, pressure and volume such that it is taken round the p–V cycle shown in the diagram below.



   The ollowing sequence o processes takes place during the cycle.

   X → Y the gas expands at constant temperature and the gas absorbs energy rom a reservoir and does 450 J o work.

   Y → Z the gas is compressed and 800 J o thermal energy is trans erred rom the gas to a reservoir.

   Z → X the gas returns to its initial stage by absorbing energy rom a reservoir.

   a) Is there a change in internal energy o the gas during the processes X → Y? Explain. [2]
   b) Is the energy absorbed by the gas during the process X → Y less than, equal to or more than 450 J? Explain. [2]
   c) Use the graph to determine the work done on the gas during the process Y → Z. [3]
   d) What is the change in internal energy o the gas during the process Y → Z? [2]
   e) How much thermal energy is absorbed by the gas during the process Z → X? Explain your answer. [2]
) What quantity is represented by the area enclosed by the graph? Estimate its value. [2]
   g) The overall e ficiency o a heat engine is defined as

   $$\text{E ficiency} = \frac{\text{net work done by the gas during a cycle}}{\text{total energy absorbed during a cycle}}$$

   I this p–V cycle represents the cycle or a particular heat engine determine the e ficiency o the heat engine. [2]

4. In a **diesel** engine, air is initially at a pressure o 1×10^5 Pa and a temperature o 27 °C. The air undergoes the cycle o changes listed below. At the end o the cycle, the air is back at its starting conditions.
   1 An **adiabatic compression** to 1/20th o its original volume.
   2 A brie **isobaric expansion** to 1/10th o its original volume.
   3 An **adiabatic expansion** back to its original volume.
   4 A cooling down at constant volume.
   a) Sketch, with labels, the cycle o changes that the gas undergoes. Accurate values are not required. [3]
   b) I the pressure a ter the **adiabatic compression** has risen to 6.6×10^6 Pa, calculate the temperature o the gas. [2]
   c) In which o the our processes:
      (i) is work done **on** the gas? [1]
      (ii) is work done **by** the gas? [1]
      (iii) does ignition o the air- uel mixture take place? [1]
   d) Explain how the 2nd law o thermodynamics applies to this cycle o changes. [2]

HL

5. With the aid o diagrams, explain
   a) What is meant by laminar flow
   b) The Bernoulli e ect
   c) Pascal's principle
   d) An ideal fluid [8]

6. Oil, o viscosity 0.35 Pa s and density 0.95 g cm^{-3}, flows through a pipe o radius 20 cm at a velocity o 2.2 m s^{-1}. Deduce whether the flow is laminar or turbulent. [4]

7. A pendulum clock maintains a constant amplitude by means o an electric power supply. The ollowing in ormation is available or the pendulum:

   Maximum kinetic energy: 5×10^{-2} J

   Frequency o oscillation: 2 Hz

   Q actor: 30

   Calculate:
   a) The driving requency o the power supply [3]
   b) The power needed to drive the clock. [3]