2.2 Forces

Understanding

- Objects as point particles
- Free-body diagrams
- Translational equilibrium
- Newton's laws of motion
- Solid friction

Nature of science

The application of mathematics enhances our understanding of force and motion. Isaac Newton was able to use the work of earlier scientists and to formalize it through the use of the calculus, a form of which he developed for this purpose. He made many insights during his scientific thinking within the topic of force and motion, but also beyond it. The story of the falling apple, whether true or not, illustrates the importance of serendipity in science and the requirement that the creative scientist can form links that go beyond what exists already.

Applications and skills

- Representing forces as vectors
- Sketching and interpreting free-body diagrams
- Describing the consequences of Newton's first law for translational equilibrium
- Using Newton's second law quantitatively and qualitatively
- Identifying force pairs in the context of Newton's third law
- Solving problems involving forces and determining resultant force
- Describing solid friction (static and dynamic) by coefficients of friction

Equations

- Newton's second law: $F = ma$
- static friction equation: $F_f \leq \mu_s R$
- dynamic friction equation: $F_f = \mu_d R$

Introduction

We depend on forces and their effects for all aspects of our life. Forces are often taught in most elementary physics courses as though they are "pushes or pulls", but forces go well beyond this simple description. Forces can change the motion of a body and they can deform the shapes of bodies. Forces can act at a distance so that there is no contact between objects or between a system that produces a force and the object on which it acts.

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Aristotle and the concept of force

Discussions about the concept of what is meant by a force go back to the dawn of scientific thought. Aristotle, a Greek philosopher who lived about 2300 years ago, had an overarching view of the world (called an Aristotelian cosmology) and he can be regarded as being an important factor in the development of science. The German philosopher Heidegger wrote that there would have been no Galileo without Aristotle before him.

Despite his importance to us, however, we would not have regarded Aristotle as a scientist in any modern sense. For one thing he is not known to have performed experiments to verify his ideas; some of his ideas seem very odd to us today. Aristotle believed in the "nature" of all objects including living things. He believed that all objects had a natural state which was to be motionless on the surface of the Earth and that all objects, if left alone, would try to attain this state. Then he distinguished between "natural motion" in which, for example, heavy objects fall downwards and "unnatural" or "forced" motion in which the objects need to have a force continually applied if they are to remain anywhere other than their natural state.

Unfortunately for those learning physics this is a very persuasive idea because we know intuitively that if we want to hold something in our hand, our muscles have to keep "working" in order to do this.

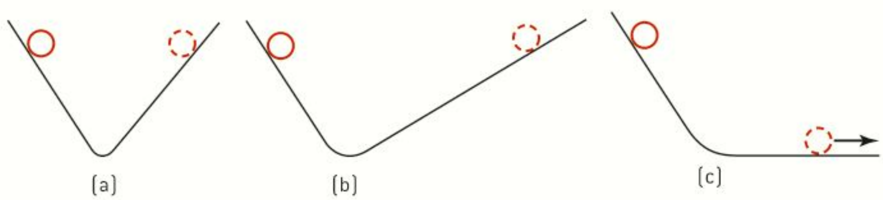
There are many other examples of Aristotelian thought and how later scientists moved our thinking forward. But it is important to remember the contribution that Aristotle made to science, even if some of his ideas are now overturned. What do you think students in 50, 100, or even a 1000 years will make of the physics of our century?

Newton's laws of motion

Newton's first law

By the time of Galileo, scientists had begun to realise that things were not as simple as the Greek philosophers such as Aristotle had thought. They were coming to the view that moving objects have **inertia**, meaning a resistance to stopping and that, once in motion, objects continue to move.

Galileo carried out an experiment with inclined planes and spheres. In fact, this may have been a thought experiment – this was often the way forward in those days – but in any event it is easy to see what Galileo was trying to suggest.



▲ Figure 1 Galileo's thought experiment.

In the first experiment (a), the two arms of the inclined plane are at the same angle and the sphere rolls the same distance up the slope as it rolled down (assuming no energy losses). In the second experiment (b), the second arm is at a lower angle than before but the sphere rolls up this plane to the same height as that from which it was released. Galileo then concluded that if the second plane is horizontal (c), the sphere will go on rolling for ever because it will never be able to climb to the original release height.

Newton included this idea in his **first law of motion** that says:

An object continues to remain stationary or to move at a constant velocity unless an external force acts on it.

Galileo had suggested that the sphere on his horizontal plane went on forever, but it was Newton who realised that there is more to say than this. Unless something from outside applies a force to change it, the velocity of an object (both its speed and its direction) must remain the same.

This was directly opposed to Aristotle's view that a force had to keep pushing constantly at a moving object for the speed to remain the same.

Newton's second law

The next step is to ask: if a force does act on an object, in what way does the velocity change?

Newton proposed that there was a fundamental equation that connected force and rate of change of velocity. This is contained in his second law of motion. This law can be written in two ways, one way is more complex than the other and we will look at the more complicated form of the law later in this topic.

Newton's second law, in its simpler form, says that

Force = mass × acceleration

As an equation this can be written

$$F = ma$$

The appropriate SI units are force in newtons (N), mass in kilograms (kg), and acceleration in metres per second^2 (m s^{-2}). As discussed in Topic 1 the newton is a derived unit in SI. We can represent it in terms of fundamental units alone as kg m s^{-2}.

Two things arise from this equation:

- Mass is a scalar, so there will not be a change in the direction of the acceleration if we multiply acceleration by the mass. The direction of the force and the direction of the acceleration must be the same. So, applying a force to a mass will change the velocity *in the same direction* as that of the force.
- One way to think about the mass in this equation is that it is the ratio of the force required per unit of acceleration for a given object. This helps us to standardize our units of force. If an object of mass 1 kg is observed to accelerate with an acceleration of 1 m s^{-2} then one unit of force (1 N) must have acted on it.

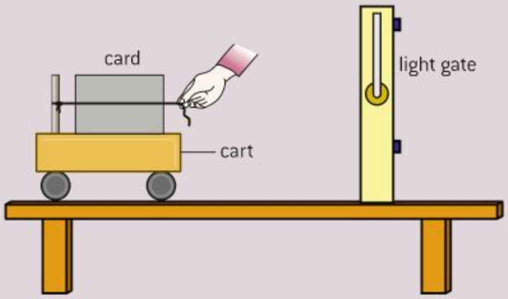
Nature of science

Inertial and gravitational mass

What exactly do we mean by *mass*? The mass that we use in Newton's second law of motion is inertial mass. Inertial mass is the property that permits an object to resist the effects of a force that is trying to change its motion. In other parts of this book, mass is used in a different context. We talk about the weight of an object and we know that this weight arises from the gravitational attraction between the mass of the object and the mass of the Earth. The two masses in this case are gravitational masses and are the response of matter to the effects of gravity. It has been experimentally verified that weight is proportional to mass to better than 3 parts in 10^{11}. It is a postulate of Einstein's general theory of relativity that inertial mass and gravitational mass are proportional.

Investigate!

Force, mass, and acceleration



Experiment 1: force and acceleration

- The idea in this experiment is to measure the acceleration of the cart (of constant mass) when it is towed by different numbers of elastic threads each one extended by the same amount.
- The timing can be done using light gates and an electronic timer as shown here. Alternatively it can be done using a data logger with, say, an ultrasound sensor and the cart moving away from the sensor.
- Practise accelerating the cart with one elastic thread attached to the rear of the cart. The thread(s) will have to be extended by the same amount for each run. A convenient point to judge is the forward end of the cart. Make sure that your hand clears the light gates if you are using this method. Your hand needs to move at the same speed as the cart so that the thread is the same length throughout the run.
- The card on the cart needs to be of a known length so that you can use the time taken to break the light beam to calculate, first, the average speed at each gate and then (using the distance apart of the gates), the acceleration of the cart. The kinematic equations are used for this. (This is why you must pull the cart with a constant force so that the acceleration is uniform.)
- Repeat the experiment with two, three, and possibly four elastic threads, all identical, all extended by the same amount. This means that you will be using one unit of force (with one thread), two units (with two threads) and so on.
- Plot a graph of calculated acceleration against number of force units. Is your graph straight? Does it go through the origin? Remember that this experiment has a number of possible uncertainties in judging the best-fit line.

Experiment 2: mass and acceleration

- The setup is essentially the same as for Experiment 1, except this time you will use a constant force (possibly two elastic threads is appropriate).
- Change the mass of your cart (some laboratory carts are specially designed to stack, one on top of another) and measure the acceleration with a constant force and varying numbers of carts.
- This time Newton's second law predicts that mass should be inversely proportional to acceleration. Plot a graph of acceleration against $\frac{1}{\text{mass}}$. Is it a straight line?

Worked examples

1 A car with a mass of 1500 kg accelerates uniformly from rest to a speed of 28 m s^{-1} (about 100 km h^{-1}) in a time of 11 s. Calculate the average force that acts on the car to produce this acceleration.

Solution

The acceleration $= a = \frac{v-u}{t} = \frac{28}{11} = 2.54$ m s^{-2}.

Force $= ma = 1500 \times 2.54 = 3.8$ kN.

2 An aircraft of mass 3.3×10^5 kg takes off from rest in a distance of 1.7 km. The maximum thrust of the engines is 830 kN. Calculate the take-off speed.

The acceleration of the aircraft is $\frac{8.3 \times 10^5}{3.3 \times 10^5} =$ 2.51 m s^{-2}.

$v^2 = u^2 + 2as$ so $v = 0 + 2 \times 2.51 \times 1700$ which leads to $v = 92.4$ m s^{-1}.

Newton's third law

Newton's third law of motion is sometimes written in such a way that the true meaning of the law has to be teased out. The law can be expressed in a number of equivalent ways.

One common way to write Newton's third law of motion is:

Every action has an equal and opposite reaction.

The first point to make is that the words "action" and "reaction" really mean "action force" and "reaction force". So, once again, Newton is referring directly to the effects of forces. A second point is that the action–reaction pair must be of the same type. So a gravitational action force must correspond to a gravitational reaction. It could not, for example, refer to an electrostatic force.

The law suggests that forces must appear in pairs, but in thinking about a particular situation it is important to identify all possible force pairs and then to pair them up correctly. Take, as an example, the situation of a rubber ball resting on a table.

At first glance the obvious action force here is the weight of the ball, that is, the gravitational pull the Earth exerts on the sphere. This force acts downwards and – if the table were not there – the ball would accelerate downwards according to Newton's second law. It would fall to the floor.

What is the reaction force here? Given that action force and reaction force must pair up like for like, the reaction must be the gravitational force that the ball exerts on the Earth. This is of exactly the same size as the pull of the Earth on the ball, but is in the opposite direction.

What prevents the ball accelerating downwards to the floor? A force must be exerted by the table on the ball – and if there are no accelerations happening, then this force is equal and opposite to the downwards gravitational pull of the Earth on the ball. But the upwards table force is *not* the reaction force to the ball's weight – we have already seen that this is the gravitation pull on the Earth. The origin of the table force is the electrostatic forces between atoms. As the ball lies on the table, it deforms the horizontal surface very slightly, rather like what happens when you push downwards with a finger on a metre ruler suspended by supports at its end – the ruler bends in a spring-like way to provide a resistance to the force acting downwards. The dent in the table surface is the response of the atoms in the table to the weight lying on it. Remove the ball and the surface will return to being flat. This upwards force that returns the table to the horizontal is pushing upwards on the ball. There is a corresponding downwards force from the deformed ball (the ball will become slightly flattened as a response to the gravitational pull). So here is the second action–reaction pair, between two forces that are electrostatic in origin.

In summary, there are four forces in this situation: the weight of the ball, the upwards "spring-like" force of the table, and the reactions to these forces which are the pull of the ball on the Earth and the downwards push of the deformed ball on the table.

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But are they really laws?

The essential question is: can Newton's laws of motion be proved? The answer is that they cannot; strictly speaking they are assertions as Newton himself recognized. In his famous *Principia* (written in Latin as was a custom in those days) he writes *Axiomata sive leges motus*, or "the axioms or laws of motion".

However, they appear to be an excellent set of rules that allow us to predict most of the motion that we undertake. They remained unchallenged as a theory for about 200 years until the two theories of relativity were formulated by Einstein at the turn of the twentieth century. Essentially (said Einstein) the rules that Newton proposed do not always apply to, for example, motion that is very fast. However, for the modest speeds at which humans travel, the rules are reliable to a high degree and are certainly good enough for our needs most of the time.

To (literally) get a feeling for this, take a one-metre laboratory ruler and suspend it between two lab stools as suggested earlier. Press down gently with your finger in the centre of the ruler so that it becomes curved. The ruler will bend, you will be able to feel it resisting your efforts to deform it too far. Remove your finger and the ruler will return to its original shape.

In explaining Newton's third law in a particular example, you must remember to emphasise the nature, the size, and the direction of the force you are describing. Another common example is that of a rocket in space. Students sometimes write that "...by Newton's third law, the rocket pushes on the atmosphere to accelerate" but this shows a poor understanding of how the propulsion actually works.

First, of course, the rocket does not "push" on anything. The fact that a rocket can accelerate in space where there is no atmosphere proves this.

What happens inside the rocket is that chemicals react together producing a gas with a very high temperature and pressure. The rocket has exhaust nozzles through which this gas escapes from the combustion chamber. At one end of the chamber inside the rocket the gas molecules rebound off the end wall and exert a force on it, as a result they reverse their direction. In principle, the rebounding molecules could then travel down the rocket and leave through the nozzles. So there is an action-reaction pair here, the force forwards that the gas molecules exert on the chamber (and therefore the rocket) and the force that the chamber exerts on the gas molecules. It is the first of these two forces that accelerates the rocket. If the chamber were completely sealed and the gas could exert an equal and opposite force at the back of the rocket, then the forward force would be exactly countered by the backwards force and no acceleration would occur.

▲ Figure 2 Chemical rockets in action.

Later we shall interpret this acceleration in a different way. But the explanation given here will still be correct at a microscopic level.

Think about the following situations and discuss them with fellow students.

- A fireman has to exert considerable force on a fire hose to keep it pointing in the direction that will send water to the correct place.
- There is a suggestion to power space travel to deep space by ejecting ions from a spaceship.
- A sailing dinghy moves forward when the wind blows into the sails.

Free-body force diagrams

As a vector quantity, a force can be represented by an arrow that gives both the scaled length and the direction of the force. In simple cases where few forces act this works well, but as the situations become more complex, diagrams that show all the arrows can become complicated.

One way to avoid this problem is to use a **free-body force diagram** to illustrate what is happening.

The rules for a free-body diagram for a body are simple, as follows.

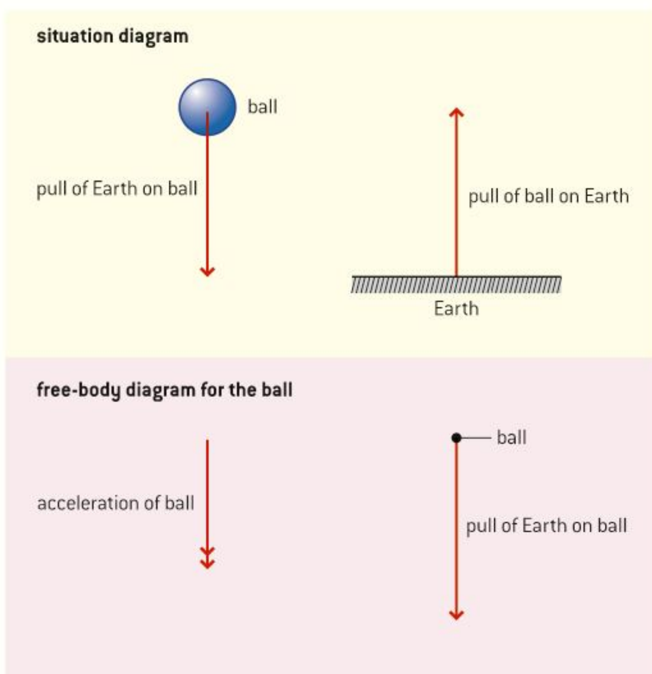
- The diagram is for one body only and the force vectors are represented as arrows.
- Only the forces acting on the body are considered.
- The force (vector) arrows are drawn to scale originating at a point that represents the centre of mass of the body.
- All forces have a clear label.

Procedure for drawing and using a free-body diagram

- Begin by sketching the general situation with all the bodies that interact in the situation.
- Select the body of interest and draw it again removed from the situation.
- Draw, to scale, and label all the forces that act on this body due to the other bodies and forces.
- Add the force vectors together (either by drawing or calculation) to give the net force acting on the body. The sum can be used later to draw other conclusions about the motion of the object.

Examples of free-body diagrams

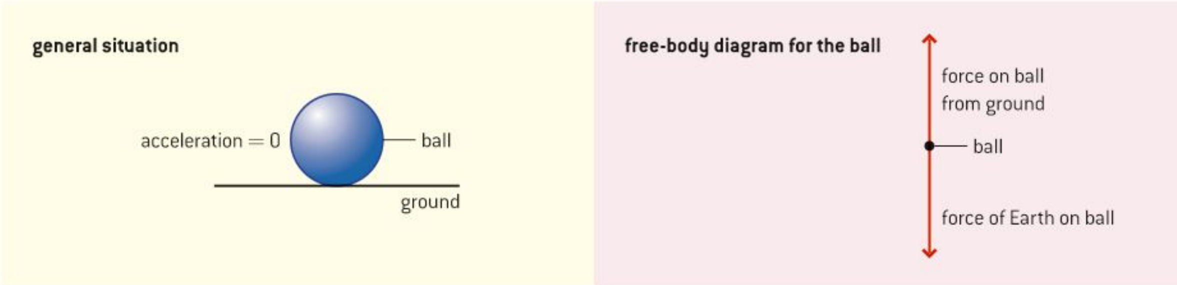
1 *A ball falling freely under gravity with no air resistance.*



▲ Figure 3 Ball falling freely under gravity.

This straightforward situation speaks for itself. There are two forces acting: the Earth pulling on the ball and the ball pulling on the Earth. For the free-body diagram of the ball we are only interested in the first of these. So the free-body diagram is a particularly simple one, showing the object and one force. Notice that the ball is not represented as a real object, but as a point that refers to the centre of mass of the object.

2 *The same ball resting on the ground.*



▲ Figure 4 Ball resting on the ground.

As we saw earlier, four forces act in this case:

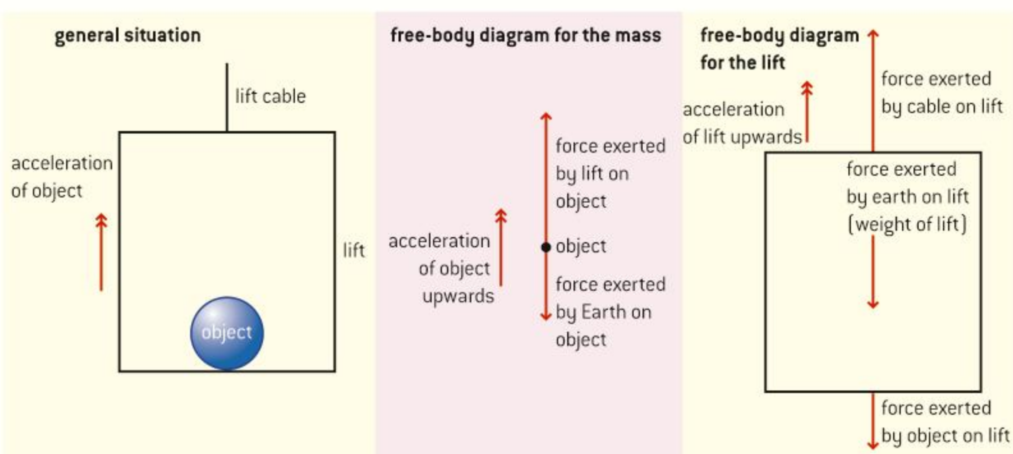
- the weight of the ball downwards
- the reaction of the Earth to this weight
- the upwards force from the ground
- the reaction of the ball to this "spring-like" force.

A free-body diagram (figure 4) definitely helps here because, by restricting ourselves to the forces acting on the ball, the four forces reduce to two: the weight of the ball and the upwards force on it due to the deformation of the ground. These two forces are equal and opposite. The net resultant (vector sum) of the forces is zero and there is therefore no acceleration.

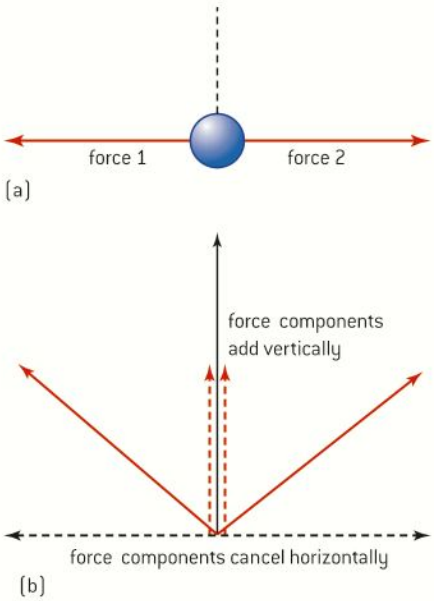
3 *An object accelerating upwards in a lift.*

The weight of the object is downwards and the size of this force is the same as though the object had been stationary on the Earth's surface. However, the upwards force of the floor of the lift on the object is now larger than the weight and the resultant force of the two has a net upwards component. The object is accelerated upwards as you would expect.

For the lift, there is an upward force in the lift cable and a downwards weight of the lift is downwards together with the weight of the object. The resultant force is upwards and is equal to the force in the cable less the weight forces of lift and object.



▲ Figure 5 Mass in a lift accelerating upwards.



▲ Figure 6 Two equal forces in different directions.

Translational equilibrium

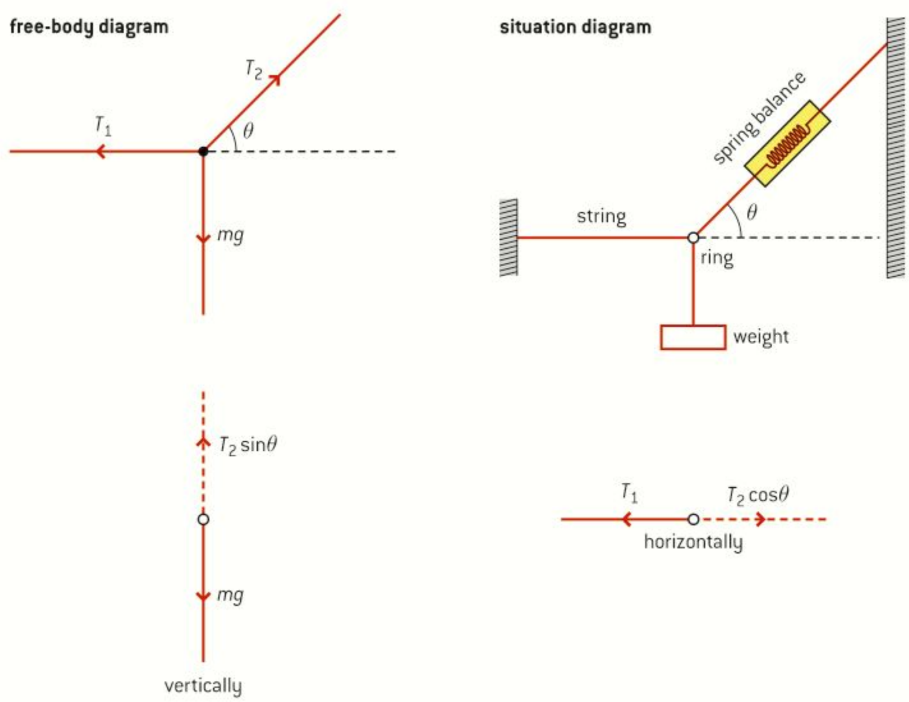
When an object is in **translational equilibrium** it is either at rest or moving at a constant velocity (not just constant *speed* in this case). Translational here means moving in a straight line. (There is also rotational equilibrium where something is at rest or rotating at a constant angular speed; this is discussed in option B).

Newton's first and second laws remind us that if there is no change of velocity then there must be zero force acting on the object. This zero force in many cases is the **resultant** (addition) of more than one force. In this section we will examine what equilibrium implies when there is more than one force.

The simplest case is that of two forces. If they are equal in size and opposite in direction, they will cancel out and be in equilibrium (Figure 6(a)).

If the forces are equal in size but not in the same direction then equilibrium is not possible. Horizontally, in figure 6(b), the two components of the force vectors are still equal and opposite and we could expect that there would be no change in this direction. But vertically the two vector components point in the same direction so that overall there will be an unbalanced vertical force and an acceleration acting on the object on which these forces act.

This gives a clue as to how we should proceed when there are three or more forces.



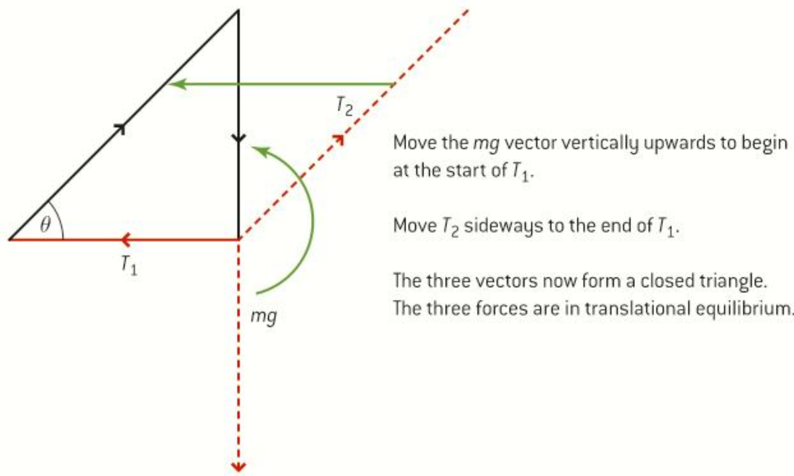
▲ Figure 7 Three forces acting on a ring.

Figure 7 shows a situation diagram and a free-body diagram for a ring on which three forces act.

For equilibrium, in whatever direction we resolve the forces, all three components must add up to zero in this direction.

Figure 7 shows that horizontal and vertical are two good directions with which to begin, because two forces are aligned with these directions and one disappears in each direction chosen. Thus, vertical force mg has no component in the horizontal direction and horizontal force T_1 has no part to play vertically. But whichever direction is chosen, if there is to be no resultant force and consequently no acceleration, then all the forces must cancel.

There is one more consequence of this idea. Figure 8 shows the forces drawn, as usual, to scale and in the correct direction (in red). The forces can be moved, as shown by the green arrows, into a new arrangement (shown in black). What is special here is that the three forces form a closed triangle where all the arrows meet.



▲ Figure 8 Making a triangle for forces.

Algebraically, this must be true because we know (ignoring the directions of the forces) that

$$T_1 = T_2 \cos\theta \text{ (horizontally) and } mg = T_2 \sin\theta \text{ (vertically)}$$

So

$$T_1^2 = T_2^2 \cos^2\theta \text{ and } (mg)^2 = T_2^2 \sin^2\theta$$

Adding these together gives

$$T_1^2 + (mg)^2 = T_2^2 (\sin^2\theta + \cos^2\theta)$$

and therefore as $\sin^2\theta + \cos^2\theta = 1$

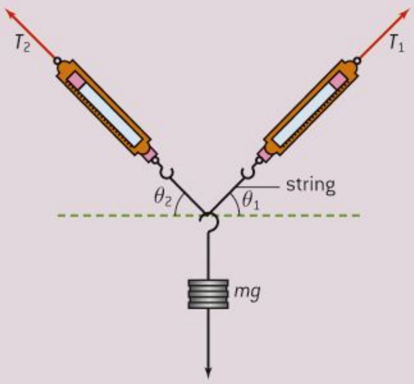
$$T_1^2 + (mg)^2 = T_2^2$$

This is equivalent to Pythagoras' theorem where $x = T_1^2$, $y = (mg)^2$ and $z = T_2^2$ so that $x^2 + y^2 = z^2$.

So, because the sum of the squares of two vector lengths equals the square of the third vector length, the three must fit together as a right-angled triangle. We have a right-angled triangle formed by the scaled lengths and directions of the three force vectors. This important figure is known as **a triangle of forces**. If you can draw the vectors for a system in this way, then the system must be in translational equilibrium.

Investigate!

Three forces in equilibrium



- If a point is in equilibrium under the influence of three forces, then the three forces must form a closed triangle.
- To verify this, use a mass and two spring balances with a knot halfway along the string between the two spring balances to act as the point object.
- Your arrangement for hanging the spring balances will depend on the resources in your school.
- Select a mass to suit the sensitivity of your spring balances.
- Place a vertical drawing board with a large sheet of paper pinned to it behind the balances so that you can mark the position of the strings. When these are marked, use a protractor to measure the angles.
- Begin with a simple case, say when T_1 is horizontal so that $\theta_1 = 90°$.
- The knot must be stationary and when this is true:
  - $T_1 \cos\theta_1 = T_2 \cos\theta_2$ (resolving horizontally)
  - $T_1 \sin\theta_1 + T_2 \sin\theta_2 = mg$ (resolving vertically)
- Construct this table to enable you to verify that both equations are true for every case you set up.

	T_1/N	θ_1/°	T_2/N	θ_2/°	mg/N	$T_1\cos\theta_1$/N	$T_1\sin\theta_1$/N	$T_2\cos\theta_2$/N	$T_2\sin\theta_2$/N
Case 1									

For each case check that the equations are correct within experimental error.

Solid friction

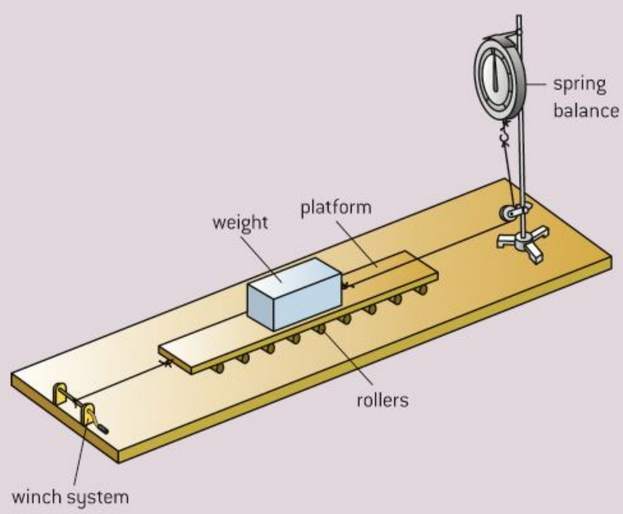
Friction is the force that occurs between two surfaces in contact. If you live in a part of the world where there is snow and ice then you will know that when the friction between your shoes and the ice disappears it can be a good thing (for skiing) or a bad thing (for falling over).

Investigate!

Solid friction

In this experiment you use a spring balance to measure the force that acts between two surfaces. It gives some unexpected results.

- Set up the platform on rollers with a suitable weight and a spring balance. A string connects the spring balance to the weight. Ensure that the weight is attached securely to the block.
- Pull the platform using the winch system with an increasing force observing, at the same time, the reading on the balance.



- Note the maximum value of the force as measured on the balance.
- Note the way the size of the force changes when the platform begins to move.
- Is this force constant? Or does it fluctuate?
- Change the size of the weights on the platform. How does the total weight affect the force measured on the balance?
- Change the type of surface at the bottom of the weights. You might use abrasive (emery) paper or cloth in different abrasive grades pinned to the platform.
- Investigate other changes: does lubrication at the bottom of the weights affect the friction?

If you carried out the *Investigate!* Solid friction experiment you may have found that:

- The force on the balance increases as you pull harder and harder, but the platform does not begin to move relative to the weights immediately.
- Eventually the platform suddenly begins to move at a particular value of force and at this instant the force, shown by the balance, drops to a new lower value.
- This new value is then maintained as the platform moves steadily.
- You may observe "stick-slip" behaviour where the platform alternately sticks and then jumps to a new sticking position. This behaviour is associated with two values of friction, but this may be too difficult to observe unless you get very suitable surfaces.
- The friction forces depend on the magnitude of the weights.

The frictional forces that occur in this experiment are described empirically as:

- **static friction** (when there is no relative movement between the surfaces)
- **dynamic friction** (when there is relative movement).

As the pulling force increases but without any slip happening, the friction is said to be **static** (because there is no motion). Eventually however, the pulling force will exceed the value of the static friction and the surfaces will start to move. As the movement continues, the friction force drops to a new value, lower than the maximum static value. This new lower value is known as the **dynamic friction**. Both static and dynamic friction forces are highly dependent on the two surfaces concerned.

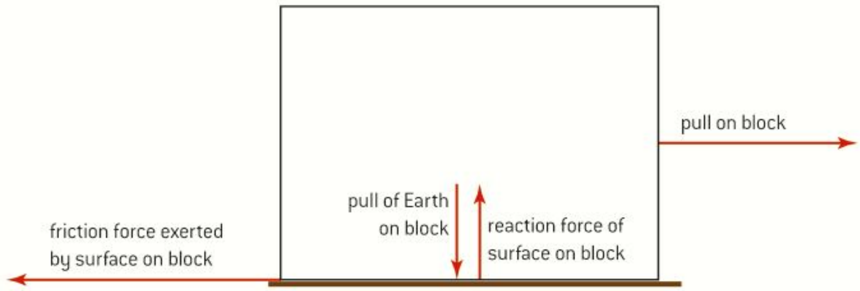
Static friction

The static friction force F_f is found to be given empirically by

$$F_f \leq \mu_s R$$

where F_f is the frictional force exerted by the surface on the block. R is the normal reaction of the surface on the block, this equals the weight of the block as there is no vertical acceleration. The symbol μ_s refers to the **coefficient of static friction**.

The "less than or equal to" symbol indicates that the static friction force can vary from zero up to a maximum value. Between these limits F_f is equal to the pull on the block. Once the pull on the block is equal to F_f, then the block is just about to move. Once the pull exceeds F_f then the block begins to slide (figure 9). For these larger forces, the friction operating is in the dynamic regime.



▲ Figure 9 Friction force acting on an object.

Dynamic friction

Dynamic friction only applies when the surfaces move relative to each other. The friction drops from its maximum static value (there is an explanation for this below) and remains at a constant value. This value depends on the total reaction force acting on the surface but (according to simple theory) is not thought to depend on the relative speed between the two surfaces. So, for dynamic friction

$$F_f = \mu_d R$$

where μ_d is the **coefficient of dynamic friction**.

The values of μ_s and μ_d vary greatly depending on the pair of surfaces being used and also the condition of the surfaces (for example, whether lubricated or not). A few typical values are given in the table. If you want to investigate a wider range of surfaces, there are many sources of the coefficient values on the Internet – search for "Coefficients of friction".

Each friction coefficient is a ratio of two forces (F_f and R) and so has no units.

Surface 1	Surface 2	μ_s	μ_d
glass	metal	0.7	0.6
rubber	concrete	1.0	0.8
rubber	wet tarmac	0.6	0.4
rubber	ice	0.3	0.2
metal	metal (lubricated)	0.15	0.06

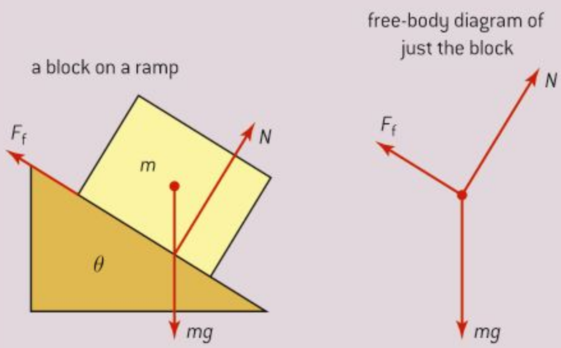
Values of μ_s and μ_d for pairs of surfaces

It is possible for the coefficients to be greater than 1 for some surface pairs. This reflects the fact that for these surfaces the friction is very strong and greater than the weight of the block. Remember that the surfaces are being pulled sideways by a horizontal force whereas the reaction force is vertical so we are not really comparing like with like in these empirical rules for friction.

Investigate!

Friction between a block and a ramp

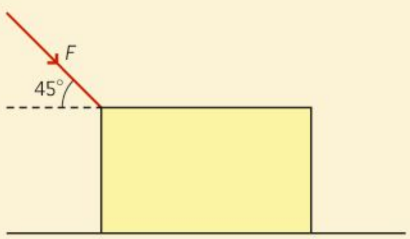
One way to measure the static coefficient of friction between two surfaces in a school laboratory is to use the two surfaces as part of a ramp system.



- One surface is the top of the ramp, the other is the base of the block.
- Resolving at 90° to plane, $N = mg\cos\theta$; resolving along the plane, $F = mg\sin\theta$
- So $\tan\theta = \frac{F_f}{N} = \mu_s$
- Start with the ramp horizontal, and then gradually raise one end until the block starts to slip.
- At this moment of slip, measure the angle of the ramp. The tangent of this angle is equal to the coefficient of static friction.

Worked examples

1 A box is pushed across a level floor at a constant speed with a force of 280 N at 45° to the floor. The mass of the box is 50 kg.



Calculate:

a) the vertical component of the force

b) the weight of the box

c) the horizontal component of the force

d) the coefficient of dynamic friction between the box and the floor.

Solution

a) the vertical component is 280 sin 45° = 198 N

b) the weight of the box = $mg = 50 \times 9.8 = 490$ N

c) the horizontal component of the force = 280 cos 45° = 198 N

d) the vertical component of the force exerted by the floor on the box = 490 + 198 N = 688 N

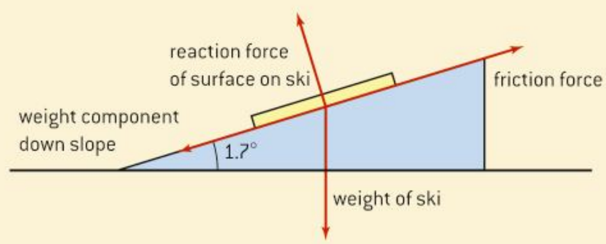
the friction force = the horizontal component (the box is travelling at a steady speed), so

$$\mu_d = \frac{198}{688} = 0.29$$

2 A skier places a pair of skis on a snow slope that is at an angle of 1.7° to the horizontal. The coefficient of static friction between the skis and the snow is 0.025.

Determine whether the skis will slide away by themselves.

Solution



Call the weight of the skis W.

The component of weight down the slope = $W\sin 1.7°$

The reaction force of the surface on the ski = $W\cos 1.7°$.

Therefore, the maximum friction force up slope = $\mu_S W\cos 1.7°$.

The skis will slide if

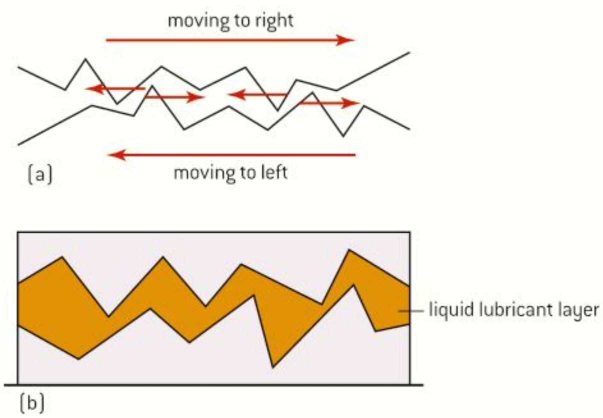
$\mu_S W\cos 1.7° < W\sin 1.7°$

in other words, if $\mu_S < \tan 1.7°$.

The value of tan 1.7° is 0.0296 and this is greater than the value of μ_S, which is 0.025, so the skis will slide away.

Origins of friction

Friction originates at the interface between the two materials, in other words at the surface where they meet. The actual causes of friction are still being investigated today and the explanation given here is highly simplified and a historical one. Leonardo da Vinci mentions friction in his notebooks and some of the next scientific writings about friction appear in works by Guillaume Amontons from around 1700.



▲ Figure 10 How solid friction arises.

One model for solid friction suggests that what are very smooth surfaces to us are not smooth at all. At the atomic level, they are actually full of peaks and troughs of atoms (figure 10(a)). When static friction occurs and two surfaces are at rest relative to each other, then the atomic peaks rest in the troughs, and it needs a certain level of force to deform or break the peaks sufficiently for sliding to begin. Once relative motion has started (so that dynamic friction occurs), then the top surface rises a little above the deformed peaks. Less force is now required to keep the motion going. Because the irregularities on the surface are very small and of atomic size, even the small forces applied in our lab experiments cause large stresses to act on the peaks. The peaks then deform like a soft plastic irrespective of whether the material is hard steel or something much softer.

Moving surfaces are often coated with a lubricant to reduce wear due to friction (figure 10(b)). The lubricant fills the space between the two surfaces and either prevents the peaks and troughs of atoms from touching or reduces the amount of contact. In either event, the atoms from the surfaces do not interact as much as before and the friction force and the coefficient are reduced.

The origins of these friction forces are bound up in the complex electronic properties of the materials that make up the surfaces. However, this simple theory should give you some understanding of friction as well as an awareness that the bulk materials we perceive on the macroscopic scale arise from microscopic properties that are operating at the atomic level.

What is important to realize is that the two equations for static and dynamic friction do not arise from a study of the interatomic forces between the surfaces; they are derived purely from experiments with bulk materials. The results are empirical not theoretical.

Fluid resistance and terminal speed

The assumption that air resistance is negligible is often unrealistic. An object that travels through a fluid (a liquid or a gas) is subject to a complex process in which, as the object travels through the material, the fluid is "stirred up" and becomes subject to a **drag force**. The process is complex, so even after introducing some simple assumptions about the resistance of a fluid, we will still not be able to give a complete analysis.

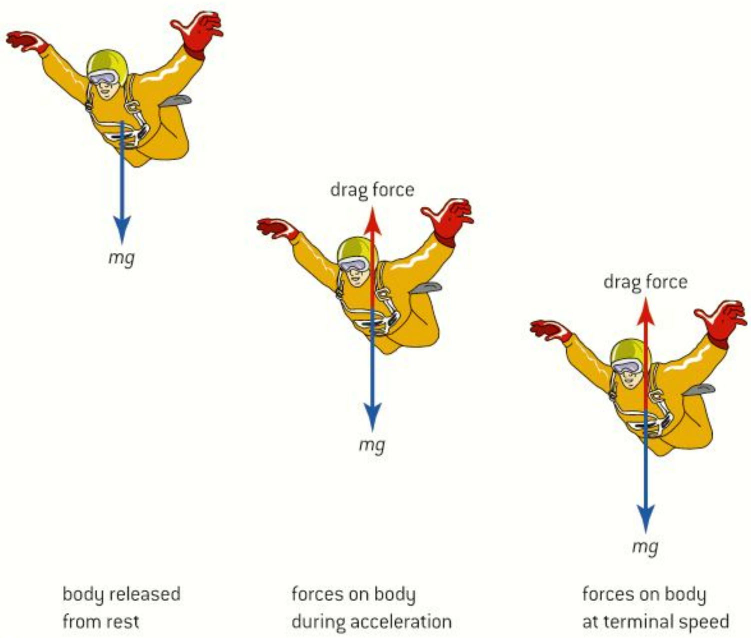
In energy terms, the action of **air resistance** is to transfer some of the energy of the moving body into the fluid through which it is moving. Some fluids absorb this energy better than others: swimming through water is much more tiring than running in the air.

For your IB Diploma Programme physics exams, you only need to describe the effects of fluid resistance without going into the mathematics.

Skydiving

In 2012, Felix Baumgartner jumped safely from a height of 39 km above New Mexico to reach a top speed of 1342 km h^{-1} – faster than the speed of sound.

A skydive from more usual heights will not take place at such high speeds, usually up to about 200 km h^{-1}. The difference is due to the variation in the resistance of the air at different heights above the Earth.

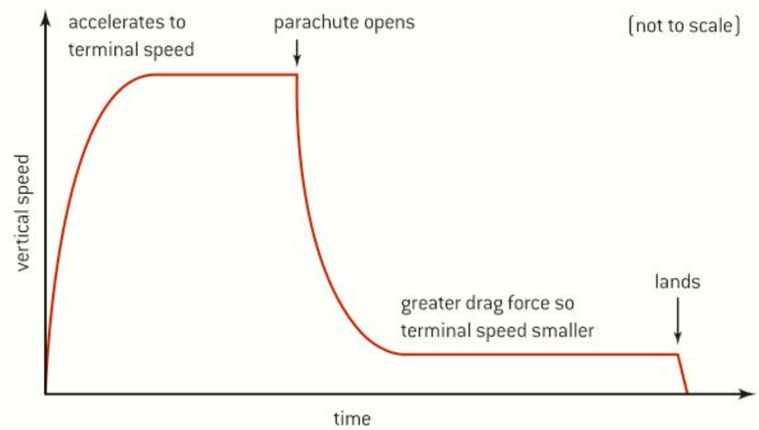


▲ Figure 11 Forces acting on skydiver.

The diagram shows the forces acting on a skydiver. The weight of the diver acts vertically downwards and is effectively constant (because there is little change in the Earth's gravitational field strength at the height of the dive). The air resistance force acts in the opposite direction to the motion of the diver and for a diver falling vertically, this will be vertical too. Other forces acting on the diver include the upwards buoyancy caused by the displacement of air by the diver.

When the skydiver initially leaves the aircraft, the diver's weight acts downwards and because the vertical speed is almost zero, there is almost no air resistance. Air resistance increases as the speed increases so that as the diver goes faster and faster the resistance force becomes larger and larger. The net force therefore decreases and consequently the acceleration of the diver downwards also decreases. Eventually the weight force downwards and the resistance force upwards are equal in magnitude and, of course, opposite in direction. At this point there is no longer any acceleration and the diver has reached a constant rate of fall known as the **terminal speed**.

The graph of vertical speed against time is as shown in figure 12.



▲ Figure 12 Speed–time for a parachute jump.

Eventually the skydiver opens the parachute. Now, because of the large surface area of the parachute envelope, the upwards resistive force is much larger than before and is greater than the weight. As a result, the directions of the net force and acceleration are also upwards; the vertical velocity decreases in magnitude. Once again a balance will be reached where the upward and downward forces are equal and opposite – but at a much lower speed than before (about 12 $m\ s^{-1}$ for a landing) and the diver reaches the ground safely.

Maximum speed of a car

The top (maximum) speed of a motor car is determined by a number of factors. The most obvious of these is the maximum force that the engine can exert through the tyres on the road surface. But, as with the skydiver, this is not the only force acting. There is a considerable drag on the vehicle due to the air and this drag force increases markedly as the speed of the car becomes larger. Typically, when the speed doubles the drag force will increase by at least a factor of four.

There is a maximum power that the car engine can produce. When the car accelerates and the speed increases towards the maximum, the power dissipated in friction also increases. When the maximum energy output of the engine every second is completely used in overcoming the energy losses, then the car cannot accelerate further and has reached its maximum speed.

Worked example

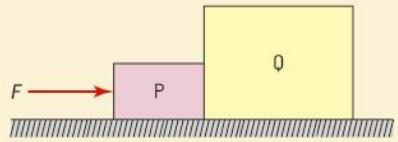
1 Calculate the upward force acting on a skydiver of mass 80 kg who is falling at a constant speed.

Solution

The weight of the skydiver is $80 \times 9.8 = 784$ N. Because the skydiver is falling at constant speed (i.e. terminal speed) the upwards drag force is equal to the downwards weight.

The upward force is 784 N.

2 P and Q are two boxes that are pushed across a rough surface at a constant velocity with a horizontal force of 30 N. The mass of P is 2.0 kg and the mass of Q is 4.0 kg.



State the resultant force on box Q.

Solution

Q is moving at constant speed. So the resultant force on this box must be 0 N.

2.3 Work, energy, and power

Understanding

- Principle of conservation of energy
- Kinetic energy
- Gravitational potential energy
- Elastic potential energy
- Work done as energy transfer
- Power as rate of energy transfer
- Efficiency

Nature of science

The theory of conservation of energy allows many areas of science to be understood at a fundamental level. It allows the explanation of natural phenomena but also means that scientists can predict the outcome of a previously unknown effect. The conservation of energy also demonstrates that paradigm shifts occur in science: the interchangeability of mass and energy as predicted by Einstein is an example of this.

Applications and skills

- Discussing the conservation of total energy within energy transformations
- Sketching and interpreting force – distance graphs
- Determining work done including cases where a resistive force acts
- Solving problems involving power
- Quantitatively describing efficiency in energy transfers

Equations

- work: $W = Fs\cos\theta$
- kinetic energy: $E_K = \frac{1}{2}mv^2$
- elastic potential energy: $E_p = \frac{1}{2}k(\Delta x)^2$
- change in gravitational potential energy: $\Delta E_p = mg\Delta h$
- power $= Fv$
- efficiency $= \frac{\text{useful workout}}{\text{total work in}} = \frac{\text{useful power out}}{\text{total power in}}$