11 ELECTROMAGNETIC INDUCTION

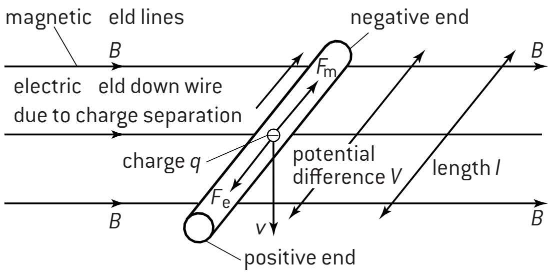
HL Induced electromotive force (emf)

INDUCED EMF

When a conductor moves through a magnetic field, an em is induced. The em induced depends on:

- The speed o the wire.
- The strength o the magnetic field.
- The length o the wire in the magnetic field.

We can calculate the magnitude o the induced em by considering an electron at equilibrium in the middle o the wire. The induced electric orce and the magnetic orce are balanced.



Electrical orce due to em , $F_e = E \times q = \left(\frac{V}{l}\right) \times q$

Magnetic orce due to movement, $F_m = Bqv$

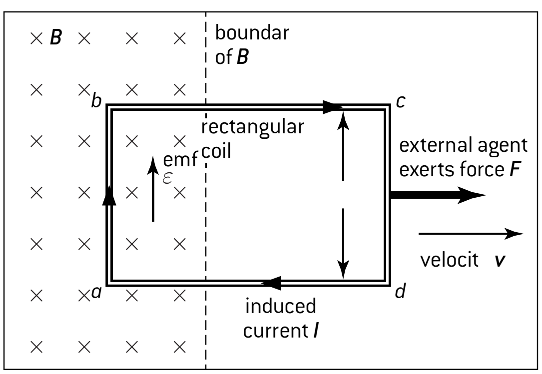
So $Bqv = \left(\frac{V}{l}\right) q$

$V = Blv$

As no current is flowing, the em ε = potential di erence

$\varepsilon = Blv$

I the wire was part o a complete circuit (outside the magnetic field), the em induced would cause a current to flow.



I this situation was repeated with a rectangular coil with N turns, each section ab would generate an em equal to Bvl. The total em generated will thus be

$\varepsilon = BvlN$

Note that in the situation above, a current only flows when one side o the coil (ab) is moving through the magnetic field and the other side (cd) is outside the field. I the whole coil was inside the magnetic field, each side would generate an em . The two em s would oppose one another and no current would flow.

PRODUCTION OF INDUCED EMF BY RELATIVE MOTION

An em is induced in a conductor whenever lines o magnetic flux are cut. But flux is more than just a way o picturing the situation; it has a mathematical definition.

I the magnetic field is perpendicular to the sur ace, the magnetic flux ϕ passing through the area A is defined in terms o the magnetic field strength B as ollows.

$\phi = B \quad A$, so $B = \frac{\phi}{A}$

In a uni orm field, $B = \frac{\phi}{A}$

An alternative name or 'magnetic field strength' is '**flux density**'.

I the area is not perpendicular, but at an angle θ to the field lines, the equation becomes

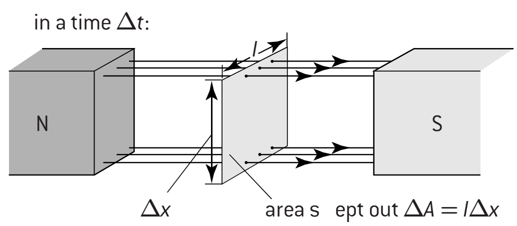
$\phi = BA\cos\theta$ (units: T m^2)

θ is the angle between **B** and the normal to the sur ace.

Flux can also be measured in webers (Wb), defined as ollows.

1 Wb = 1 T m^2

These relationships allow us to calculate the induced em ε in a moving wave is terms o flux.



$\varepsilon = Blv$ since $v = \frac{x}{t}$ then $\varepsilon = \frac{Bl \quad x}{t}$

but $l \quad x = \quad A$, the area 'swept out' by the conductor in a time t so $\varepsilon = \frac{B \quad A}{t}$

but $B \quad A = \quad \phi$ so $\varepsilon = \frac{\phi}{t}$

In words, 'the em induced is equal to the rate o cutting o flux'. I the conductor is kept stationary and the magnets are moved, the same e ect is produced.

EXAMPLE

An aeroplane flies at 200 m s^{-1}. Estimate the maximum pd that can be generated across its wings.

Vertical component o Earth's magnetic field = 10^{-5} T (approximately)

Length across wings = 30 m (estimated)

em = $10^{-5} \times 30 \times 200$

= 6×10^{-2} V

= 0.06 V

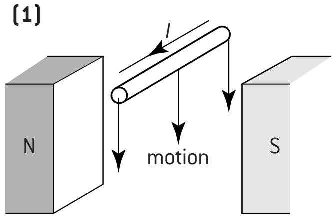
Lenz's law and Faraday's law

LENZ'S LAW

Lenz's law states that

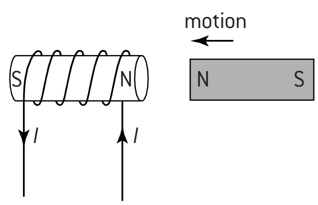
'The direction o the induced em is such that i an induced current were able to flow, it would oppose the change which caused it.'

(1)



Current induced in this direction, the force would be upwards (left-hand rule)
∴ original motion would be opposed.

(2)



If current were induced this way, the induced eld would repel the magnet — opposing motion.

Lenz's law can be explained in terms o the conservation o energy. The electrical energy generated within any system must result rom work being done on the system. When a conductor is moved through a magnetic field and an induced current flows, an external orce is needed to keep the conductor moving (the external orce balances the opposing orce that Lenz's law predicts). The external orce does work and this provides the energy or the current to flow.

Put another way, i the direction o an induced current did not oppose the change that caused it, then it would be acting to support the change. I this was the case, then a orce would be generated that urther accelerated the moving object which would generate an even greater em – electrical energy would be generated without work being done.

TRANSFORMER-INDUCED EMF

An em is also produced in a wire i the magnetic field changes with time.

I the amount o flux passing through one turn o a coil is ϕ, then the total **flux linkage** with all N turns o the coil is given by

Flux linkage $= N\phi$

The universal rule that applies to all situations involving induced em can now be stated as

'The magnitude o an induced em is proportional to the rate o change o flux linkage.'

This is known as **Faraday's law** $\varepsilon = N\dfrac{\Delta\phi}{\Delta t}$

Faraday's law and Lenz's law can be combined together in the ollowing mathematical statement or the em , ε, generated in a coil o N turns with a rate o change o flux through the coil o $\frac{\Delta\phi}{\Delta t}$:

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

The dependence on the rate o change o flux and the number o turns is Faraday's law and the negative sign (opposing the change) is Lenz's law.

APPLICATION OF FARADAY'S LAW TO MOVING AND ROTATING COILS

There are many situations involving magnetic fields with moving or rotating coils. To decide whether or not an em is generated and, i it is, to calculate its value, the ollowing procedure can be used:

- Choose the period o time, Δt, over which the motion o the coil is to be considered.
- At the beginning o the period, work out the flux passing through one turn o the coil, $\phi_{initial}$. Note that the shape o the coil is not relevant just the cross-sectional area.

  $\phi = BA\cos\theta$.
- At the end o the period, work out the flux passing through one turn o the coil ϕ_{final} using the equation above. Note that the sense o the magnetic field is important. I the magnitude o the field is the same but it is passing through the coil in the opposite direction, then

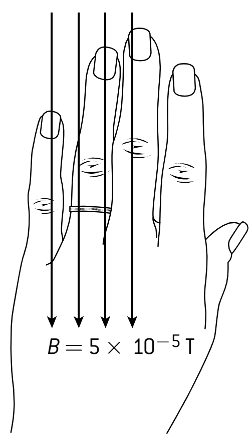
  $\phi_{final} = -\phi_{initial}$
- Determine the change in flux, $\Delta\phi$:

  $\Delta\phi = \phi_{final} - \phi_{initial}$
- I there is no overall change o flux then, overall, no em will be induced. I there is a change in flux then the em induced in a coil o N turns will be:

  $$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

Example:

A physicist holds her hand so that the magnetic field o the Earth (50 μT) passes through a ring on her hand.



In 0.1 s, she quickly turns her hand through 90° so that the magnetic field o the Earth no longer goes through the ring. Estimate the em generated in the ring.

Answer:

Estimate o cross-sectional area o ring, $A \approx 1\ \text{cm}^2 = 10^{-4}\ \text{m}^2$

$\phi_{initial} = 5 \times 10^{-5} \times 10^{-4}\cos(0) = 5 \times 10^{-9}$ Wb

$\phi_{final} = 0$

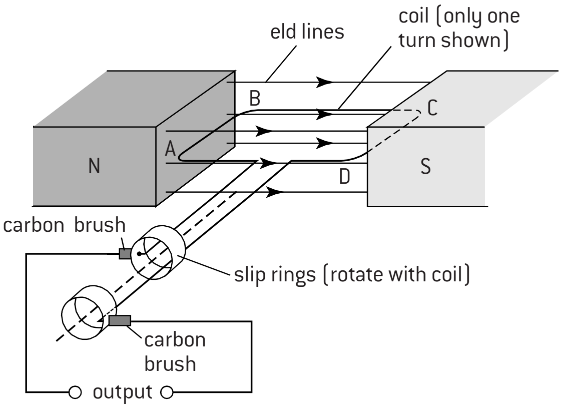
$\therefore\ \Delta\phi = 5 \times 10^{-9}$ Wb

magnitude o $\varepsilon = N\dfrac{\Delta\phi}{\Delta t} = \dfrac{5 \times 10^{-9}}{10^{-1}} = 5 \times 10^{-8}$ V

Alternating current (1)

COIL ROTATING IN A MAGNETIC FIELD – AC GENERATOR

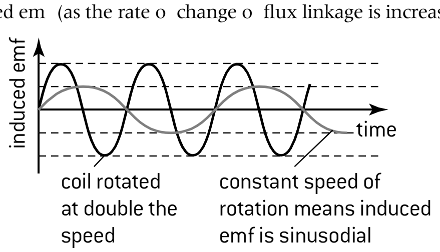
The structure o a typical ac generator is shown below.



ac generator

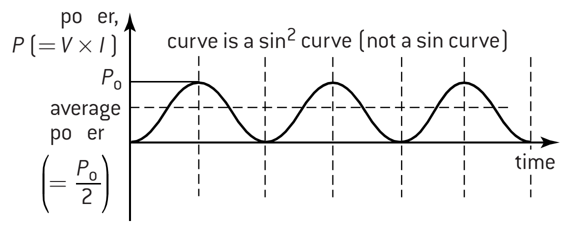
The coil o wire rotates in the magnetic field due to an external orce. As it rotates the flux linkage o the coil changes with time and induces an em (Faraday's law) causing a current to flow. The sides AB and CD o the coil experience a orce opposing the motion (Lenz's law). The work done rotating the coil generates electrical energy.

A coil rotating at constant speed will produce a sinusoidal induced em . Increasing the speed o rotation will reduce the time period o the oscillation a d increase the amplitude o the induced em (as the rate o change o flux linkage is increased).



RMS VALUES

I the output o an ac generator is connected to a resistor an alternating current will flow. A sinusoidal potential di erence means a sinusoidal current.



The graph shows that the average power dissipation is hal the peak power dissipation or a sinusoidal current.

$$\text{Average power } \overline{P} = \frac{I_0{}^2 R}{2} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R$$

Thus the e ective current through the resistor is √ (mean value o I^2) and it is called the **root mean square** current or **rms** current, I_{rms}.

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \text{ (or sinusoidal currents)}$$

When ac values or voltage or current are quoted, it is the root mean square value that is being used. In Europe this value is 230 V, whereas in the USA it is 120 V.

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$

$$\overline{P} = V_{\text{rms}} I_{\text{rms}} = \frac{1}{2} I_0 V_0$$

$$P_{\text{max}} = I_0 V_0$$

$$R = \frac{V}{I} = \frac{V_0}{I_0} = \frac{V_{\text{rms}}}{I_{\text{rms}}}$$

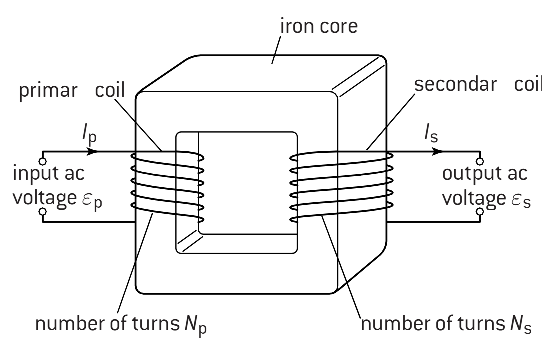
TRANSFORMER OPERATION

An **alternating** potential di erence is put into the trans ormer, and an **alternating** potential di erence is given out. The value o the output potential di erence can be changed (increased or decreased) by changing the **turns ratio**. A **step-up** trans ormer increases the voltage, whereas a **step-do n** trans ormer decreases the voltage.

The ollowing sequence o calculations provides the correct method or calculating all the relevant values.

- The output voltage is fixed by the input voltage and the turns ratio.
- The value o the load that you connect fixes the output current (using $V = I\,R$).
- The value o the output power is fixed by the values above ($P = V\,I$).
- The value o the input power is equal to the output power or an ideal trans ormer.
- The value o the input current can now be calculated (using $P = V\,I$).

So how does the trans ormer manage to alter the voltages in this way?



$$\frac{\varepsilon_p}{\varepsilon_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p}$$

Trans ormer structure

- The alternating pd across the primary creates an ac within the coil and hence an alternating magnetic field in the iron core.
- This alternating magnetic field links with the secondary and induces an em . The value o the induced em depends on the rate o change o flux linkage, which increases with increased number o turns on the secondary. The input and output voltages are related by the turns ratio.

Al erna ing curren (2)

TRANSMISSION OF ELECTRICAL POWER

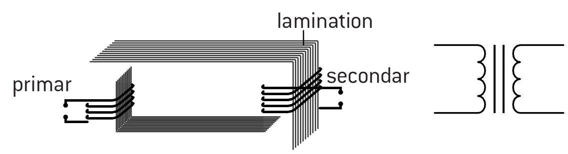
Transformers play a very important role in the safe and efficient transmission of electrical power over large distances.

- If large amounts of power are being distributed, then the currents used will be high. (Power = I)
- The wires cannot have zero resistance. This means they must dissipate some power
- Power dissipated is $P = I^2 R$. If the current is large then the (current)^2 will be very large.
- Over large distances, the power wasted would be very significant.
- The solution is to choose to transmit the power at a very high potential difference.
- Only a small current needs to flow.
- A very high potential difference is much more efficient, but very dangerous to the user.
- Use step-up transformers to increase the voltage for the transmission stage and then use step-down transformers for the protection of the end user.

LOSSES IN THE TRANSMISSION OF POWER

In addition to power losses associated with the resistance of the power supply lines, which cause the power lines to warm up, there are also losses associated with non-ideal transformers:

- **Resistance o the windings (joule heating)** of a transformer result in the transformer warming up.
- **Eddy currents** are unwanted currents induced in the iron core. The currents are reduced by **laminating** the core into individually electrically insulated thin strips.
- **Hysteresis** losses cause the iron core to warm up as a result of the continued cycle of changes to its magnetism.
- **Flux** losses are caused by magnetic 'leakage'. A transformer is only 100% efficient if all of the magnetic flux that is produced by the primary links with the secondary.

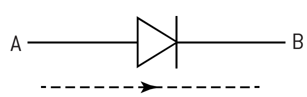


DIODE BRIDGES

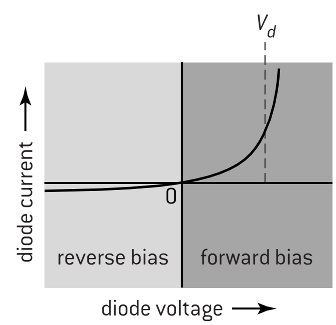
The efficient transmission of electrical power is best achieved using alternating current (ac) and transformers can ensure the appropriate V_{rms} is supplied. Many electrical devices are, however, designed to operate using direct current (dc). The conversion from ac into dc is called **rectification** which relies on diodes.

A **diode** is a two-terminal electrical device that has different electrical characteristics depending on which way around it is connected. An ideal diode allows current to flow in the forward direction (negligible resistance with forward bias) but does not allow current to flow in the reverse direction (infinite resistance with reverse bias).

Symbol:



Current is allowed to flow from A to B (A is positive and B is negative) but is prevented from following from B to A (A is negative and B is positive).



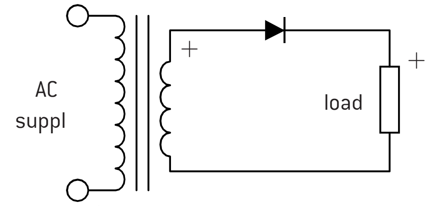
t pical diode characteristics

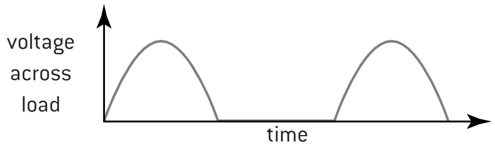
Rectification and smoothing circuits

RECTIFICATION

1. Hal -wave rectification

   A single diode will convert ac into a pulsating dc:

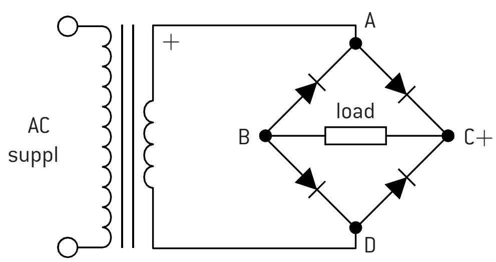


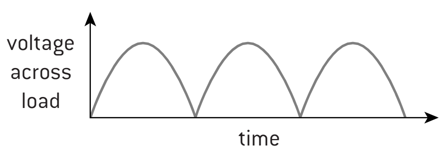


   In hal -wave rectification, electrical energy that is available in the negative cycle o the ac is not utilized.

2. Full-wave rectification

   A diode bridge (using our diodes) can utilize all the electrical energy that is available during a complete cycle as shown below.





   In the positive hal o the cycle, current flows through the diode bridge rom A→C→B→D.

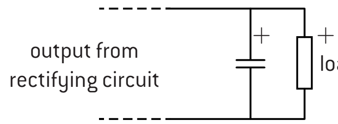
   In the negative hal o the cycle, current flows through the diode bridge rom D→C→B→A.

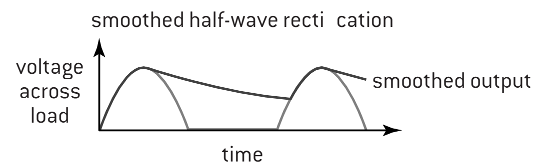
   Note that:

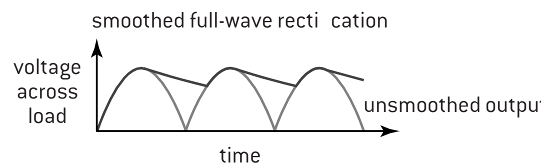
   - Current always flows through the load resistor in the same direction. (C→B)
   - Diodes on parallel sides point in the same directions.
   - The ac signal is ed to the points where opposite ends o two diodes join.
   - The positive output is taken rom the junction o the negative side o two diodes.
   - The negative output is taken rom the junction o the positive side o two diodes.
   - During each hal -cycle one set o parallel-side diodes conducts.

SMOOTHING CIRCUITS

Diode-bridge circuits provide a current that flows in one direction (dc) but still pulsates. In order to achieve a steady pd, a **smoothing** device is required. One possibility is a **capacitor** (see page 117 or more details).







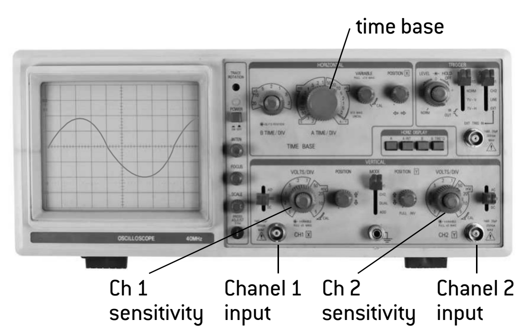
Note that:

- The output is still fluctuating slightly; this is known as the output **ripple**.
- The capacitor is acting as a short-term store o electrical energy.
- The capacitor is constantly charging and discharging.
- In order to ensure a slow discharge, the value o the capacitor *C* needs to be chosen to ensure that the time constant (see page 118) is su ficiently large.

INVESTIGATING A DIODE-BRIDGE RECTIFICATION CIRCUIT EXPERIMENTALLY

The display o the varying pd across the load is best achieved using a cathode ray oscilloscope (CRO).

The **y-input control**, allows the sensitivity o the CRO to appropriately display a changing pd on the y-axis. The **time-base** controls allows an appropriate calibration o the x-axis to match the time period o the oscillations.



time base set at 2.5 mS cm^{-1}

1 oscillation = 8 cm on screen = 20 mS

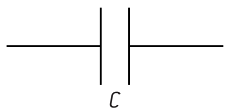
$\therefore$ requency $= \frac{1}{0.02} = 50$ Hz

HL Capaci ance

CAPACITANCE

Capacitors are devices that can store charge. The charge stored is proportional to the pd across the capacitor *V* and the constant of proportionality is called the capacitance *C*.

Symbol:



charge in coulombs

$C = \frac{}{V}$

capacitance in farads

pd in volts

The farad (F) is a very large unit and practical capacitances are measured in μF, nF or pF.

1 F = 1 C V^{-1}

A measurement of the pd across a capacitance allows the charge stored to be calculated.

The capacitance of a parallel plate capacitor depends on three different factors:

- The area of each plate, *A*. Each plate is assumed to have the same area *A* and the plates overlap one another completely.
- The separation of the plates, *d*
- The material between the plates which is called the **dielectric material**. Different materials will have different values of a constant called its **permittivity**, ε. The permittivity of air is effectively the same as the permittivity of a vacuum (free space), $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}. The permittivity of all substances is greater than ε_0.

The relationship is:

$$C = \frac{\varepsilon A}{d}$$

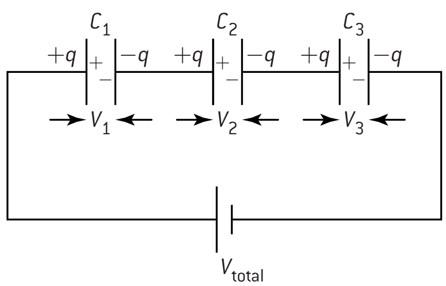
when a dielectric material is introduced, change separation across the dielectric is induced. This increases the capacitance.

CAPACITORS IN SERIES AND PARALLEL

The effective total capacitance, C_{total}, of the combination of capacitors (C_1, C_2, C_3, etc.) in a circuit depends on whether the capacitors are joined together in series or in parallel. The capacitor equation can be used on individual capacitors or on the combination.

$$C_{total} = \frac{_{total}}{V_{total}} \text{ and } C_1 = \frac{_1}{V_1},\ C_2 = \frac{_2}{V_2}, \text{ etc.}$$

1. In series



The charge stored in each capacitor is the same, and the pds across the individual capacitors add together to give the total pd

$$_{total} = {}_1 = {}_2 = {}_3 =$$

$$V_{total} = V_1 + V_2 + V_3$$

$$\therefore \frac{_{total}}{C_{total}} = \frac{_1}{C_1} + \frac{_2}{C_2} + \frac{_3}{C_3}$$

$$\therefore \frac{}{C_{series}} = \frac{}{C_1} + \frac{}{C_2} + \frac{}{C_3}$$

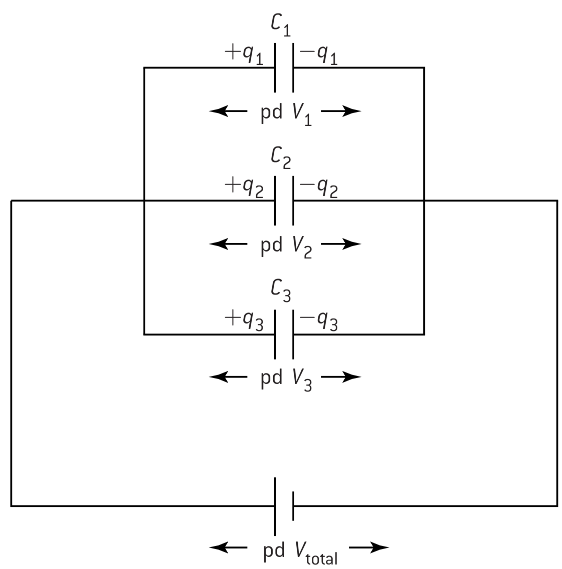
$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} +$$

e.g. if three capacitors 5 F, 10 F and 20 F are added in series, the combined capacitance is:

$$\frac{1}{C_{series}} = \frac{1}{5} + \frac{1}{10} + \frac{1}{20} = \frac{7}{20} \quad F^{-1}$$

$$\therefore C_{series} = \frac{20}{7} = 2.86 \quad F$$

2. In parallel



The pd across each capacitor is the same, *V* and the charges stored in each of the individual capacitors add together to give the total charge stored.

$$V_{total} = V_1 = V_2 = V_3 = V$$

$$_{total} = {}_1 + {}_2 + {}_3$$

$$\therefore C_{total}\, V_{total} = C_1 V_1 + C_2 V_2 + C_3 V_3$$

$$\therefore C_{parallel}\, V = C_1 V + C_2 V + C_3 V$$

$$\therefore C_{parallel} = C_1 + C_2 +$$

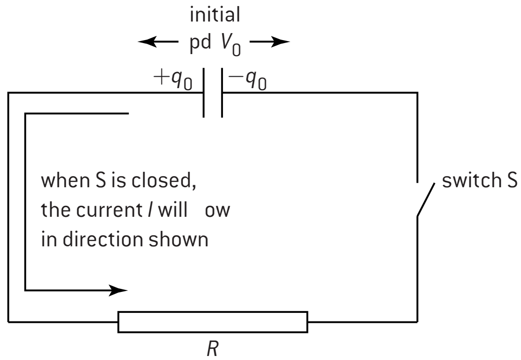
e.g. if three capacitors 5 F, 10 F and 20 F are added in parallel, the combined capacitance is:

$$C_{parallel} = 5 + 10 + 20 = 35 \quad F$$

Capaci or discharge

CAPACITOR (RC) DISCHARGE CIRCUITS

I the two ends o a charged capacitor are joined together with a resistor, a current will flow until the capacitor is discharged.



During the discharge process:

- the value o the discharge current, I, drops rom an initial maximum I_0 down to zero
- the value o the stored charge, q, drops rom an initial maximum q_0 down to zero
- the value o the pd across the capacitor (which is also the pd across the resistor), V, drops rom an initial maximum V_0 down to zero.

Applying Kircho 's law around the loop gives

$$0 = IR + \frac{q}{C}$$

Since I is the rate o flow o charge, $\frac{dq}{dt}$,

$$0 = R\frac{dq}{dt} + \frac{q}{C}$$

$$\frac{dq}{dt} = -\frac{q}{RC}$$

This has the rate o flow o charge proportional to the charge stored. The solution is an exponential decrease o charge stored given by:

$$q = q_0\, e^{-\frac{t}{RC}}$$

time (s)
capacitance (F)
resistance (Ω)
charge remaining
original charge

The product o RC is called the **time constant** or the circuit and is given the symbol τ (the Greek letter tau).

$$\tau = RC$$

The SI unit or τ will be seconds (NB: care needed with SI multipliers).

$$q = q_0\, e^{-\frac{t}{\tau}}$$

Since the current I and the pd V are both proportional to the charge, the ollowing equations also apply:

$$I = I_0\, e^{-\frac{t}{\tau}}$$

$$V = V_0\, e^{-\frac{t}{\tau}}$$

Where

$$I_0 = \frac{q_0}{RC} = \frac{V_0}{R}$$

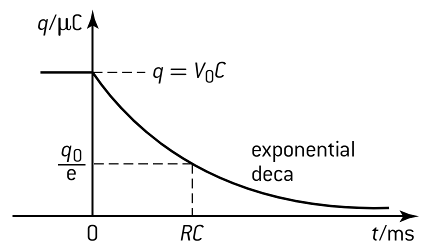
Example

A 10 μF capacitor is discharged through a 20 kΩ resistor. Calculate (a) the time constant τ or the circuit and (b) the raction o charge remaining a ter one time constant

a) $\tau = RC = 10\ \mu\text{F} \times 20\ \text{k}\Omega = 200$ ms

b) A ter one time constant,

$$q = q_0 e^{-1} = 0.37 q_0$$



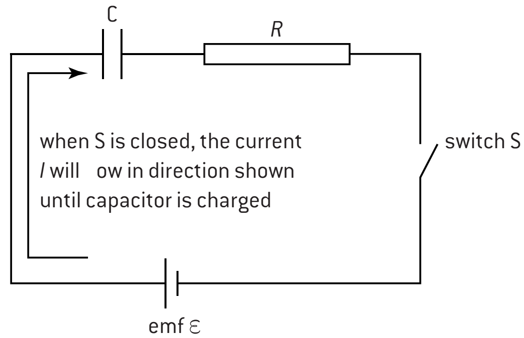
time	charge
0	100%
1RC	37%
2RC	14%
3RC	5%
4RC	2%
5RC	<1%

A ter 5 time constents, the capacitor is e ectively discharged

Capaci or charge

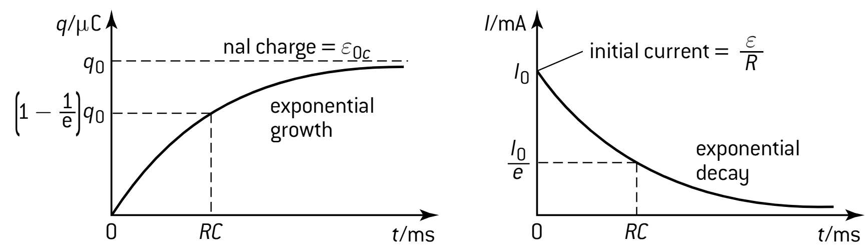
CAPACITOR CHARGING CIRCUITS

I the two ends o an uncharged capacitor are joined together with a resistor, a current will flow until the capacitor is charged.



During the charging process:,

- the value o the charging current, I, drops rom an initial maximum I_0 down to zero
- the value o the stored charge, q, increases rom zero up to a final maximum value, q_0
- the value o the pd across the capacitor, V, increases rom zero up to a final maximum value, ε
- the value o the pd across the resistor drops rom an initial maximum ε down to zero.

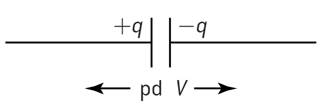


The equation or the increase o charge on the capacitor (which does not need to be memorized) is:

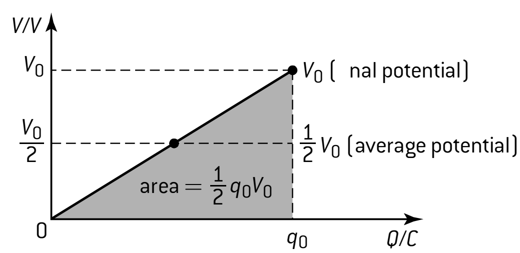
$q = q_0 \left(1 - e^{-\frac{t}{\tau}}\right)$

ENERGY STORED IN A CHARGED CAPACITOR

A charged capacitor can provide a temporary store o electrical energy when there is a potential di erence V across the capacitor. The charge, q, that is stored is distributed with $+q$ on one plate and $-q$ on the other plate as shown below. There is an electric field between the plates.



In the charging process, as more charge is added to the capacitor, the pd across it also increases proportionally. The graph (right) shows how the pd across the capacitor varies with charge stored in the capacitor during the charging process. The total energy stored, E, is represented by the area under the graph.



$$E = \frac{1}{2} qV = \frac{1}{2} \frac{q^2}{C} = \frac{1}{2} CV^2$$

Note that both charging and discharging are exponential processes. I a circuit is arranged in which a capacitor spends equal time charging and discharging through the same value resistor, then in one complete cycle, more charge will be added to the capacitor during the charging time than it loses during the discharging time. The result over several cycles will be or the capacitor to charge up to the same pd as the power supply.

IB Quest ons – electromagnet c nduct on

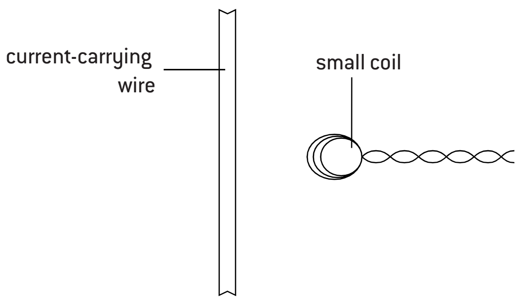
1. The **primary** o an ideal trans ormer has 1000 turns and the **secondary** 100 turns. The current in the primary is 2 A and the input power is 12 W.

   Which **one** o the ollowing about the **secondary current** and the **secondary power output** is true?

	secondary current	secondary power output
A.	20 A	1.2 W
B.	0.2 A	12 W
C.	0.2 A	120 W
D.	20 A	12 W

2. This question is about electromagnetic induction.

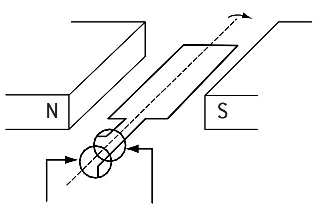
   A small coil is placed with its plane parallel to a long straight current-carrying wire, as shown below.



   a) (i) State Faraday's law o electromagnetic induction. [2]

   (ii) Use the law to explain why, when the current in the wire changes, an em is induced in the coil. [1]

3. The diagram shows a simple generator with the coil rotating between magnetic poles. Electrical contact is maintained through two brushes, each touching a slip ring.



   At the instant when the rotating coil is oriented as shown, the voltage across the brushes

   A. is zero.

   B. has its maximum value.

   C. has the same constant value as in all other orientations.

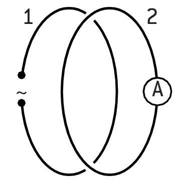
   D. reverses direction.

4. The rms current rating o an electric heater is 4A. What direct current would produce the same power dissipation in the electric heater? [2]

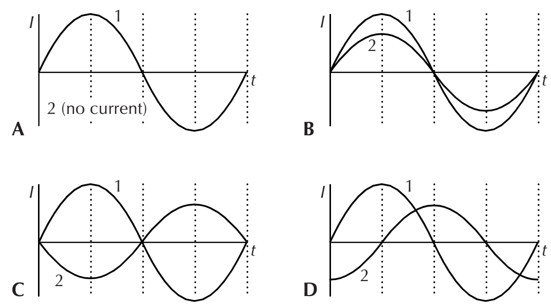
   A. $\frac{4}{\sqrt{2}}$A B. 4A

   C. $4\sqrt{2}$A D. 8A

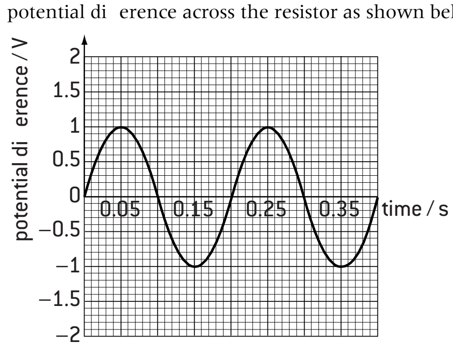
5. Two loops o wire are next to each other as shown here. There is a source o alternating em connected to loop 1 and an ammeter in loop 2.



   The variation with time o the current in loop 1 is shown as line 1 in each o the graphs below. In which graph does line 2 best represent the current in loop 2?



6. A loop o wire o negligible resistance is rotated in a magnetic field. A 4 Ω resistor is connected across its ends. A cathode ray oscilloscope measures the varying induced potential di erence across the resistor as shown below.



   a) I the coil is rotated at twice the speed, show on the axes above how potential di erence would vary with time. [2]

   b) What is the rms value o the induced potential di erence, V_{rms}, at the **original** speed o rotation? [1]

   c) Draw a graph showing how the power dissipated in the resistor varies with time, at the **original** speed o rotation. [3]

7. a) A 3μF capacitor is charged to 240 V. Calculate the charge stored. [1]

   b) Estimate the amount o time it would take or the charge you have calculated in (**a**) to flow through a 60 W light bulb connected to the 240 V mains electricity. [2]

   c) The charged capacitor in (**a**) is discharged through a 60 W 240 V light bulb.

   (i) Explain why the current during its discharge will not be constant. [2]

   (ii) Estimate the time taken or the capacitor to discharge through the light bulb. [2]

   (iii) Will the bulb light during discharge? Explain your answer. [2]