HL Potential (gravitational and electric)

DESCRIBING FIELDS: g and E

The concept o field lines can be used to visually represent:

- the gravitational field, g, around a mass (or collection o masses)
- the electric field, E, around a charge (or collection o charges).

Magnetic fields can also be represented using field lines (see page 61). In all cases the field is the orce per unit test point object placed at a particular point in the field with:

- gravitational field = orce per unit test point mass (units: N kg^{-1})
- electric field = orce per unit test point positive charge (units: N C^{-1})

Forces are vectors and field lines represent both the magnitude and the direction o the orce that would be elt by a test object.

- The **magnitude** o the orce is represented by how close the field lines are to one another (or an example o a more precise definition, see definition o magnetic flux on page 112).
- The **direction** o the orce is represented by the direction o the field lines.

This means that, or both gravitational and electric fields, as a test object is moved:

- **along** a field line, work will be done (orce and distance moved are in the same direction)
- **at right angles** to a field line, no work will be done (orce and distance moved are perpendicular).

An alternative method o mapping the fields around an object is to consider the energy needed to move between points in the field. This defines the new concepts o electric potential and gravitational potential (see below).

POTENTIAL, *V* (GRAVITATIONAL OR ELECTRIC)

The *field* (gravitational or electric) is defined as the orce per unit test point object placed at a particular point in the field. In an analogous definition, the *potential* (gravitational, V_g, or electric, V_e) is defined as the energy per unit test point object that the object has as a result o the field. The ull mathematical relationships are shown on page 110.

Gravitational potential, $V_g = \dfrac{\text{energy}}{\text{mass}}$

Units o $V_g = \text{J kg}^{-1}$

Electric potential, $V_e = \dfrac{\text{energy}}{\text{charge}}$

Units o $V_e = \text{J C}^{-1}$ (or volts)

POTENTIAL DIFFERENCE Δ*V* (ELECTRIC AND GRAVITATIONAL)

Potential is the energy per unit test object. In general, moving a mass between two points, A and B, in a gravitational field (or moving a charge between two points, A and B, in an electric field) means that work is done. When work is done, the potential at A and the potential at B will be di erent. Between the points A and B, there will be a potential di erence, ΔV.

- I positive work is done *on* a test object as it moves between two points then the potential between the two points must increase.
- I work is done *b* the test object as it moves between the two points then the potential between the two points must decrease.

Gravitational potential di erence between two points,

$$\Delta V_g = \frac{\text{work done moving a test mass}}{\text{test mass}}$$

Units o $\Delta V_g = \text{J kg}^{-1}$

Electric potential di erence between two points,

$$\Delta V_e = \frac{\text{work done moving a test charge}}{\text{test charge}}$$

Units o $\Delta V_e = \text{J C}^{-1}$ or V (volts)

Thus to calculate the work done, W, in moving a charge q or a mass m between two points in a field we have:

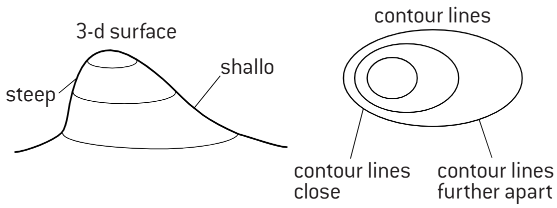
$W = q\Delta V_e$

$W = m\Delta V_g$

Equipotentials

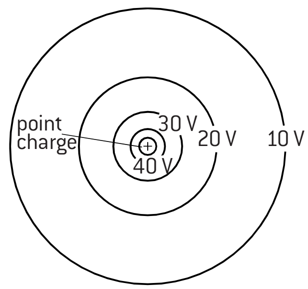
EQUIPOTENTIAL SURFACES

The best way of representing how the electric potential varies around a charged object is to identify the regions where the potential is the same. These are called **equipotential** surfaces. In two dimensions they would be represented as lines of equipotential. A good way of visualizing these lines is to start with the contour lines on a map.



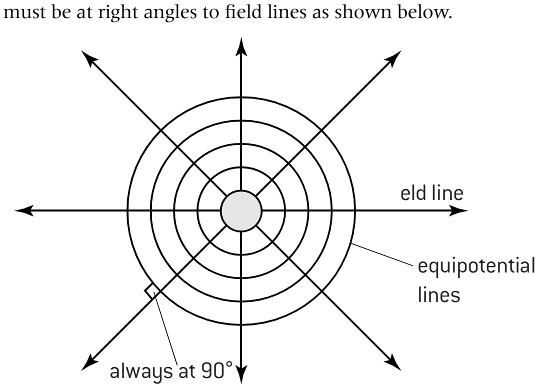
The contour diagram on the right represents the changing heights of the landscape on the left. Each line joins up points that are at the same height. Points that are high up represent a high value of gravitational potential and points that are low down represent a low gravitational potential. Contour lines are lines of equipotential in a gravitational field.

The same can be done with an electric field. Lines are drawn joining up points that have the same electric potential. The situation right shows the equipotentials for an isolated positive point charge.



RELATIONSHIP TO FIELD LINES

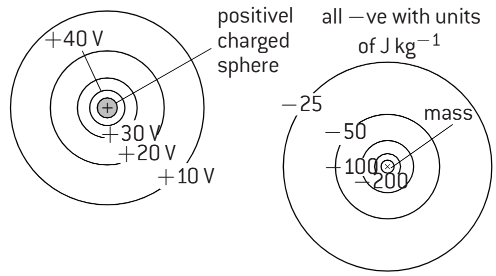
There is a simple relationship between electric field lines and lines of equipotential – they are always at right angles to one another. Imagine the contour lines. If we move along a contour line, we stay at the same height in the gravitational field. This does not require work because we are moving at right angles to the gravitational force. Whenever we move along an electric equipotential line, we are moving between points that have the same electric potential – in other words, no work is being done. Moving at right angles to the electric field is the only way to avoid doing work in an electric field. Thus equipotential lines must be at right angles to field lines as shown below.



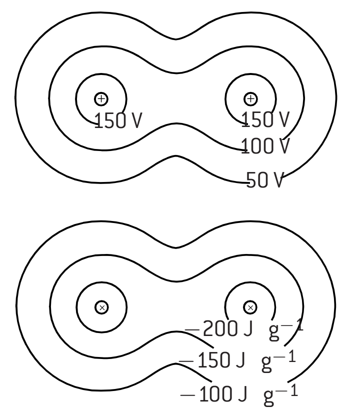
Field lines and equipotentials are at right angles

EXAMPLES OF EQUIPOTENTIALS

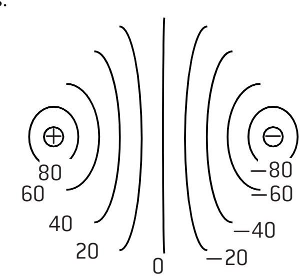
The diagrams below show equipotential lines for various situations.



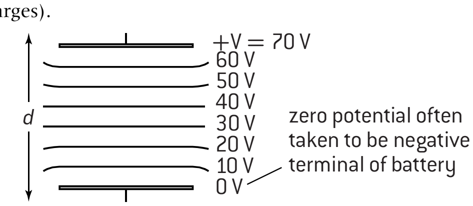
Equipotentials outside a charge-conducting sphere and a point mass.



Equipotentials for two point charges (same charge) and two point masses.



Equipotentials for two point charges (equal and opposite charges).



Equipotential lines between charged parallel plates.

It should be noted that although the correct definition of zero potential is at infinity, most of the time we are not really interested in the actual value of potential, we are only interested in the value of the difference in potential. This means that in some situations (such as the parallel conducting plates) it is easier to imagine the zero at a different point. This is just like setting sea level as the zero for gravitational contour lines rather than correctly using infinity for the zero.

Gravitational potential energ and potential

GRAVITATIONAL POTENTIAL ENERGY

It is easy to work out the di erence in gravitational energy when a mass moves between two di erent heights near the Earth's sur ace.

The di erence in energies $= mg(h_2 - h_1)$

There are two important points to note:

- this derivation has assumed that the gravitational field strength g is constant. However, Newton's theory o universal gravitation states that the field MUST CHANGE with distance. **This equation can only be used if the vertical distance we move is not very large.**
- the equation assumes that the gravitational potential energy gives zero PE at the sur ace o the Earth. This works or everyday situations but it is not undamental.

The true zero o gravitational potential energy is taken as infinity.

I the potential energy o the mass, m, was zero at infinity, and it lost potential energy moving in towards mass M, the potential energy must be **negative** at a given point, P.

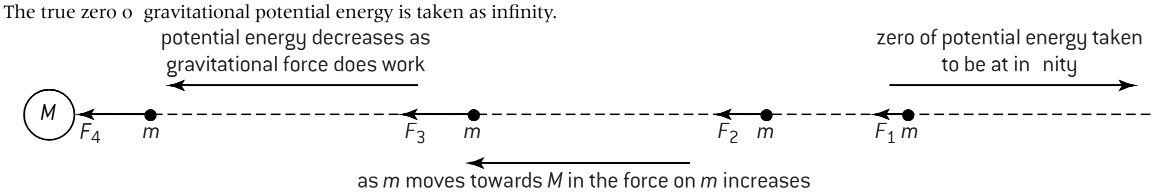
The value o gravitational potential energy o a mass at any point in space is defined as the work done in moving it rom infinity to that point. The mathematics needed to work this out is not trivial since the orce changes with distance.

It turns out that

$$\text{Gravitational potential energy o mass m} = -\frac{G\,M\,m}{r}$$

(due to M)

This is a scalar quantity (measured in joules) and is independent o the path taken rom infinity.



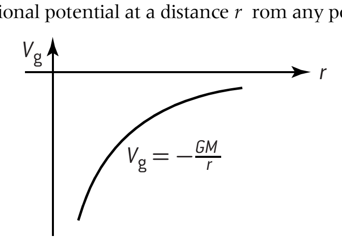
GRAVITATIONAL POTENTIAL

We can define the **gravitational potential** V_g that measures the energy per unit test mass.

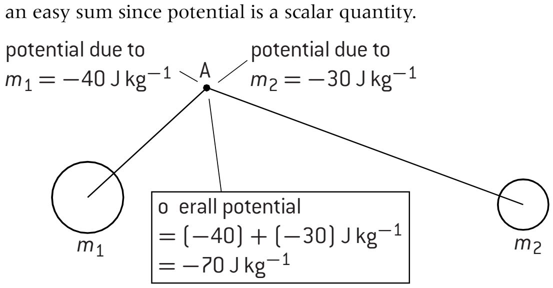
$$V_g = \frac{W}{m} \quad \frac{(\ \text{ork done})}{(\text{test mass})}$$

The SI units o gravitational potential are J kg^{-1}. It is a scalar quantity.

Using Newton's law o universal gravitation, we can work out the gravitational potential at a distance r rom any point mass.



This ormula and the graph also works or spherical masses (planets etc.). The gravitational potential as a result o lots o masses is just the addition o the individual potentials. This is an easy sum since potential is a scalar quantity.



Once you have the potential at one point and the potential at another, the di erence between them is the energy you need to move a unit mass between the two points. It is independent o the path taken.

ESCAPE SPEED

The escape speed o a rocket is the speed needed to be able to escape the gravitational attraction o the planet. This means getting to an infinite distance away.

We know that gravitational potential at the sur ace o a planet $= -\frac{GM}{R_p}$.

(where R_p is the radius o the planet)

This means that or a rocket o mass m, the di erence between its energy at the sur ace and at infinity $= \frac{GMm}{R_p}$

There ore the minimum kinetic energy needed $= \frac{GMm}{R_p}$

In other words,

$$\frac{1}{2}m\,(v_{esc})^2 = \frac{GMm}{R_p}$$

so

$$v_{esc} = \sqrt{\left(\frac{2GM}{R_p}\right)}$$

This derivation assumes the planet is isolated.

EXAMPLE

The escape speed rom an isolated planet like Earth (radius o Earth $R_E = 6.37 \times 10^6$ m) is calculated as ollows:

$$v_{esc} = \sqrt{\left(\frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.37 \times 10^6 \text{ m}}\right)}$$

$$= \sqrt{(1.25 \times 10^8)} \text{ m s}^{-1}$$

$$= 1.12 \times 10^4 \text{ m s}^{-1}$$

$$\approx 11 \text{ km s}^{-1}$$

The vast majority o rockets sent into space are destined to orbit the Earth so they leave with a speed that is less than the escape speed.

Orbi al mo ion

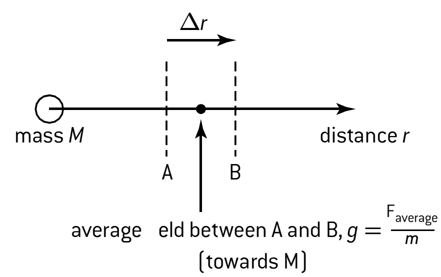
GRAVITATIONAL POTENTIAL GRADIENT

In the diagram below, a point test mass m moves in a gravitational field rom point A to point B.

The di erence in gravitational potential, ΔV_g

$= \frac{\text{average orce} \times \text{distance moved}}{m} = -g \times \Delta r$

The negative sign is because g is directed towards M, but the orce doing the work is directed away rom M and thus in the opposite direction rom g. Since the gravitational orce is attractive, work has to be done in going rom A to B, so the potential at A < potential at B.



$\Delta V_g = -g \times \Delta r$

$g = -\frac{\Delta V}{\Delta r}$

$\frac{\Delta V}{\Delta r}$ is called the potential gradient. It has units o J kg^{-1} m^{-1} (which are the same as N kg^{-1} or m s^{-2}).

The gravitational field strength is equal to minus the potential gradient. The equivalent relationship also applies or electric fields (see page 109).

ENERGY OF AN ORBITING SATELLITE

We already know that the gravitational energy $= -\frac{GMm}{r}$

The kinetic energy $= \frac{1}{2} m v^2$ but $v = \sqrt{\left(\frac{GM}{r}\right)}$ (Circular motion)

kinetic energy $= \frac{1}{2}\ m \frac{GM}{r} = \frac{1}{2}\ \frac{GMm}{r}$

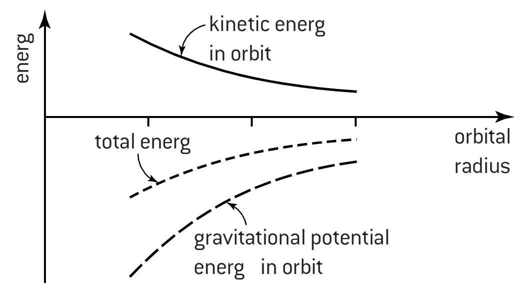
So total energy = KE + PE

$= \frac{1}{2}\ \frac{GMm}{r} - \frac{GMm}{r} = -\frac{1}{2}\ \frac{GMm}{r}$

Note that:

- In the orbit the magnitude o the KE $= \frac{1}{2}$ magnitude o the PE.
- The overall energy o the satellite is negative. (A satellite must have a total energy less than zero otherwise it would have enough energy to escape the Earth's gravitional field.)
- In order to move rom a small radius orbit to a large radius orbit, the total energy must increase. To be precise, an increase in orbital radius makes the total energy go rom a large negative number to a smaller negative number – this is an increase.

This can be summarized in graphical orm.

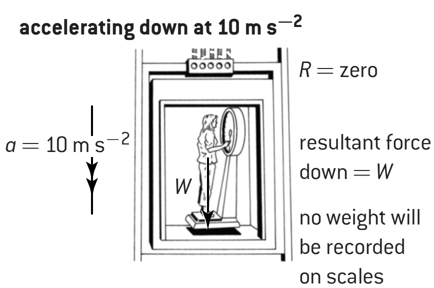


WEIGHTLESSNESS

One way o defining the weight o a person is to say that it is the value o the orce recorded on a supporting scale.

I the scales were set up in a li t, they would record di erent values depending on the **acceleration** o the li t.

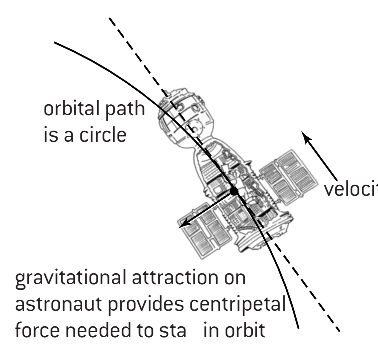
An extreme version o these situations occurs i the li t cable breaks and the li t (and passenger) accelerates down at 10 m s^{-2}.



The person would appear to be weightless or the duration o the all. Given the possible ambiguity o the term 'weight', it is better to call this situation the **apparent   eightlessness** o objects in ree- all together.

An astronaut in an orbiting space station would also appear weightless. The space station and the astronaut are in ree- all together.

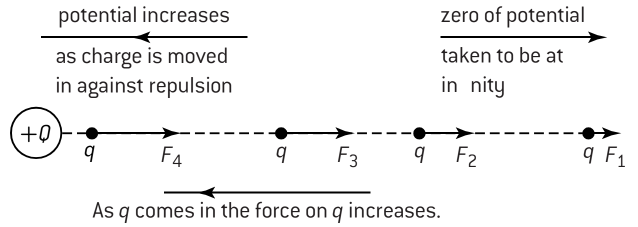
In the space station, the gravitational pull on the astronaut provides the centripetal orce needed to stay in the orbit. This resultant orce causes the centripetal acceleration. The same is true or the gravitational pull on the satellite and the satellite's acceleration. There is no contact orce between the satellite and the astronaut so, once again, we have apparent weightlessness.



HL Elec ric po en ial energy and po en ial

POTENTIAL AND POTENTIAL DIFFERENCE

The concept o electrical potential di erence between two points was introduced on page 105. As the name implies, potential di erence is just the di erence between the potential at one point and the potential at another. Potential is simply a measure o the total electrical energy per unit charge at a given point in space. The definition is very similar to that o gravitational potential.



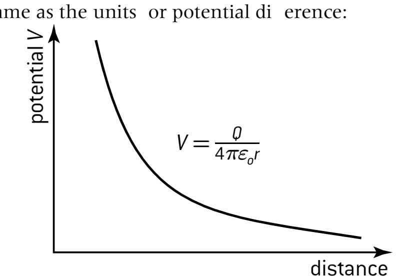
I the total work done in bringing a positive test charge q rom infinity to a point in an electric field is W, then the electric potential at that point, V, is defined to be

$$V = \frac{W}{q}$$

The units or potential are the same as the units or potential di erence: J C^{-1} or volts.

$$V = \frac{Q}{4\pi\varepsilon_o r}$$

This equation only applies to a single point charge.



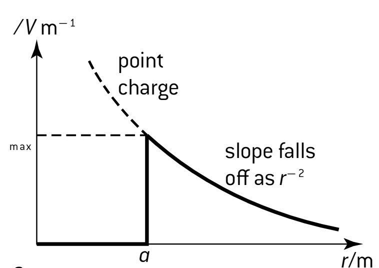
POTENTIAL INSIDE A CHARGED SPHERE

Charge will distribute itsel uni ormly on the outside o a conducting sphere.

- **Outside the sphere**, the field lines and equipotential sur aces are the same as i all the charge was concentrated at a point at the centre o the sphere.
- **Inside the sphere**, there is no net contribution rom the charges outside the sphere and the electric field is zero. The potential gradient is thus also zero meaning that every point inside the sphere is at the same potential – the potential at the sphere's sur ace.

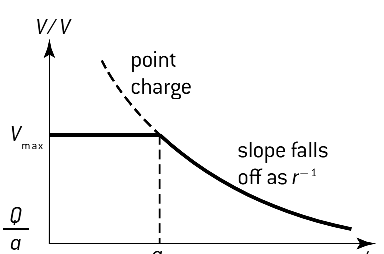
The graphs below show how field and potential vary or a sphere o radius a.

(a)



$$_{max} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Q}{a^2}$$

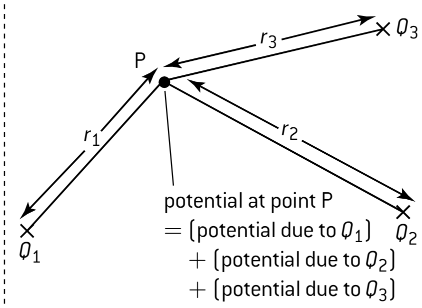
(b)



$$V_{max} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Q}{a}$$

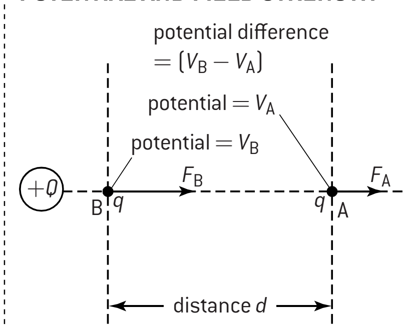
POTENTIAL DUE TO MORE THAN ONE CHARGE

I several charges all contribute to the total potential at a point, it can be calculated by adding up the individual potentials due to the individual charges.

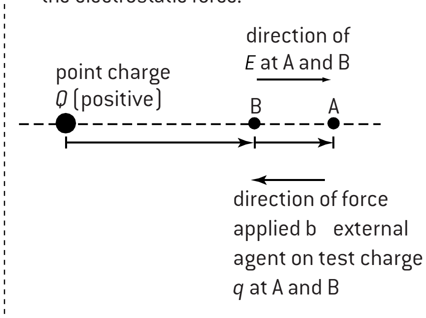


The electric potential at any point outside a charged conducting sphere is exactly the same as i all the charge had been concentrated at its centre.

POTENTIAL AND FIELD STRENGTH



Bringing a positive charge from A to B means work needs to be done against the electrostatic force.



The work done $\delta W = -E q \delta$ [the negative sign is because the direction o the orce needed to do the work is opposite to the direction o E]

There ore $E = -\frac{1}{q}\ \frac{\delta W}{\delta}$

$= -\frac{\delta V}{\delta}\ \left[\text{since } \delta V = \frac{\delta W}{q}\right]$

In words,

electric field = − potential gradient

Units = $\frac{\text{volt}}{\text{metre}}$ (V m^{-1}), N C^{-1}

HL Electric and gravitational fields compared

COMPARISON BETWEEN ELECTRIC & GRAVITATIONAL FIELD

Electrostatics	Gravitational
Force can be attractive or repulsive Coulomb's law – or point charges $F_E = \frac{q_1 q_2}{4\pi\varepsilon_o r^2} = k\frac{q_1 q_2}{r^2}$	Force always attractive Newton's law – or point masses $F = G\frac{m_1 m_2}{r^2}$
Electric field $E = \frac{F}{q_2} = \frac{q_1}{4\pi\varepsilon_0\ ^2} = k\frac{q_1}{^2}$ (electric eld: E; test charge: q_2; charge producing eld: q_1)	Gravitational field $g = \frac{F}{m_2} = \frac{Gm_1}{^2}$ (gravitational eld: g; test mass: m_2; mass producing eld: m_1)
Electric potential due to a point charge $V_e = \frac{q_1}{4\pi\varepsilon_o r} = k\frac{q_1}{r}$	Gravitational potential due to a point mass, m_1 $V_g = -\frac{Gm_1}{r}$
Electric potential gradient $E = -\frac{V_e}{r}$	Gravitational potential gradient $g = -\frac{V_g}{r}$
Electric potential energy $E_p = qV_e = \frac{q_1 q_2}{4\pi\varepsilon_o r} = k\frac{q_1 q_2}{r}$	Gravitational potential energy $E_p = mV_g = -\frac{GMm}{r}$

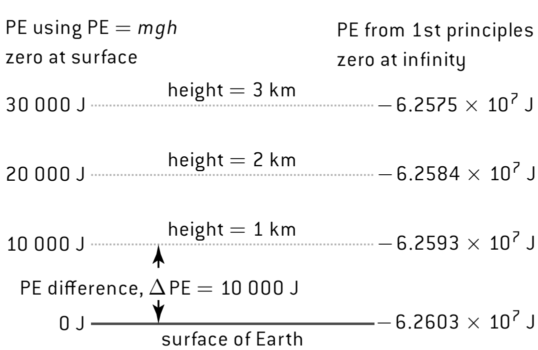
UNIFORM FIELDS

Field strength is equal to minus the potential gradient.
A constant field thus means:

- A constant potential gradient i.e. a given increase in distance will equate to a fixed change in potential.
- In 3D this means that equipotential sur aces will be flat planes that are equally spaced apart. In 2D equipotential lines will be equally spaced.
- Field lines (perpendicular to equipotential sur aces) will be equally spaced parallel lines.

1. Constant gravitational field

   The gravitational field near the sur ace o a planet is e ectively constant. At the sur ace o the Earth, the field lines will be perpendicular to the Earth's sur ace. Since $g = 9.81$ m s^{-2}, the potential gradient must also be 9.81 J kg^{-1} m^{-1}. Equipotential sur aces that are 1,000 m apart represent changes o potential approximately equal to 10 kJ kg^{-1}.

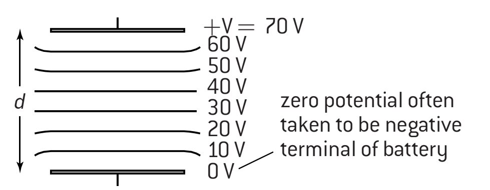


2. Constant electrical field

   The electric field in between charged parallel plates (e.g. a capacitor – see page 52) is e ectively constant in the middle section.

   In the diagram below, the potential di erence across the plates is V and the separation o the plates is d. Thus the electric potential gradient is $\frac{V}{d}$ and the constant field in the centre o the plates, $E = \frac{V}{d}$. The units V m^{-1} and N C^{-1} are equivalent and can both be used or E.

   Strictly, the electric field between two charged parallel plates cannot remain uni orm throughout the plates and there will be an edge e ect. It is straight orward to show that at the edge, the field must have dropped to hal the value in the centre, but modelling the field as constant everywhere between the parallel plates with the edge e ects occurring beyond the limits o the plates can be acceptable.

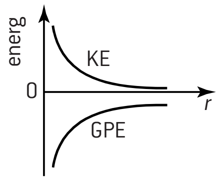


Equipotentials lines between charged parallel plates.

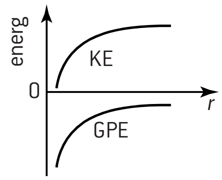
HL IB Questions – fie ds

1. Which **one** o the ollowing graphs best represents the variation o the kinetic energy, KE, and o the gravitational potential energy, GPE, o an orbiting satellite at a distance r rom the centre o the Earth?

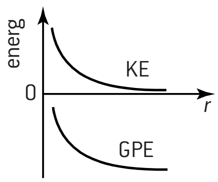
A.



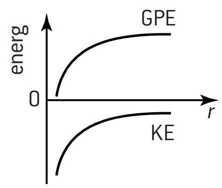
B.



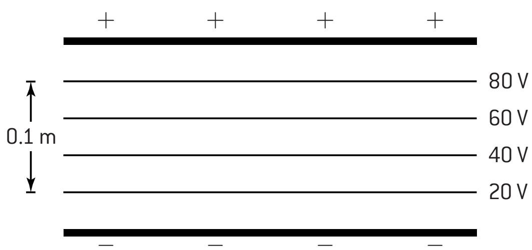
C.



D.



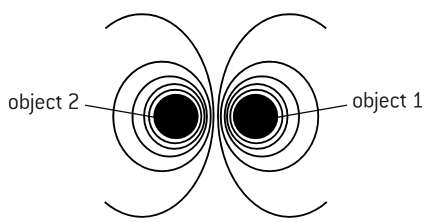
2. The diagram below illustrates some equipotential lines between two charged parallel metal plates.



The electric field strength between the plates is

A. 6 NC^{-1}
B. 8 NC^{-1}
C. 600 NC^{-1}
D. 800 NC^{-1}

3. The diagram shows equipotential lines due to two objects



The two objects could be

A. electric charges o the same sign only.
B. masses only.
C. electric charges o opposite sign only.
D. masses or electric charges o any sign. [1]

4. The Space Shuttle orbits about 300 km above the sur ace o the Earth. The shape o the orbit is circular, and the mass o the Space Shuttle is 6.8×10^4 kg. The mass o the Earth is 6.0×10^{24} kg, and radius o the Earth is 6.4×10^6 m.

a) (i) Calculate the change in the Space Shuttle's gravitational potential energy between its launch and its arrival in orbit. [3]

(ii) Calculate the speed o the Space Shuttle whilst in orbit. [2]

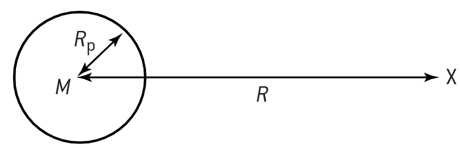
(iii) Calculate the energy needed to put the Space Shuttle into orbit. [2]

b) (i) What orces, i any, act on the astronauts inside the Space Shuttle whilst in orbit? [1]

(ii) Explain why astronauts aboard the Space Shuttle eel weightless. [2]

c) Imagine an astronaut 2 m outside the exterior walls o the Space Shuttle, and 10 m rom the centre o mass o the Space Shuttle. By making appropriate assumptions and approximations, calculate how long it would take or this astronaut to be pulled back to the Space Shuttle by the orce o gravity alone. [7]

5. a) The diagram below shows a planet o mass M and radius R_p.



The gravitational potential V due to the planet at point X distance R rom the centre o the planet is given by

$$V = -\frac{GM}{R}$$

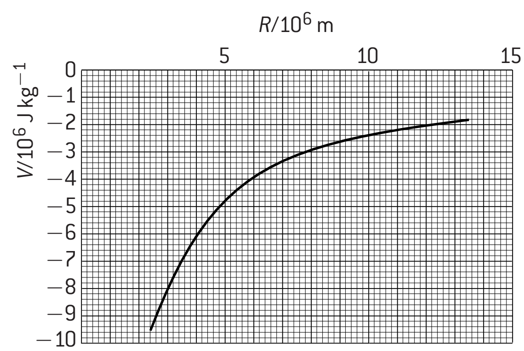
where G is the universal gravitational constant.

Show that the gravitational potential V can be expressed as

$$V = -\frac{g_0 R_p^{\,2}}{R}$$

where g_0 is the acceleration o   ree- all at the sur ace o the planet. [3]

b) The graph below shows how the gravitational potential V due to the planet varies with distance R rom the centre o the planet or values o R greater than R_p, where $R_p = 2.5 \times 10^6$ m.



Use the data rom the graph to

(i) determine a value o g_0. [2]

(ii) show that the minimum energy required to raise a satellite o mass 3000 kg to a height 3.0×10^6 m above the **surface** o the planet is about 1.7×10^{10} J. [3]