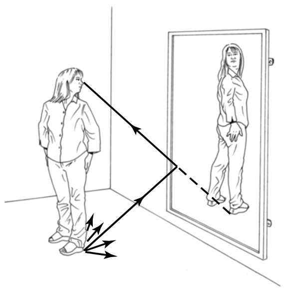
Image forma ion

RAY DIAGRAMS

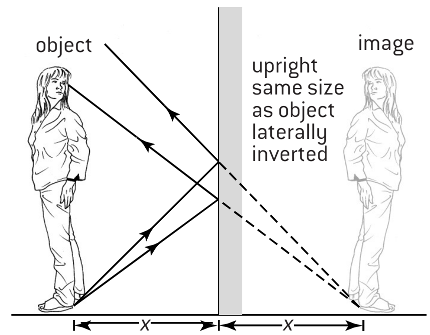
If an object is placed in front of a plane mirror, an image will be formed.



The process is as follows:

- Light sets off in all directions from every part of the object. (This is a result of diffuse reflections from a source of light.)
- Each ray of light that arrives at the mirror is reflected according to the law of reflection.
- These rays can be received by an observer.
- The location of the image seen by the observer arises because the rays are assumed to have travelled in straight lines.

In order to find the location and nature of this image a ray diagram is needed.

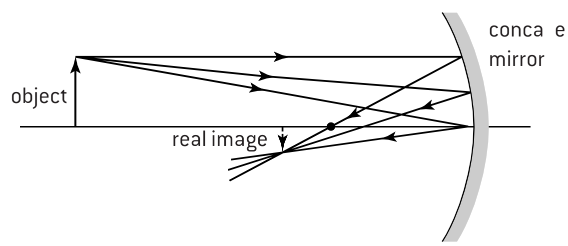


The image formed by reflection in a plane mirror is always:

- **the same distance behind the mirror as the object is in front**
- **upright** (as opposed to being inverted)
- **the same size as the object** (as opposed to being magnified or diminished)
- **laterally inverted** (i.e. left and right are interchanged)
- **virtual** (see below).

REAL AND VIRTUAL IMAGES

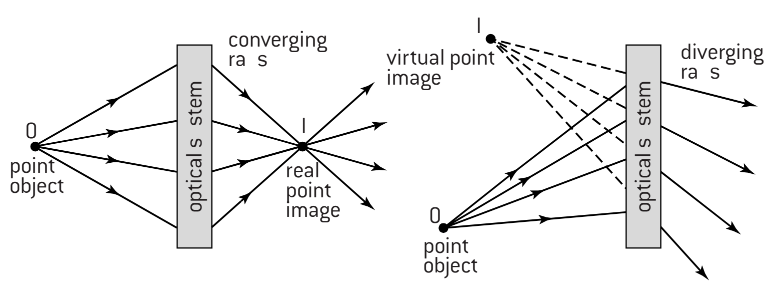
The image formed by reflection in a plane mirror is described as a **virtual image**. This term is used to describe images created when rays of light **seem** to come from a single point but in fact they do not pass through that point. In the example above, the rays of light seem to be coming from behind the mirror. They do not, of course, actually pass behind the mirror at all.



The opposite of a virtual image is a **real image**. In this case, the rays of light do actually pass through a single point. Real images cannot be formed by plane mirrors, but they can be formed by concave mirrors or by lenses. For example, if you look into the concave surface of a spoon, you will see an image of yourself. This particular image is

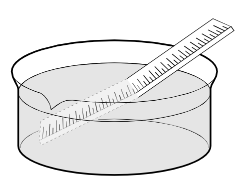
- Upside down
- Diminished
- Real.

(a) real image **(b) virtual image**

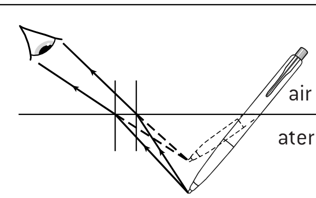


STICK IN WATER

The image formed as a result of the refraction of light leaving water is so commonly seen that most people forget that the objects are made to seem strange. A straight stick will appear bent if it is placed in water. The brain assumes that the rays that arrive at one's eyes must have been travelling in a straight line.



A straight stick appears bent when placed in water



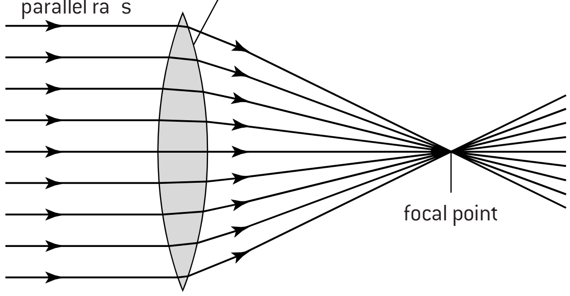
The image of the end of the pen is:

- Nearer to the surface than the pen actually is.
- Virtual.

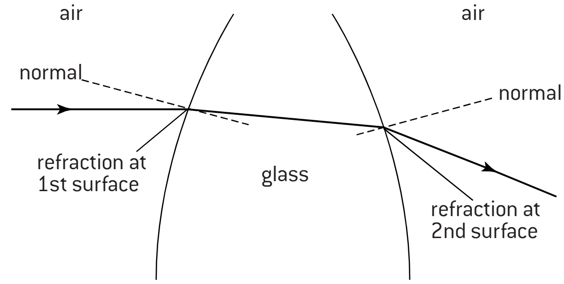
Converging lenses

CONVERGING LENSES

A converging lens brings parallel rays into one ocus point.



The reason that this happens is the re raction that takes place at both sur aces o the lens.



The rays o light are all brought together in one point because o the particular shape o the lens. Any one lens can be thought o as a collection o di erent-shaped glass blocks. It can be shown that any thin lens that has sur aces ormed rom sections o spheres will converge light into one ocus point.

A converging lens will always be thicker at the centre when compared with the edges.

POWER OF A LENS

The power o a lens measures the extent to which light is bent by the lens. A higher power lens bends the light more and thus has a smaller ocal length. The definition o the power o a lens, P, is the reciprocal o the ocal length, f:

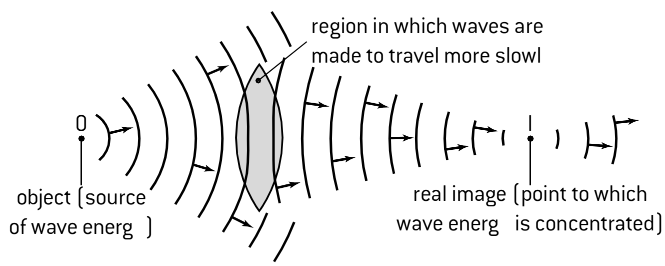
$$P = \frac{1}{f}$$

f is the ocal length measured in m

P is the power o the lens measured in m^{-1} or **dioptres** (dpt)

A lens o power = +5 dioptre is converging and has a ocal length o 20 cm. When two thin lenses are placed close together their powers approximately add.

WAVE MODEL OF IMAGE FORMATION



Formation o a real image by re raction (ignoring di raction)

DEFINITIONS

When analysing lenses and the images that they orm, some technical terms need to be defined.

- The curvature o each sur ace o a lens makes it part o a sphere. The **centre o curvature** or the lens sur ace is the centre o this sphere.
- The **principal axis** is the line going directly through the middle o the lens. Technically it joins the centres o curvature o the two sur aces.
- The **ocal point** (principal ocus) o a lens is the point on the principal axis to which rays that were parallel to the principal axis are brought to ocus a ter passing through the lens. A lens will thus have a ocal point on each side.
- The **ocal length** is the distance between the centre o the lens and the ocal point.
- The **linear magnification**, m, is the ratio between the size (height) o the image and the size (height) o the object. It has no units.

$$\text{linear magnification, } m = \frac{\text{image size}}{\text{object size}} = \frac{h_i}{h_o}$$

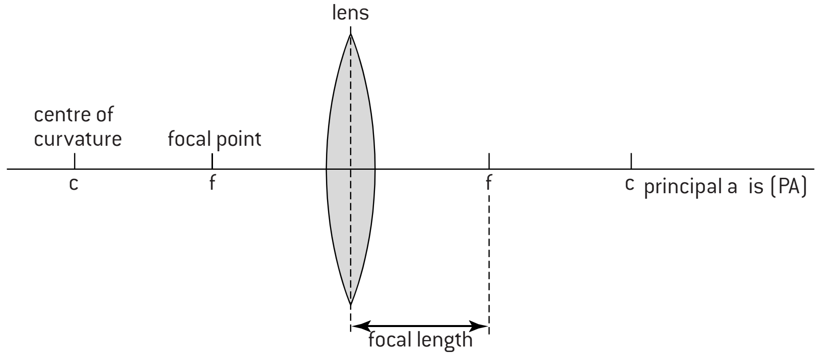


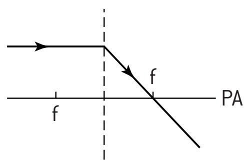
Image forma ion in convex lenses

IMPORTANT RAYS

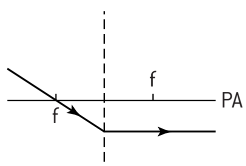
In order to determine the nature and position of the image created of a given object, we need to construct a scaled **ray diagram** of the set-up. In order to do this, we concentrate on the paths taken by three particular rays. As soon as the paths taken by two of these rays have been constructed, the paths of all the other rays can be inferred. These important rays are described below.

Converging lens

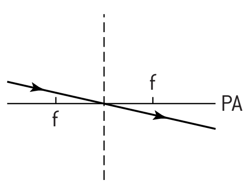
1. Any ray that was travelling parallel to the principal axis will be refracted towards the focal point on the other side of the lens.

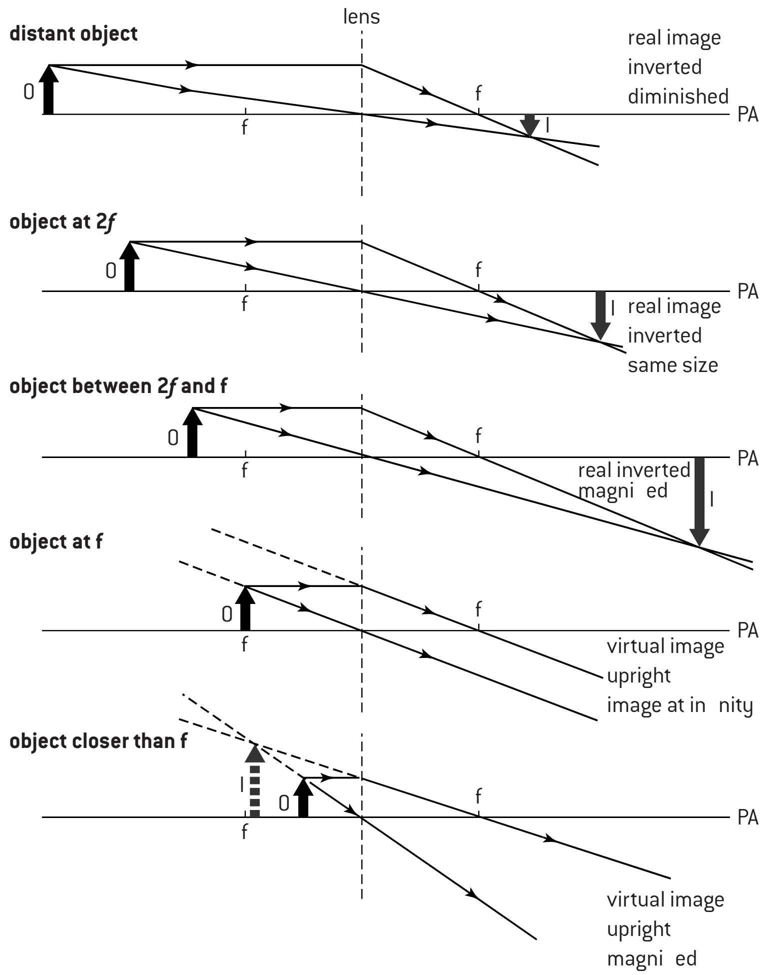


2. Any ray that travelled through the focal point will be refracted parallel to the principal axis.



3. Any ray that goes through the centre of the lens will be undeviated.





Converging lens images

POSSIBLE SITUATIONS

A ray diagram can be constructed as follows:

- An upright arrow on the principal axis represents the object.
- The paths of two important rays from the top of the object are constructed.
- This locates the position of the top of the image.
- The bottom of the image must be on the principal axis directly above (or below) the top of the image.

A full description of the image created would include the following information:

- if it is real or virtual
- if it is upright or inverted
- if it is magnified or diminished
- its exact position.

It should be noted that the important rays are just used to locate the image. The real image also consists of all the other rays from the object. In particular, the image will still be formed even if some of the rays are blocked off.

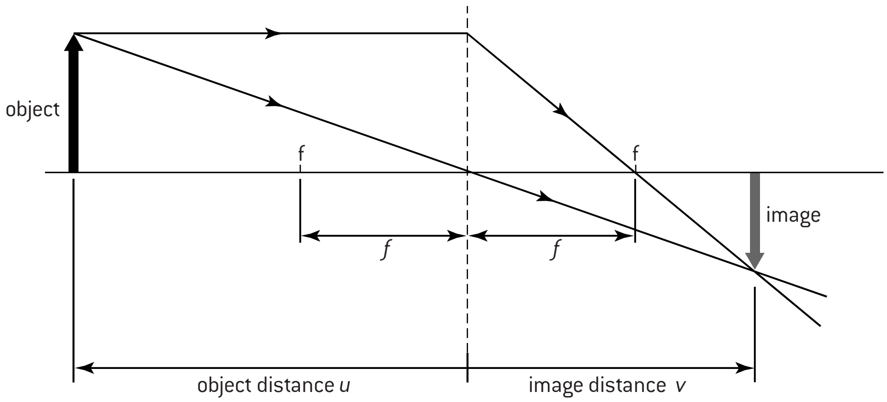
An observer receiving parallel rays sees an image located in the far distance (at infinity).

Thin lens equation

LENS EQUATION

There is a mathematical method o locating the image ormed by a lens. An analysis o the angles involved shows that the ollowing equation can be applied to thin spherical lenses:

$$\frac{1}{f} = \frac{1}{\ } + \frac{1}{u}$$



Suppose $u = 25$ cm

$f = 10$ cm

This would mean that $\frac{1}{\ } = \frac{1}{f} - \frac{1}{u} = \frac{1}{10} - \frac{1}{25} = \frac{5}{50} - \frac{2}{50} = \frac{3}{50}$

In other word, $\ = \frac{50}{3} = 16.7$ cm i.e. image is real

In this case $m = \frac{-16.7}{10} = -1.67$ and inverted.

LINEAR MAGNIFICATION

In all cases, linear magnification,

$$m = \frac{h_i}{h_o} = -\frac{\ }{u}$$

$$m = \frac{\text{height o image}}{\text{height o object}}$$

$$= \frac{h_i}{h_o}$$

$$= -\frac{\ }{u}$$

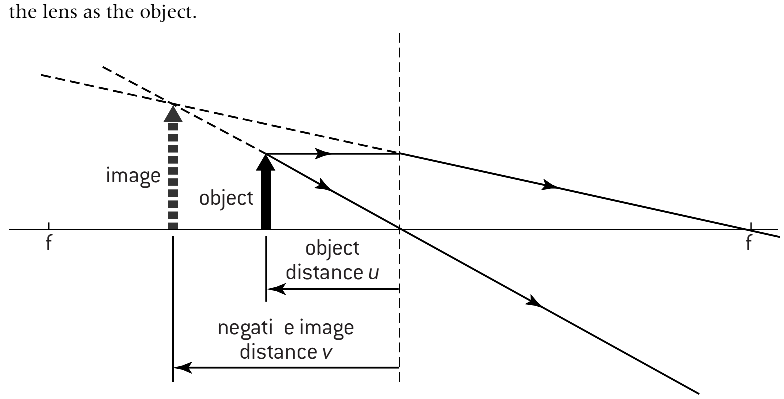
For real images, m is negative and image is inverted

For virtual images m is positive and image is upright

REAL IS POSITIVE

Care needs to be taken with virtual images. The equation does work but or this to be the case, the ollowing convention has to be ollowed:

- Distances are taken to be **positive** i actually traversed by the light ray (i.e. distances to real object and image).
- Distances are taken to be **negative** i apparently traversed by the light ray (distances to virtual objects and images).
- Thus a virtual image is represented by a negative value or   – in other words, it will be on the same side o the lens as the object.



Suppose $u = 10$ cm

$f = 25$ cm

This would mean that $\frac{1}{\ } = \frac{1}{f} - \frac{1}{u} = \frac{1}{25} - \frac{1}{10} = \frac{2}{50} - \frac{5}{50} = -\frac{3}{50}$

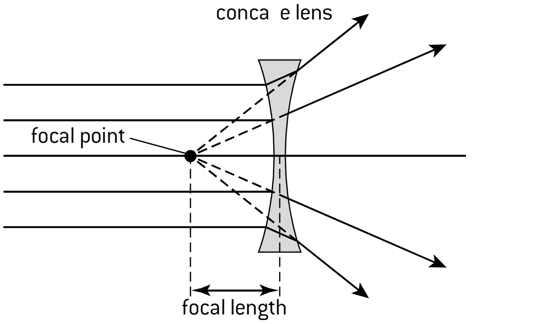
In other word, $\ = -\frac{50}{3} = -16.7$ cm i.e. image is virtual

In this case $m = +\frac{16.7}{10} = +1.67$ and upright

Diverging lenses

DIVERGING LENSES

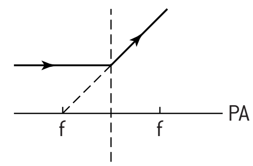
A diverging lens spreads parallel rays apart. These rays appear to all come rom one ocus point on the other side o the lens.



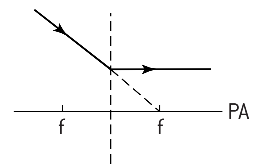
The reason that this happens is the re raction that takes place at both sur aces. A diverging lens will always be thinner at the centre when compared with the edges.

When constructing ray diagrams or diverging lenses, the important rays whose paths are known (and rom which all other ray paths can be in erred) are:

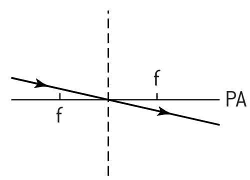
1. Any ray that was travelling parallel to the principal axis will be re racted away rom a ocal point on the incident side o the lens.



2. Any ray that is heading towards the ocal point on the other side o the lens, will be re racted so as to be parallel to the principal axis.



3. Any ray that goes through the centre o the lens will be undeviated.



DEFINITIONS AND IMPORTANT RAYS

Diverging lenses have the same analogous definitions as converging lenses or all o the ollowing terms:

Centre of curvature, principal axis, focal point, focal length, linear magnification.

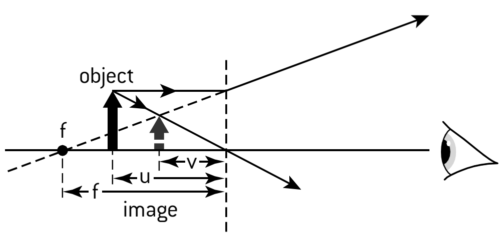
Note that:

- The ocal point is the point on the principal axis **from hich** rays that were parallel to the principal axis **appear to come** a ter passing through the lens.
- As the ocal point is behind the diverging lens, **the focal length of a diverging lens is negative**.

IMAGES CREATED BY A DIVERGING LENS

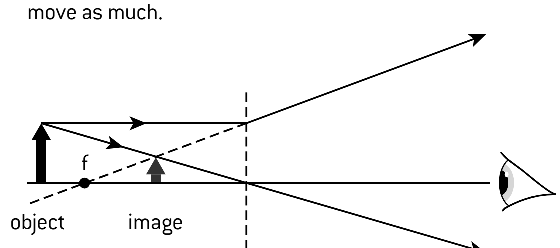
Whatever the position o the object, a diverging lens will always create an upright, diminished and virtual image located between the ocal point and the lens on the same side o the lens as the object.

If ou look at an object through a concave lens, it will look smaller and closer.



object inside focal length

If ou move the object further out, the image will not move as much.



object outside focal length

The thin lens equation will still work providing one remembers the negative ocal length o a diverging lens.

For example, i an object is placed at a distance $2l$ away rom a diverging lens o ocal length l, the image can be calculated as ollows:

Given: $u = 2l$, $f = -l$, $v = ?$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-l} - \frac{1}{2l} = \frac{-3}{2l}$$

$$\therefore v = -\frac{2l}{3}$$

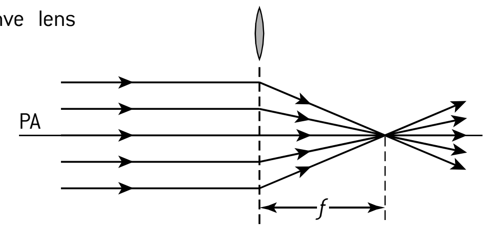
This is a virtual diminished and upright image with $m = +\frac{1}{3}$

Converging and diverging mirrors

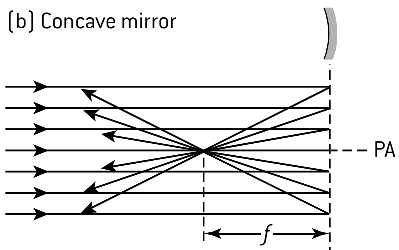
GEOMETRY OF MIRRORS AND LENSES

The geometry of the paths of rays after reflection by a spherical concave or convex mirror is exactly analogous to the paths of rays through converging or diverging lenses. The only difference is that mirrors reflect all rays backwards whereas rays pass through lenses and continue forwards.

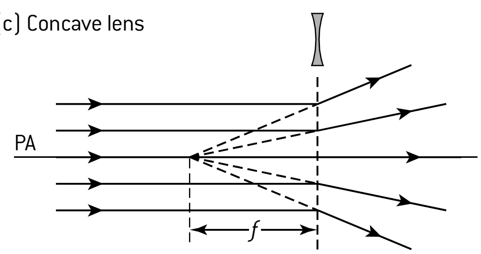
(a) Conve lens



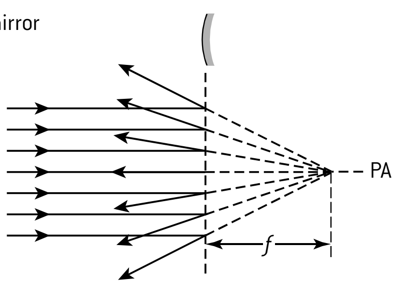
(b) Concave mirror



(c) Concave lens



(d) Conve mirror



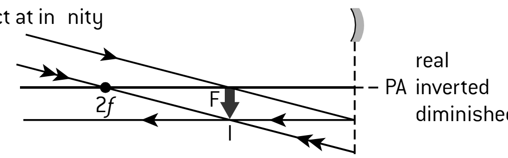
This analogous behaviour means that all the definitions and equations for lenses can be used (with suitable attention to detail with the sign conventions) with mirrors.

An additional important ray for mirrors is the ray that travels through (or towards) the centre of curvature of the mirror (located at twice the focal length). This ray will be reflected back along the same path.

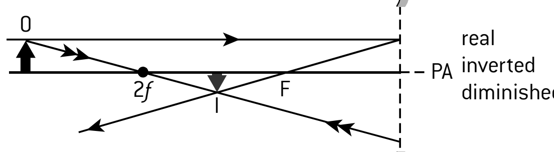
IMAGE FORMATION IN MIRRORS

(1) Conca e

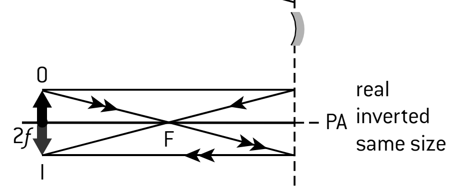
object at in nity



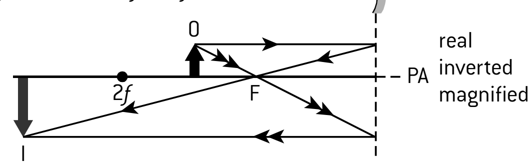
object between in nity and 2f



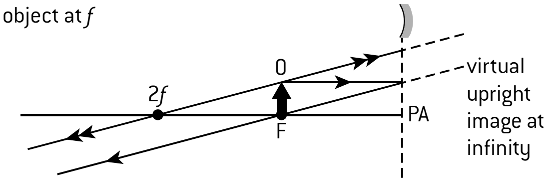
object at 2f



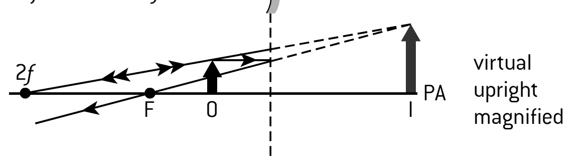
object between 2f and f



object at f

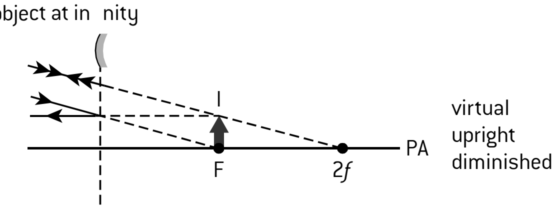


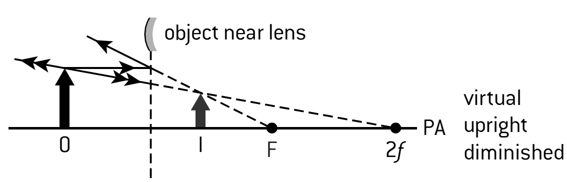
object between f and mirror



(2) Convex

object at in nity





The simple magnifying glass

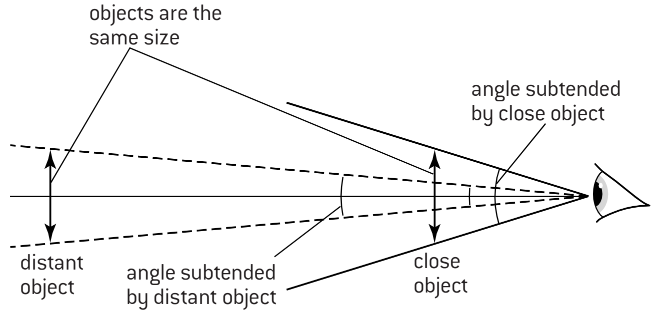
NEAR AND FAR POINT

The human eye can ocus objects at di erent distances rom the eye. Two terms are use ul to describe the possible range o distances – the **near point** and the **ar point** distance.

- The distance to the **near point** is the distance between the eye and the nearest object that can be brought into clear ocus (without strain or help rom, or example, lenses). It is also known as the 'least distance o distinct vision'. By convention it is taken to be 25 cm or normal vision.
- The distance to the **ar point** is the distance between the eye and the urthest object that can be brought into ocus. This is taken to be infinity or normal vision.

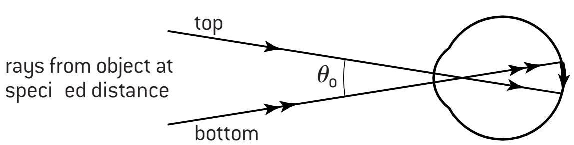
ANGULAR SIZE

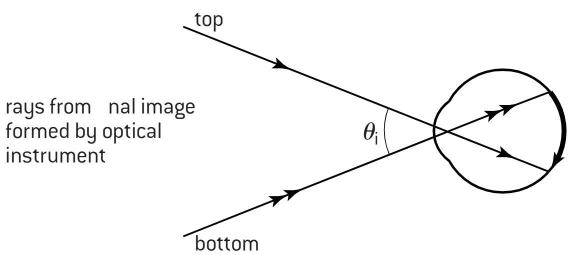
I we bring an object closer to us (and our eyes are still able to ocus on it) then we see it in more detail. This is because, as the object approaches, it occupies a bigger visual angle. The technical term or this is that the object **subtends** a larger angle.



ANGULAR MAGNIFICATION

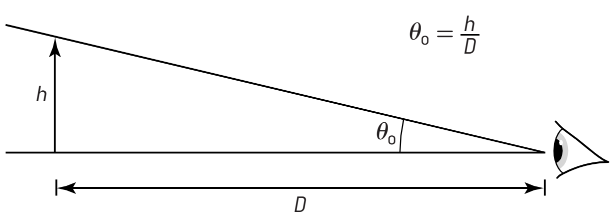
The angular magnification, *M*, o an optical instrument is defined as the ratio between the angle that an object subtends normally and the angle that its image subtends as a result o the optical instrument. The 'normal' situation depends on the context. It should be noted that the angular magnification is not the same as the linear magnification.





Angular magnification, $M = \frac{\theta_i}{\theta_o}$

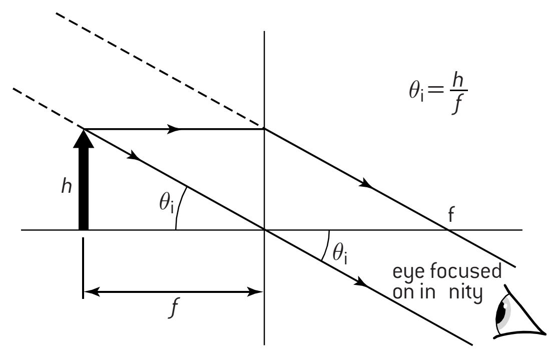
The largest visual angle that an object can occupy is when it is placed at the near point. This is o ten taken as the 'normal' situation.



A simple lens can increase the angle subtended. It is usual to consider two possible situations.

1. **Image ormed at infinity**

   In this arrangement, the object is placed at the ocal point. The resulting image will be ormed at infinity and can be seen by the relaxed eye.



   In this case the angular magnification would be

$$M_{\text{infinity}} = \frac{\theta_i}{\theta_o} = \frac{\frac{h}{f}}{\frac{h}{D}} = \frac{D}{f}$$

   This is the smallest value that the angular magnification can be.

2. **Image ormed at near point**

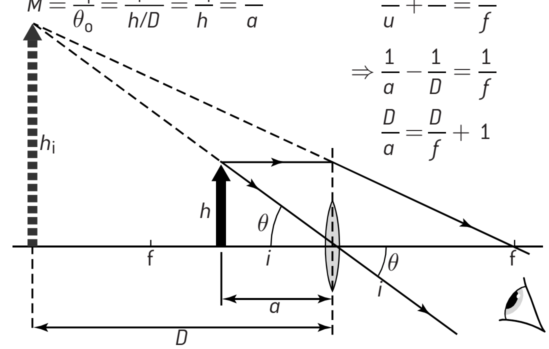
   In this arrangement, the object is placed nearer to the lens. The resulting virtual image is located at the near point. This arrangement has the largest possible angular magnification.

$$M = \frac{\theta_i}{\theta_o} = \frac{h_i/D}{h/D} = \frac{h_i}{h} = \frac{D}{a}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{a} - \frac{1}{D} = \frac{1}{f}$$

$$\frac{D}{a} = \frac{D}{f} + 1$$



So the magnitude o $M_{\text{near point}} = \frac{D}{f} + 1$

Aberra ions

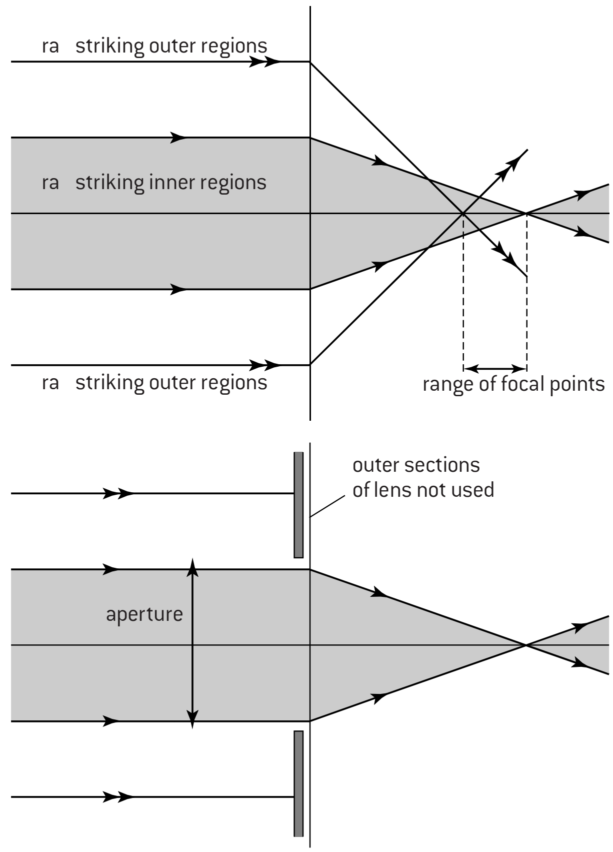
SPHERICAL

A lens is said to have an **aberration** if, for some reason, a point object does not produce a perfect point image. In realit , lenses that are spherical do not produce perfect images. **Spherical aberration** is the term used to describe the fact that ra s striking the outer regions of a spherical lens will be brought to a slightl different focus point from those striking the inner regions of the same lens. This is not to be confused with **barrel distortion**.

In general, a point object will focus into a small circle of light, rather than a perfect point. There are several possible wa s of reducing this effect:

- the shape of the lens could be altered in such a wa as to correct for the effect. The lens would, of course, no longer be spherical. A particular shape onl works for objects at a particular distance awa .
- the effect can be reduced for a given lens b decreasing the aperture. The technical term for this is **stopping do n** the aperture. The disadvantage is that the total amount of light is reduced and the effects of diffraction (see page 46) would be made worse.

The effect for mirrors can be eliminated for all point objects on the axis b using a parabolic (as opposed to a spherical) mirror. For mirrors, the effect can again be reduced b using a smaller aperture.



Spherical aberration

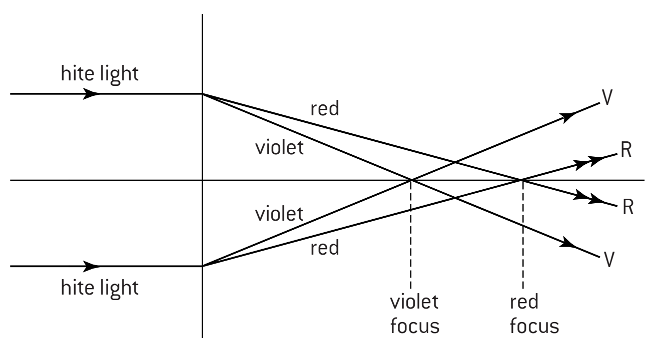
CHROMATIC

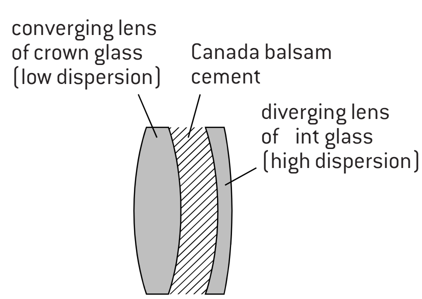
Chromatic aberration is the term used to describe the fact that ra s of different colours will be brought to a slightl different focus point b the same lens. The refractive index of the material used to make the lens is different for different frequencies of light.

A point object will produce a blurred image of different colours.

The effect can be eliminated for two given colours (and reduced for all) b using two different materials to make up a compound lens. This compound lens is called an **achromatic doublet**. The two t pes of glass produce equal but opposite dispersion.

Mirrors do not suffer from chromatic aberration.



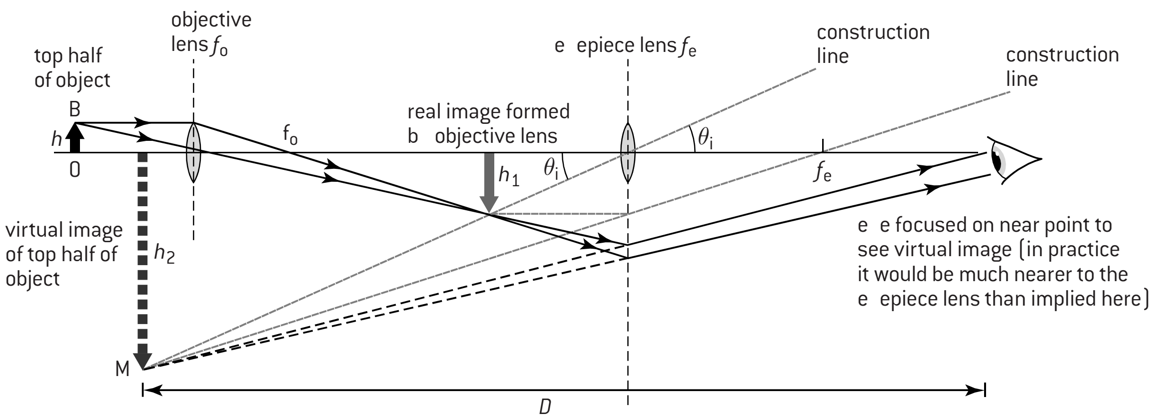


Achromatic doublet

The compound microscope and astronomical telescope

COMPOUND MICROSCOPE

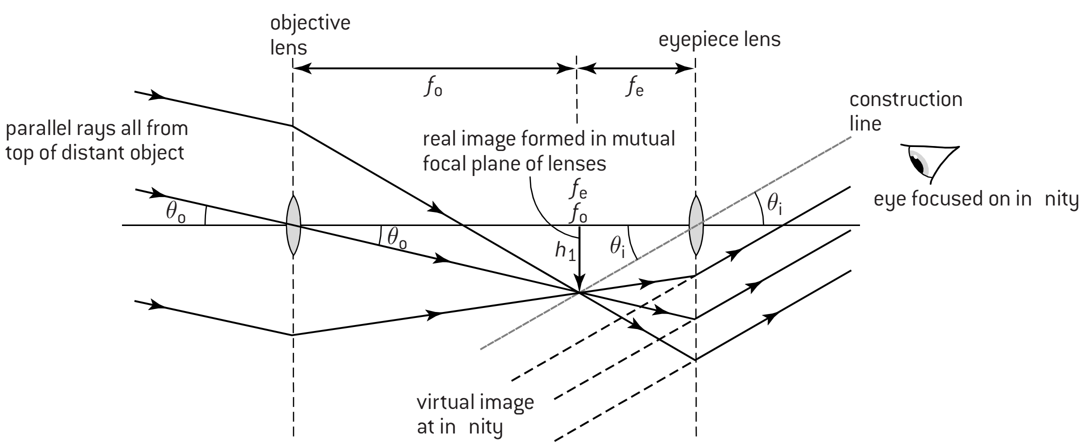
A compound microscope consists o two lenses – the **objective lens** and the **eyepiece lens**. The first lens (the objective lens) orms a **real magnified** image o the object being viewed. This real image can then be considered as the object or the second lens (the eyepiece lens) which acts as a magni ying lens. The rays rom this real image travel into the eyepiece lens and they orm a **virtual magnified** image. In normal adjustment, this virtual image is arranged to be located at the near point so that maximum angular magnification is obtained.



$M = \frac{\theta_i}{\theta_o} = \frac{\frac{\ _2}{D}}{\frac{\ }{D}} = \frac{\ _2}{\ } = \frac{\ _2}{\ _1} \cdot \frac{\ _1}{\ }$ = linear magnification produced by eyepiece × linear magnification produced by objective

ASTRONOMICAL TELESCOPE

An astronomical telescope also consists o two lenses. In this case, the objective lens orms a **real** but **diminished** image o the distant object being viewed. Once again, this real image can then be considered as the object or the eyepiece lens acting as a magni ying lens. The rays rom this real image travel into the eyepiece lens and they orm a **virtual magnified** image. In normal adjustment, this virtual image is arranged to be located at infinity.



$M = \frac{\theta_i}{\theta_o} = \frac{\frac{\ _1}{f_e}}{\frac{\ _1}{f_o}} = \frac{f_o}{f_e}$

The length o the telescope $\approx f_o + f_e$.

As ronomical reflec ing elescopes

COMPARISON OF REFLECTING AND REFRACTING TELESCOPES

A refracting telescope uses an objective (converging) lens to form a real diminished image of a distant object. This image is then viewed by the eyepiece lens (converging) which, acting as a simple magnifying glass, produces a virtual but magnified final image.

In an analogous way, a reflecting telescope uses a concave mirror set up so as to form a real, diminished image of a distant object. This image, however, would be difficult to view as it would be produced in front of the concave mirror. Thus mirrors are used to produce a viewable image that can, like the refracting telescope, be viewed by the eyepiece lens (converging). Once again the eyepiece acts as a simple magnifying glass and produces the virtual, but magnified, final image. Two common mountings for reflecting telescopes are the **Ne tonian mounting** and the **Cassegrain mounting.**

All telescopes are made to have large apertures in order to:

a) reduce diffraction effects, and

b) collect enough light to make bright images of low power sources.

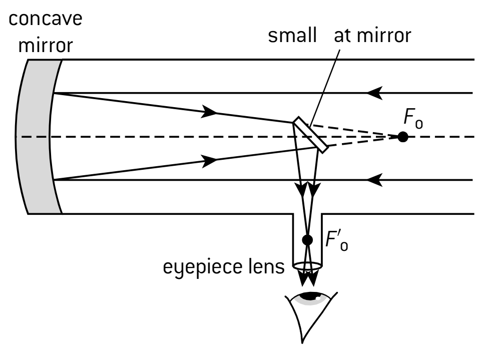
Large telescopes are reflecting because:

- Mirrors do not suffer from chromatic aberration
- It is difficult to get a uniform refractive index throughout a large volume of glass
- Mounting a large lens is harder to achieve than mounting a large mirror.
- Only one surface needs to be the right shape.

Reflecting telescopes can easily suffer damage to the mirror surface.

NEWTONIAN MOUNTING

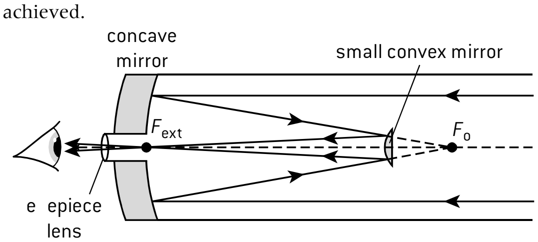
A small flat mirror is placed on the principal axis of the mirror to reflect the image formed to the side:



CASSEGRAIN MOUNTING

A small convex mirror is mounted on the principal axis of the mirror. The mirror has a central hole to allow the image to be viewed.

The convex mirror will add to the angular magnification achieved.

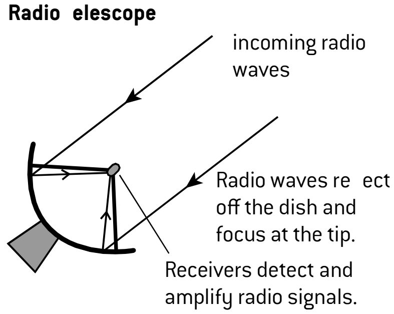


Radio elescopes

SINGLE DISH RADIO TELESCOPES

A single dish radio telescope operates in a very similar way to a reflecting telescope. Rather than reflecting visible light to form an image, the much longer wavelengths of radio waves are reflected by the curved receiving dish. The antenna that is the receiver of the radio waves can be tuned to pick up specific wavelengths under observation and are used to study naturally occurring radio emission from stars, galaxies, quasars and other astronomical objects between wavelengths of about 10 m and 1mm.

Radio elescope



Diffraction effects can significantly limit the accuracy with which a radio telescope can locate individual sources of radio signals. Increasing the diameter of a radio telescope improves the telescope's ability to resolve different sources and ensure that more power can be received (see resolution on page 101).

RADIO INTERFEROMETRY TELESCOPES

The angular resolution of a radio telescope can be improved using a principle called interferometry. This process analyses signals received at two (or more) radio telescopes that are some distance apart but pointing in the same direction. This effectively creates a virtual radio telescope that is much larger than any of the individual telescopes.

The technique is complex as it involves collecting signals from two or more radio telescopes (an **array telescope**) in one central location. The arrival of each signal at an individual antenna needs to be carefully calibrated against a single shared reference signal so that different signals can be combined as though they arrived at one single antenna. When the signals from the different telescopes are added together, they interfere. The result is to create a combined telescope that is equivalent in resolution (though not in sensitivity) to a single radio telescope whose diameter is approximately equal to the maximum separations of the antennae.

The principle can be extended, in a process called **Very Long Baseline Interferometry**, to allow recordings of radio signals (originally made hundreds of km apart) to be synchronized to within a millionth of a second thus allowing scientists from different countries to collaborate to create a virtual radio telescope of huge size and high resolving power.

COMPARATIVE PERFORMANCE OF EARTH-BOUND AND SATELLITE-BORNE TELESCOPES

The following points about Earth-based (EB) and satellite-borne (SB) telescopes can be made:

- SB observations are free from interference and/or absorptions due to the Earth's atmosphere that hinder EB observations, giving better resolution for SB telescopes.
- Modern computer techniques can effectively correct for many atmospheric effects making new ground-based telescopes similar in resolution to some SB telescopes.
- Many significant wavelengths of EM radiation (UV, IR and long wavelength radio) are absorbed by the Earth's atmosphere so SB telescopes are the only possibility in their wavelengths.
- SB observations do not suffer from light pollution / radio interference as a result of nearby human activity.
- SB facilities are not subject to continual wear and tear as a result of the Earth's atmosphere (storms etc.).
- The possibility of damage from space debris exists for SB telescopes.
- There is a great deal of added cost in getting the telescope into orbit and controlling it remotely, meaning that SB telescopes are significantly more expensive to build and this places a limit on their size and weight.
- There is an added difficultly of effecting repairs / alterations to a SB telescope once operational.
- SB telescopes need to withstand wider temperature variations than EB telescopes.
- EB optical telescopes can only operate at night whereas SB telescopes can operate at all times.

Fibre op ics

OPTIC FIBRE

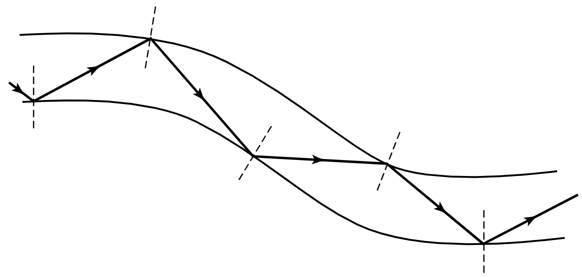
Optic fibres use the principle o total internal reflection (see page 45) to guide light along a certain path. The idea is to make a ray o light travel along a transparent fibre by bouncing between the walls o the fibre. So long as the incident angle o the ray on the wall o the fibre is always greater than the critical angle, the ray will always remain within the fibre even i the fibre is bent (see right).

As shown on page 45, the relation between critical angle, *c*, and re ractive index *n* is given by

$$n = \frac{1}{\sin c}$$

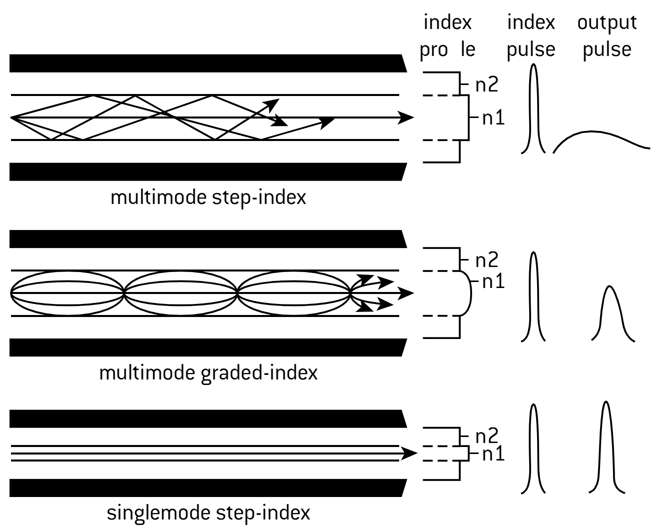
Two important uses o optic fibres are:

- In the communication industry. Digital data can be encoded into pulses o light that can then travel along the fibres. This is used or telephone communication, cable TV etc.
- In the medical world. Bundles o optic fibres can be used to carry images back rom inside the body. This instrument is called an endoscope.
- This type o optic fibre is known as a step-index optic fibre. Cladding o a material with a lower re ractive index surrounds the fibre. This cladding protects and strengthens the fibre.



TYPES OF OPTIC FIBRES

The simplest fibre optic is a step-index fibre. Technically this is known as a **multimode step-index fibre**. Multimode re ers to the act that light can take di erent paths down the fibre which can result in some distortion o signals (see waveguide dispersion, page 183). The (multimode) **graded-index** fibre is an improvement. This uses a graded re ractive index profile in the fibre meaning that rays travel at di erent speeds depending on their distance rom the centre. This has the e ect o reducing the spreading out o the pulse. Most fibres used in data communications have a graded index. The optimum solution is to have a very narrow core – a **singlemode step-index fibre**.

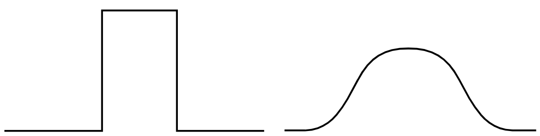


Dispersion, attenuation and noise in optical fibres

MATERIAL DISPERSION

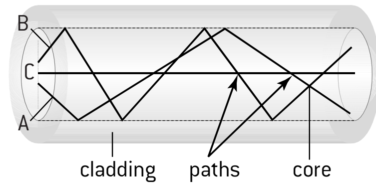
The re ractive index o any substance depends on the requency o electromagnetic radiation considered. This is the reason that white light is dispersed into di erent colours when it passes through a triangular prism.

As light travels along an optical fibre, di erent requencies will travel at slightly di erent speeds. This means that i the source o light involves a range o   requencies, then a pulse that starts out as a square wave will tend to spread out as it travels along the fibre. This process is known as **material dispersion**.



WAVEGUIDE DISPERSION

I the optical fibre has a significant diameter, another process called **waveguide dispersion** that can cause the stretching o a pulse is **multipath** or **modal dispersion**. The path length along the centre o a fibre is shorter than a path that involves multiple reflections. This means that rays rom a particular pulse will not all arrive at the same time because o the di erent distances they have travelled.



The problems caused by modal dispersion have led to the development o **monomode** (or **singlemode**) **step-index fibres**. These optical fibres have very narrow cores (o the same order o magnitude as the wavelength o the light being used (approximately 5 μm) so that there is only one e ective transmission path – directly along the fibre.

ATTENUATION

As light travels along an optic fibre, some o the energy can be scattered or absorbed by the glass. The intensity o the light energy that arrives at the end o the fibre is less than the intensity that entered the fibre. The signal is said to be **attenuated**.

The amount o attenuation is measured on a logarithmic scale in decibels (dB). The attenuation is given by

$$\text{attenuation (dB)} = 10\log \frac{I}{I_o}$$

I is the intensity o the output power measured in W

I_o is the intensity o the original input power measured in W

A negative attenuation means that the signal has been reduced in power. A positive attenuation would imply that the signal has been amplified.

See page 188 or another example o the use o the decibel scale.

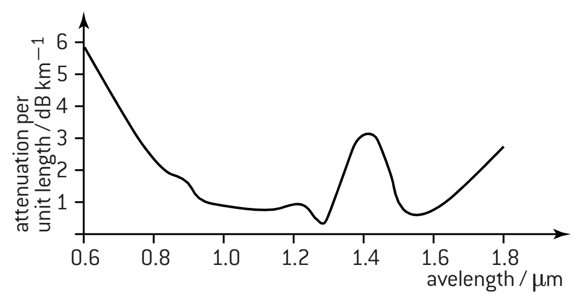
It is common to quote the attenuation per unit length as measured in dB km^{-1}. For example, 5 km o fibre optic cable causes an input power o 100 mW to decrease to 1 mW. The attenuation per unit length is calculated as ollows:

$$\text{attenuation} = 10 \log (10^{-3}/10^{-1}) = 10 \log (10^{-2}) = -20 \text{ dB}$$

$$\text{attenuation per unit length} = -20 \text{ dB}/5 \text{ km} = -4 \text{ dB km}^{-1}$$

The attenuation o a 10 km length o this fibre optic cable would there ore be −40 dB. The overall attenuation resulting rom a series o   actors is the algebraic sum o the individual attenuations.

The attenuation in an optic fibre is a result o several processes: those caused by impurities in the glass, the general scattering that takes place in the glass and the extent to which the glass absorbs the light. These last two actors are a ected by the wavelength o light used. A typical the overall attenuation is shown below:



CAPACITY

Attenuation causes an upper limit to the amount o digital in ormation that can be sent along a particular type o optical fibre. This is o ten stated in terms o its capacity.

capacity o an optical fibre = bit rate × distance

A fibre with a capacity o 80 Mbit s^{-1} km can transmit 80 Mbit s^{-1} along a 1 km length o fibre but only 20 Mbit s^{-1} along a 4 km length.

NOISE, AMPLIFIERS AND RESHAPERS

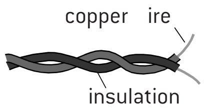
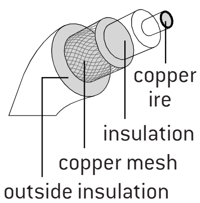
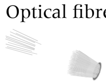
Noise is inevitable in any electronic circuit. Any dispersions or scatterings that take place within an optical fibre will also add to the noise.

An amplifier increases the signal strength and thus will tend to correct the e ect o attenuation – these are also sometimes called regenerators. An amplifier will also increase any noise that has been added to the electrical signal.

A reshaper can reduce the e ects o noise on a digital signal by returning the signal to a series o 1s and 0s with sharp transitions between the allowed levels.

Channels of communication

The table below shows some common communication links.

	Options for communication	Uses	Advantages and disadvantages
Wire pairs (twisted pair) 	Two wires can connect the sender and receiver of information. For example a simple link between a microphone, an amplifier and a loudspeaker.	Very simple communication systems e.g. intercom	Very simple and cheap. Susceptible to noise and interference. Unable to transfer information at the highest rates.
Coaxial cables 	This arrangement of two wires reduces electrical interference. A central wire is surrounded by the second wire in the form of an outer cylindrical copper tube or mesh. An insulator separates the two wires. Wire links can carry frequencies up to about 1 GHz but the higher frequencies will be attenuated more for a given length of wire. A typical 100 MHz signal sent down low-loss cable would need repeaters at intervals of approximately 0.5 km. The upper limit for a single coaxial cable is approximately 140 Mbit s^{-1}.	Coaxial cables are used to transfer signals from TV aerials to TV receivers. Historically they are standard for underground telephone links.	Simple and straightforward. Less susceptible to noise compared to simple wire pair but noise still a problem.
Optical fibres 	Laser light can be used to send signals down optical fibres with approximately the same frequency limit as cables – 1 GHz. The attenuation in an optical fibre is less than in a coaxial cable. The distance between repeaters can easily be tens (or even hundreds) of kilometres.	Long-distance telecommunication and high volume transfer of digital data including video data.	Compared to coaxial cables with equivalent capacity, optical fibres: • have a higher transmission capacity • are much smaller in size and weight • cost less • allow for a wider possible spacing of regenerators • offer immunity to electromagnetic interference • suffer from negligible cross talk (signals in one channel affecting another channel) • are very suitable for digital data transmission • provide good security • are quiet – they do not hum even when carrying large volumes of data. There are some disadvantages: • the repair of fibres is not a simple task • regenerators are more complex and thus potentially less reliable.

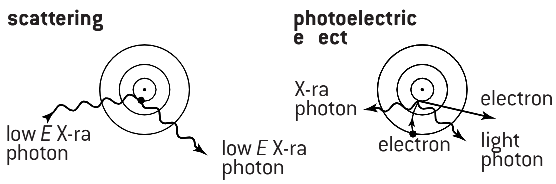
X-ra s

INTENSITY, QUALITY AND ATTENUATION

The effects of X-rays on matter depend on two things, the **intensity** and the **quality** of the X-rays.

- The intensity, I, is the amount of energy per unit area that is carried by the X-rays.
- The quality of the X-ray beam is the name given to the spread of wavelengths that are present in the beam. Low-energy photons will be absorbed by all tissues and potentially cause harm without contributing to forming the image. It is desirable to remove these from the beam.

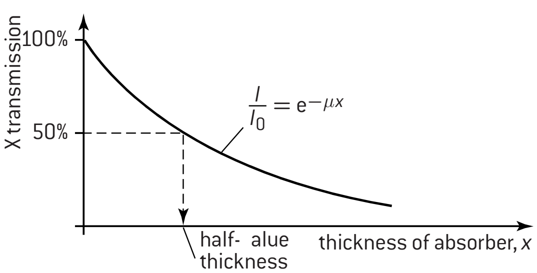
If the energy of the beam is absorbed, then it is said to be **attenuated**. If there is nothing in the way of an X-ray beam, it will still be attenuated as the beam spreads out. Two processes of attenuation by matter, **simple scattering** and the **photoelectric e ect** are the dominant ones for low-energy X-rays.



Simple scattering affects X-ray photons that have energies between zero and 30 keV.

- In the photoelectric effect, the incoming X-ray has enough energy to cause one of the inner electrons to be ejected from the atom. It will result in one of the outer electrons 'falling down' into this energy level. As it does so, it releases some light energy. This process affects X-ray photons that have energies between zero and 100 keV.

Both attenuation processes result in a near exponential transmission of radiation as shown in the diagram below. For a given energy of X-rays and given material there will be a certain thickness that reduces the intensity of the X-ray by 50%. This is known as the **hal -value thickness**.



The **attenuation coe ficient** μ is a constant that mathematically allows us to calculate the intensity of the X-rays given any thickness of material. The equation is as follows:

$$I = I_0 e^{-\mu x}$$

The relationship between the attenuation coefficient and the half-value thickness is

$$\mu x_{\frac{1}{2}} = \ln 2$$

$x_{\frac{1}{2}}$ The half-value thickness of the material (in m)

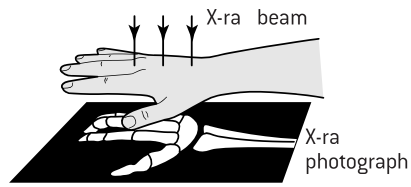
ln 2 The natural log of 2. This is the number 0.6931

μ The attenuation coefficient (in m^{-1})

μ depends on the wavelength of the X-rays – short wavelengths are highly penetrating and these X-rays are **hard**. **So t** x-rays are easily attenuated and have long wavelengths.

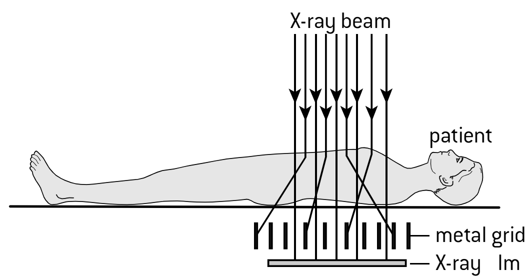
BASIC X-RAY DISPLAY TECHNIQUES

The basic principle of X-ray imaging is that some body parts (for example bones) will attenuate the X-ray beam much more than other body parts (for example skin and tissue). Photographic film darkens when a beam of X-rays is shone on them so bones show up as white areas on an X-ray picture.



The sharpness of an X-ray image is a measure of how easy it is to see edges of different organs or different types of tissue.

X-ray beams will be scattered in the patient being scanned and the result will be to blur the final image and to reduce the contrast and sharpness. To help reduce this effect, a metal filter grid is added below the patient:



Alternatively computer software can be used to detect and enhance edges.

Since X-rays cause ionizations, they are dangerous. This means that the intensity used needs to be kept to an absolute minimum. This can be done by introducing something to **intensi y** (to enhance) the image. There are two simple techniques of **enhancement**:

- When X-rays strike an intensifying screen the energy is re-radiated as visible light. The photographic film can absorb this extra light. The overall effect is to darken the image in the areas where X-rays are still present (see page 187).
- In an image-intensifier tube, the X-rays strike a fluorescent screen and produce light. This light causes electrons to be emitted from a photocathode. These electrons are then accelerated towards an anode where they strike another fluorescent screen and give off light to produce an image.

MASS ATTENUATION COEFFICIENT

An alternative way of writing the equation for the attenuation coefficient is shown below:

$$I = I_0 e^{-\left(\frac{\mu}{\ }\right) x}$$

Where is the density of the substance. In this format, $\frac{\mu}{\ }$ is known as the **mass attenuation coe ficient** $\frac{\mu}{\ }$, and x is known as the **area density** or **mass thickness**.

Units of mass attenuation coefficient, $\frac{\ }{\ } = m^2\ kg^{-1}$

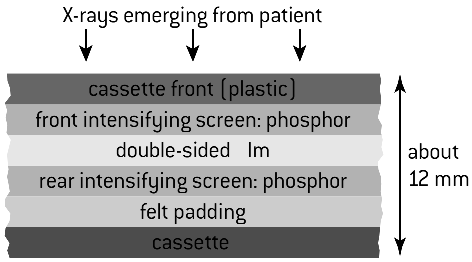
Units of area density, $x = kg\ m^{-2}$

$$I = I_0 e^{-\left(\frac{\mu}{\ }\right) x}$$

X-ra   imaging techniques

1) Intensifying screens

The arrangement of the intensifying screens described on page 185 are shown below.



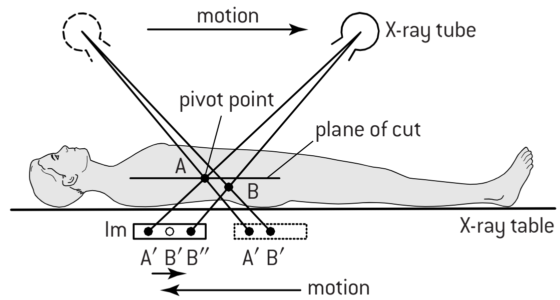
With a simple X-ray photograph it is hard to identify problems within soft tissue, for example in the gut. There are two general techniques aimed at improving this situation.

2) Barium meals

In a **barium meal**, a dense substance is swallowed and its progress along the gut can be monitored. The contrast between the gut and surrounding tissue is increased. Typically the patient is asked to swallow a harmless solution of barium sulfate. The result is an increase in the sharpness of the image.

3) Tomography

Tomography is a technique that makes the X-ray photograph focus on a certain region or 'slice' through the patient. All other regions are blurred out of focus. This is achieved by moving the source of X-rays and the film together.



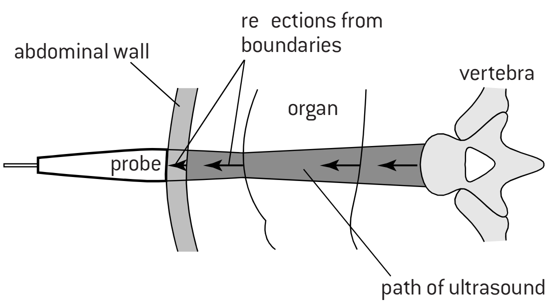
An extension of basic tomography is the **computed tomography scan** or **CT scan**. In this set-up a tube sends out a pulse of X-rays and a set of sensitive detectors collects information about the levels of X-radiation reaching each detector. The X-ray source and the detectors are then rotated around a patient and the process is repeated. A computer can analyse the information recorded and is able to reconstruct a 3-dimensional 'map' of the inside of the body in terms of X-ray attenuation.

Ul rasonic imaging

ULTRASOUND

The limit of human hearing is about 20 kHz. Any sound that is of higher frequency than this is known as **ultrasound**. Typically ultrasound used in medical imaging is just within the MHz range. The velocity of sound through soft tissue is approximately 1500 m s^{-1} meaning that typical wavelengths used are of the order of a few millimetres.

Unlike X-rays, ultrasound is not ionizing so it can be used very safely for imaging inside the body – with pregnant women for example. The basic principle is to use a probe that is capable of emitting and receiving pulses of ultrasound. The ultrasound is reflected at any boundary between different types of tissue. The time taken for these reflections allows us to work out where the boundaries must be located.



ACOUSTIC IMPEDANCE

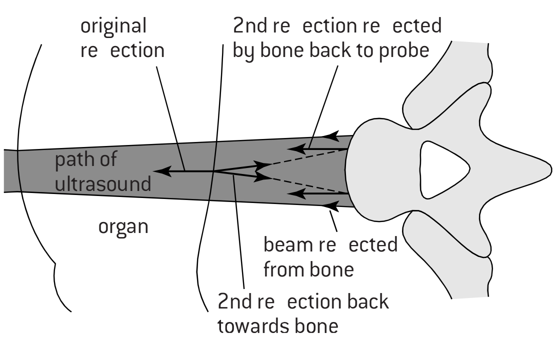
The acoustic impedance of a substance is the product of the density, ρ, and the speed of sound, c.

$Z = \rho c$

unit of Z = kg m^{-2} s^{-1}

Very strong reflections take place when the boundary is between two substances that have very different acoustic impedances. This can cause some difficulties.

- In order for the ultrasound to enter the body in the first place, there needs to be no air gap between the probe and the patient's skin. An air gap would cause almost all of the ultrasound to be reflected straight back. The transmission of ultrasound is achieved by putting a gel or oil (of similar density to the density of tissue) between the probe and the skin.
- Very dense objects (such as bones) can cause a strong reflection and multiple images can be created. These need to be recognized and eliminated.

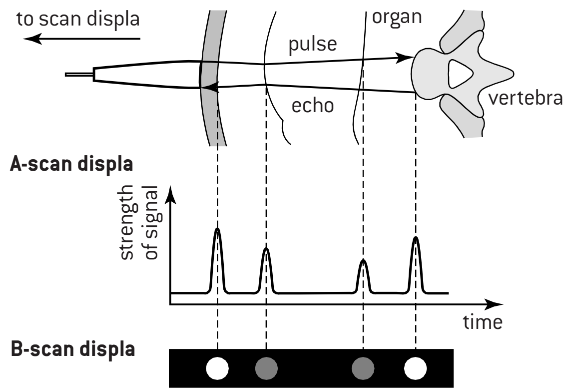


PIEZOELECTRIC CRYSTALS

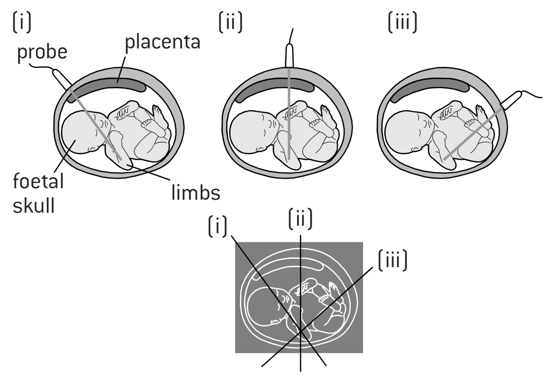
These quartz crystals change shape when an electric current flows and can be used with an alternating pd to generate ultrasound. They also generate pds when receiving sound pressure waves so one crystal is used for generation and detection.

A- AND B-SCANS

There are two ways of presenting the information gathered from an ultrasound probe, the **A-scan** or the **B-scan**. The A-scan (amplitude-modulated scan) presents the information as a graph of signal strength versus time. The B-scan (brightness-modulated scan) uses the signal strength to affect the brightness of a dot of light on a screen.



A-scans are useful where the arrangement of the internal organs is well known and a precise measurement of distance is required. If several B-scans are taken of the same section of the body at one time, all the lines can be assembled into an image which represent a section through the body. This process can be achieved using a large number of transducers.



Building a picture from a series of B-scan lines

CHOICE OF FREQUENCY

The choice of frequency of ultrasound to use can be seen as the choice between resolution and attenuation.

- Here, the resolution means the size of the smallest object that can be imaged. Since ultrasound is a wave motion, diffraction effects will be taking place. In order to image a small object, we must use a small wavelength. If this was the only factor to be considered, the frequency chosen would be as large as possible.
- Unfortunately attenuation increases as the frequency of ultrasound increases. If very high frequency ultrasound is used, it will all be absorbed and none will be reflected back. If this was the only factor to be considered, the frequency chosen would be as small as possible.

On balance the frequency chosen has to be somewhere between the two extremes. It turns out that the best choice of frequency is often such that the part of the body being imaged is about 200 wavelengths of ultrasound away from the probe.

Imaging continued

RELATIVE INTENSITY LEVELS OF ULTRASOUND

The relative intensity levels o ultrasound between two points are compared using the decibel scale (dB). As its name suggests, the decibel unit is simply one tenth o a base unit that is called the bel (B). The decibel scale is logarithmic.

Mathematically,

Relative intensity level in bels,

$$L_I = \log \frac{\text{intensity level o ultrasound at measurement point}}{\text{intensity level o ultrasound at re erence point}}$$

or Relative intensity level in bels $= \log \frac{I_1}{I_0}$

Since 1 bel = 10 dB,

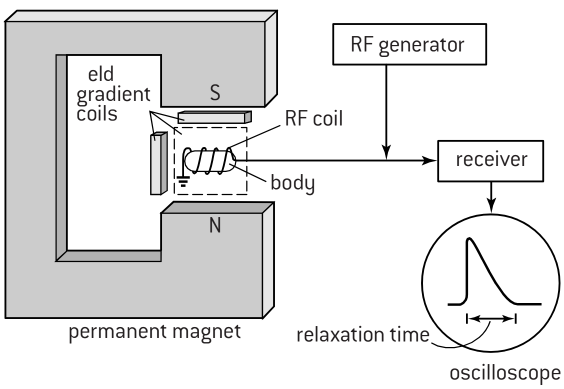
Relative intensity level in decibels, $L_I = 10 \log \frac{I_1}{I_0}$

NMR

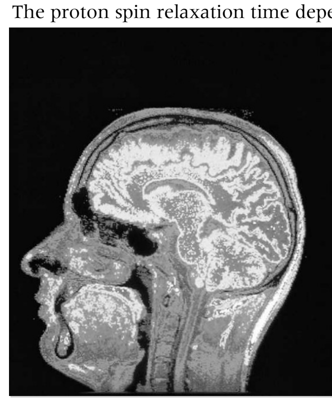
Nuclear Magnetic Resonance (NMR) is a very complicated process but one that is extremely use ul. It can provide detailed images o sections through the body without any invasive or dangerous techniques. It is o particular use in detecting tumours in the brain. It involves the use o a non-uni orm magnetic field in conjuction with a large uni orm field.

In outline, the process is as ollows:

- The nuclei o atoms have a property called spin.
- The spin o these nuclei means that they can act like tiny magnets.
- These nuclei will tend to line up in a strong magnetic field.



- They do not, however, per ectly line up – they oscillate in a particular way that is called **precession**. This happens at a very high requency – the same as the requency o radio waves.
- The particular requency o precession depends on the magnetic field and the particular nucleus involved. It is called the **Larmor frequency**.
- I a pulse o radio waves is applied at the Larmor requency, the nuclei can absorb this energy in a process called **resonance**. The protons make a spin transition.
- A ter the pulse, the nuclei return to their lower energy state by emitting radio waves.
- The time over which radio waves are emitted is called the **relaxation time**.
- The radio waves emitted and their relaxation times can be processed to produce the NMR scan image.
- The signal analysis is targeted at the hydrogen nuclei (protons) present.
- The number o H nuclei varies with the chemical composition so di erent tissues extract di erent amounts o energy rom the applied signal.
- Thus RF signal orces protons to make a spin transition and
  - ◊ The gradient field allows determination o the point rom which the photons are emitted.
  - ◊ The proton spin relaxation time depends on the type o …n is emitted.



COMPARISON BETWEEN ULTRASOUND AND NMR

The ollowing points can be noted:

- NMR imaging is very expensive when compared with ultrasound equipment and is very bulky – patient needs to be brought to the NMR machine and process is time consuming.
- Ultrasound measurements are easy to per orm (equipment can be brought to patient at the point o care) and can be repeated as required but quality o image can rely on skill o operator.
- NMR produces a 3-dimensional scan, ultrasound typically produces a 2-dimensional scan.
- Detail produced by NMR is greater than by ultrasound.
- NMR particularly use ul or very delicate areas o body e.g. brain.
- NMR patients have to remain very still, ultrasound images can be more dynamic.
- Ultrasound waves do not enter the body easily and multiple reflections can reduce the clarity o the image.
- Both wave energies carry energy but the energy associated with the ultrasound is greater that the energy associated with the radio requencies used in NMR.
- At the radio requencies used in NMR there is no danger o resonance but some ultrasound energy can cause heating.
- Ultrasound can cause cavitation – the production o small gas bubbles which will absorb energy and can damage surrounding tissue. The requencies and intensities used or diagnostics avoid this possibility as much as possible.
- The strong magnetic fields used in NMR present problems or patients with surgical implants and / or pacemakers.

IB Questions – option C – imaging

1. For each of the following situations, locate and describe the final image formed. Solutions should found using scale diagrams and mathematically.

   a) An object is placed 7 cm in front of a concave mirror of focal length 14 cm. [4]

   b) A diverging lens of focal length 12.0 cm is placed at the focal point of a converging lens of focal length 8.0 cm. An object is placed 16.0 cm in front of the converging lens. [4]

   c) An object is placed 18.0 cm in front of a convex lens of focal length 6.0 cm. A second convex lens of focal length 3.0 cm is an additional 18 cm behind the first lens. [4]

2. A student is given two converging lenses, A and B, and a tube in order to make a telescope.

   a) Describe a simple method by which she can determine the focal length of each lens. [2]

   b) She finds the focal lengths to be as follows:

      Focal length of lens A   10 cm

      Focal length of lens B   50 cm

      Draw a diagram to show how the lenses should be arranged in the tube in order to make a telescope. Your diagram should include:

      (i) labels for each lens;

      (ii) the focal points for each lens;

      (iii) the position of the eye when the telescope is in use. [4]

   c) On your diagram, mark the location of the intermediate image formed in the tube. [1]

   d) Is the image seen through the telescope upright or upside-down? [1]

   e) Approximately how long must the telescope tube be? [1]

3. Explain what is meant by

   a) Material dispersion

   b) Waveguide dispersion

   c) Spherical aberrations

   d) Chromatic aberrations

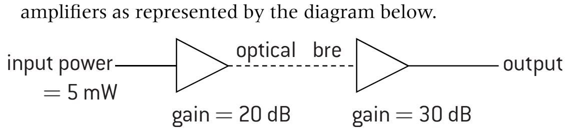
   e) A Cassegrain mounting

   f) Total Internal reflection

   g) Step-index fibres

   [2 each]

4. A 15 km length of optical fibre has an attenuation of 4 dB km^{-1}. A 5 mW signal is sent along the wire using two amplifiers as represented by the diagram below.



   Calculate

   a) the overall gain of the system

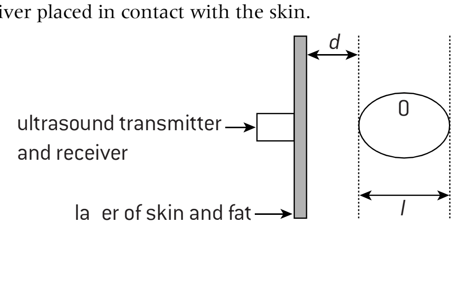
   b) the output power. [2]

HL

5. This question is about ultrasound scanning.

   a) State a typical value for the frequency of ultrasound used in medical scanning. [1]

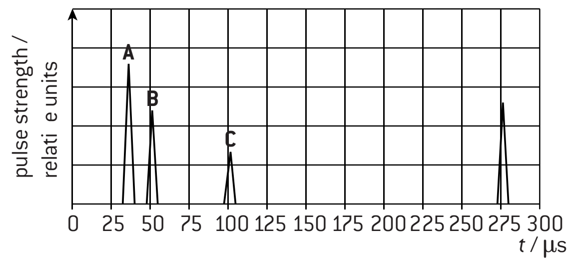
   The diagram below shows an ultrasound transmitter and receiver placed in contact with the skin.



   The purpose of this particular scan is to find the depth *d* of the organ labelled O below the skin and also to find its length, *l*.

   b) (i) Suggest why a layer of gel is applied between the ultrasound transmitter/receiver and the skin. [2]

   On the graph below the pulse strength of the reflected pulses is plotted against the time lapsed between the pulse being transmitted and the time that the pulse is received, *t*.



   (ii) Indicate on the diagram the origin of the reflected pulses A, B and C and D. [2]

   (iii) The mean speed in tissue and muscle of the ultrasound used in this scan is 1.5×10^3 ms^{-1}. Using data from the above graph, estimate the depth *d* of the organ beneath the skin and the length *l* of the organ O. [4]

   c) The above scan is known as an A-scan. State **one** way in which a B-scan differs from an A-scan. [1]

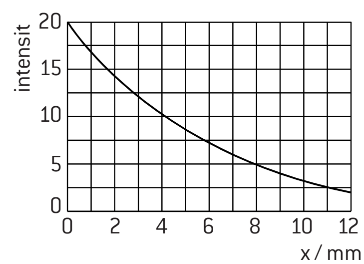
   d) State **one** advantage and **one** disadvantage of using ultrasound as opposed to using X-rays in medical diagnosis. [2]

6. a) State and explain which imaging technique is normally used

   (i) to detect a broken bone [2]

   (ii) to examine the growth of a fetus. [2]

   The graph below shows the variation of the intensity *I* of a parallel beam of X-rays after it has been transmitted through a thickness *x* of lead.



   b) (i) Define *half-value thickness*, $x_{\frac{1}{2}}$. [2]

   (ii) Use the graph to estimate $x_{\frac{1}{2}}$ for this beam in lead. [2]

   (iii) Determine the thickness of lead required to reduce the intensity transmitted to 20% of its initial value. [2]

   (iv) A second metal has a half-value thickness $x_{\frac{1}{2}}$ for this radiation of 8 mm. Calculate what thickness of this metal is required to reduce the intensity of the transmitted beam by 80%. [3]