1.3 Vectors and scalars

Understanding

- Vector and scalar quantities
- Combination and resolution of vectors

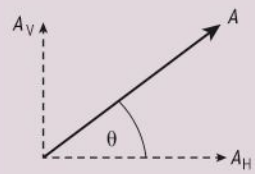
Applications and skills

- Solving vector problems graphically and algebraically.

Equations

The horizontal and vertical components of vector A:

- $A_H = A \cos\theta$
- $A_V = A \sin\theta$



Nature of science

All physical quantities that you will meet on the course are classified as being vectors or scalars. It is important to know whether any quantity is a vector or a scalar since this will affect how the quantity is treated mathematically. Although the concept of adding forces is an intuitive application of vectors that has probably been used by sailors for millennia, the analytical aspect of it is a recent development. In the *Philosophiæ Naturalis Principia Mathematica*, published in 1687, Newton used quantities which we now call vectors, but never generalized this to deal with the concepts of vectors. At the start of the 19th century vectors became an indispensible tool for representing three-dimensional space and complex numbers. Vectors are now used as a matter of course by physicists and mathematicians alike.

Vector and scalar quantities

Scalar quantities are those that have magnitude (or *size*) but no direction. We treat scalar quantities as numbers (albeit with units) and use the rules of algebra when dealing with them. Distance and time are both scalars, as is speed. The average speed is simply the distance divided by the time, so if you travel 80 m in 10 s the speed will always be $8\,\text{m}\,\text{s}^{-1}$. There are no surprises.

Vector quantities are those which have both magnitude and direction. We must use vector algebra when dealing with vectors since we must take into account direction. The vector equivalent of distance is called displacement (i.e., it is a distance in a specified direction). The vector equivalent of speed is velocity (i.e., it is the speed in a specified direction). Time, as we have seen, is a scalar. Average velocity is defined as being displacement divided by time.

Dividing a vector by a scalar is the easiest operation that we need to do involving a vector. To continue with the example that we looked at with scalars, suppose the displacement was 80 m due north and the time was, again, 10 s. The average velocity would be 8 $m s^{-1}$ due north. So, to generalize, when we divide a vector by a scalar we end up with a new vector that has the direction of the original one, but which will be of magnitude equal to that of the vector divided by that of the scalar.

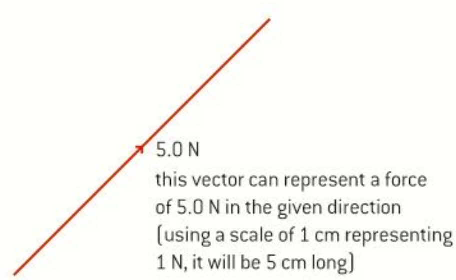
Commonly used vectors and scalars		
Vectors	**Scalars**	**Comments**
force (F)	mass (m)	$\vec{F}$
displacement (s)	length/distance (s, d, etc.)	displacement used to be called "space" – now that means something else!
velocity (v or u)	time (t)	
momentum (p)	volume (V)	
acceleration (a)	temperature (T)	
gravitational field strength (g)	speed (v or u)	velocity and speed often have the same symbol
electric field strength (E)	density (ρ)	the symbol for density is the Greek "rho" not the letter "p"
magnetic field strength (B)	pressure (p)	
area (A)	energy/work (W, etc.)	the direction of an area is taken as being at right angles to the surface
	power (P)	
	current (I)	with current having direction you might think that it should be a vector but it is not (it is the ratio of two scalars, charge and time, so it cannot be a vector). In more advanced work you might come across *current density* which is a vector.
	resistance (R)	
	gravitational potential (V_G)	the subscripts tell us whether it is gravitational or electrical
	electric potential (V_E)	
	magnetic flux (Φ)	flux is often thought as having a direction – it doesn't!

Representing vector quantities

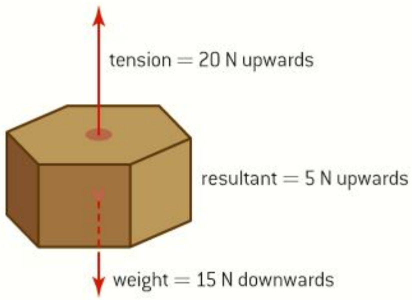
A vector quantity is represented by a line with an arrow.

- The direction the arrow points represents the direction of the vector.
- The length of the line represents the magnitude of the vector to a chosen scale.

When we are dealing with vectors that act in one dimension it is a simple matter to assign one direction as being positive and the opposite direction as being negative. Which direction is positive and which negative really doesn't matter as long as you are consistent. So, if one force acts upwards on an object and another force acts downwards, it is a simple matter to find the resultant by subtracting one from the other.



▲ Figure 1 Representing a vector.

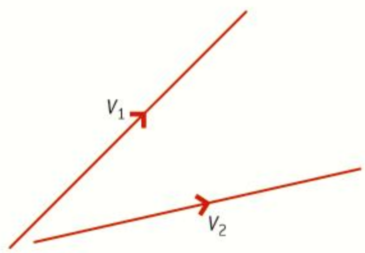


▲ Figure 2 Two vectors acting on an object.

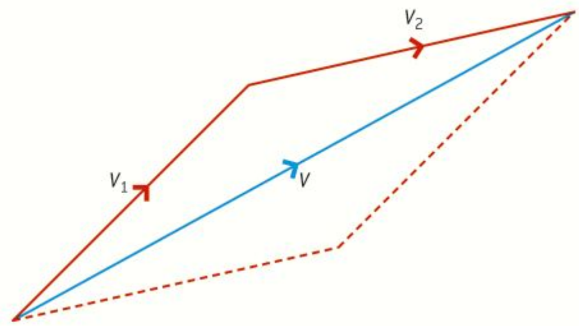
Figure 2 shows an upward tension and downward weight acting on an object – the upward line is longer than the downward line since the object is not in equilibrium and has an upward resultant.

Adding and subtracting vectors

When adding and subtracting vectors, account has to be taken of their direction. This can be done either by a scale drawing (graphically) or algebraically.



▲ Figure 3 Two vectors to be added.



▲ Figure 4 Adding the vectors.

Scale drawing (graphical) approach

Adding two vectors V_1 and V_2 which are not in the same direction can be done by forming a parallelogram to scale.

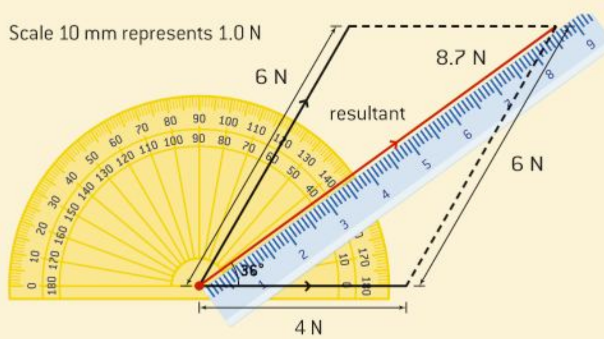
- Make a rough sketch of how the vectors are going to add together to give you an idea of how large your scale needs to be in order to fill the space available to you. This is a good idea when you are adding the vectors mathematically too.
- Having chosen a suitable scale, draw the scaled lines in the direction of V_1 and V_2 (so that they form two adjacent sides of the parallelogram).
- Complete the parallelogram by drawing in the remaining two sides.
- The blue diagonal represents the **resultant vector** in both magnitude and direction.

Worked example

Two forces of magnitude 4.0 N and 6.0 N act on a single point. The forces make an angle of 60° with each other. Using a scale diagram, determine the resultant force.

Solution

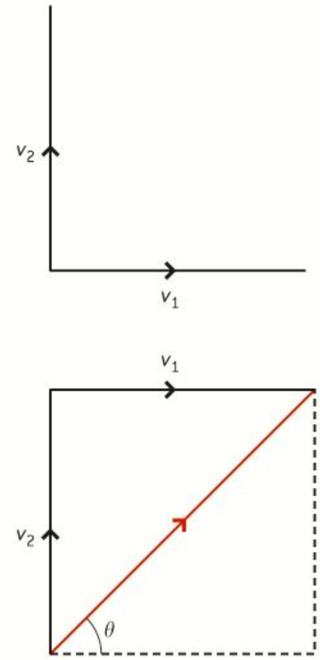
Don't forget that the vector must have a magnitude and a direction; this means that the angle is just as important as the size of the force.



length of resultant = 87 mm so the force = 8.7 N
angle resultant makes with 4 N force = 36°

Algebraic approach

Vectors can act at any angle to each other but the most common situation that you are going to deal with is when they are at right angles to each other. We will deal with this first.



▲ Figure 5 Adding two perpendicular vectors.

Adding vector quantities at right angles

Pythagoras' theorem can be used to calculate a resultant vector when two perpendicular vectors are added (or subtracted). Assuming that the two vector quantities are horizontal and vertical *but the principle is the same as long as they are perpendicular*.

Figure 5 shows two perpendicular velocities v_1 and v_2; they form a parallelogram that is a rectangle.

The magnitude of the resultant velocity $= \sqrt{v_1^2 + v_2^2}$

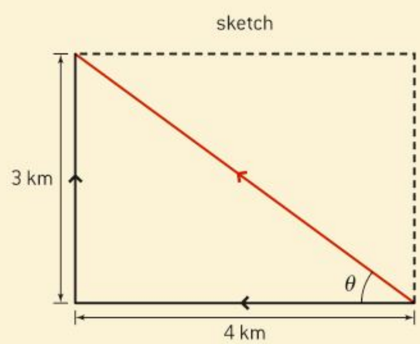
- The resultant velocity makes an angle θ to the horizontal given by

  $$\tan\theta = \left(\frac{v_1}{v_2}\right) \text{ so that } \theta = \tan^{-1}\left(\frac{v_1}{v_2}\right)$$

- Notice that the order of adding the two vectors makes no difference to the length or the direction of the resultant.

Worked example

A walker walks 4.0 km due west from his starting point. He then stops before walking 3.0 km due north. At the end of his journey, how far is the walker from his starting point?



Solution

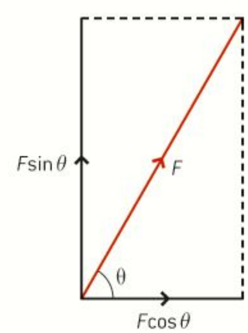
resultant $= \sqrt{4^2 + 3^2} = 5$ km

angle $\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.9°$

Before we look at adding vectors that are not perpendicular, we need to see how to resolve a vector – i.e. split it into two components.

Resolving vectors

We have seen that adding two vectors together produces a **resultant vector**. It is sensible, therefore, to imagine that we could split the resultant into the two vectors from which it was formed. In fact this is true for any vector – it can be divided into components which, added together, make the resultant vector. There is no limit to the number of vectors that can be added together and, consequently, there is no limit to the number of components that a vector can be divided into. However, we most commonly divide a vector into two components that are perpendicular to one another. The reason for doing this is that perpendicular vectors have no affect on each other as we will see when we look at projectiles in Topic 2.



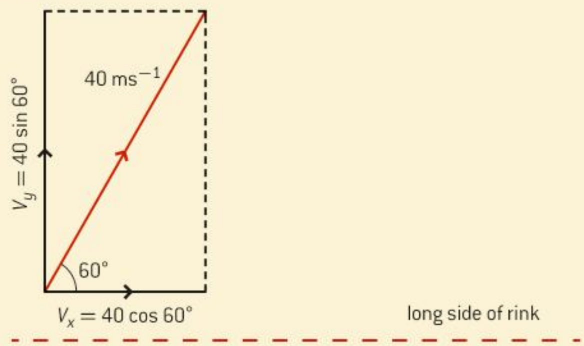
▲ Figure 6 Resolving a force.

The force F in figure 6 has been resolved into the horizontal component equal to $F\cos\theta$ and a vertical component equal to $F\sin\theta$. (The component opposite to the angle used is always the sine component.)

Worked example

An ice-hockey puck is struck at a constant speed of 40 m s^{-1} at an angle of 60° to the longer side of an ice rink. How far will the puck have travelled in directions **a)** parallel and **b)** perpendicular to the long side after 0.5 s?

Solution



a) Resolving parallel to longer side:

$v_x = v \cos 60°$

$v_x = 40 \cos 60° = 20$ m s^{-1}

distance travelled $(x) = v_x t = 20 \times 0.5$
$= 10$ m

b) Resolving parallel to shorter side:

$v_x = v \sin 60°$

$v_y = 40 \cos 60° = 34.6$ m s^{-1}

distance travelled $(y) = v_y t = 34.6 \times 0.5$
$= 17$ m

Just to demonstrate that resolving is the reverse of adding the components we can use Pythagoras' theorem to add together our two components giving:

total speed $= \sqrt{20^2 + 34.6^2} = 39.96$ m s^{-1}

as the value for v_y was rounded this gives the expected 40 m s^{-1}

Adding vector quantities that are not at right angles

You are now in a position to add any vectors.

- Resolve each of the vectors in two directions at right angles – this will often be horizontally and vertically, but may be parallel and perpendicular to a surface.
- Add all the components in one direction to give a single component.
- Add all the components in the perpendicular direction to give a second single component.
- Combine the two components using Pythagoras' theorem, as for two vector quantities at right angles.

V_1 and V_2 are the vectors to be added.

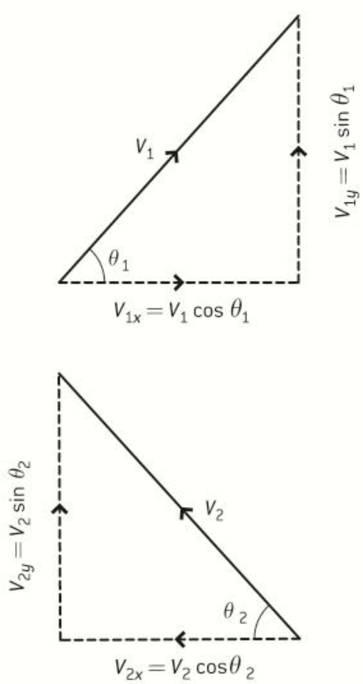
Each vector is resolved into components in the x and y directions. Note that since the x component of V_2 is to the left it is treated as being negative (the y component of each vector is in the same direction ... so upwards is treated as positive).

Total x component $V_x = V_{1x} + V_{2x} = V_1 \cos \theta_1 - V_2 \cos \theta_2$

Total y component $V_y = V_{1y} + V_{2y} = V_1 \sin \theta_1 + V_2 \sin \theta_2$

Having calculated V_x and V_y we can find the resultant by using Pythagoras so $V = \sqrt{V_x^2 + V_y^2}$

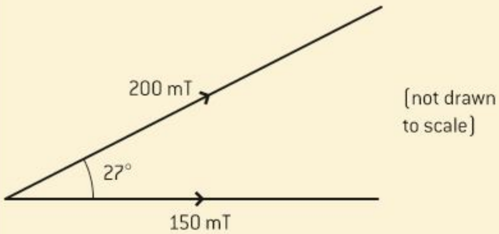
and the angle θ made with the horizontal $= \tan^{-1}\left(\frac{V_y}{V_x}\right)$.



▲ Figure 7 Finding the resultant of two vectors that are not perpendicular.

Worked example

Magnetic fields have strength 200 mT and 150 mT respectively. The fields act at 27° to one another as shown in the diagram.



Calculate the resultant magnetic field strength.

Solution

The 150 mT field is horizontal and so has no vertical component.

Vertical component of the 200 mT field = 200 sin 27° = 90.8 mT

This makes the total vertical component of the resultant field.

Horizontal component of the 200 mT field = 200 cos 27° = 178.2 mT

Total horizontal component of resultant field = (150.0 + 178.2) mT = 328.2 mT

Resultant field strength $\sqrt{90.8^2 + 328.2^2} = 340$ mT

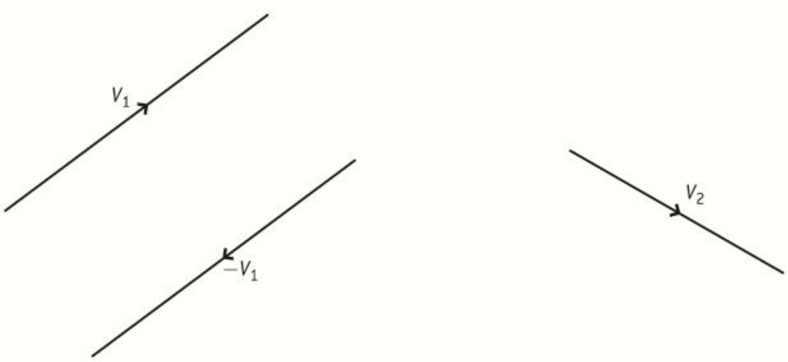
The resultant field makes an angle of:

$\tan^{-1}\left(\frac{90.8}{328.2}\right) = 15°$ with the150 mT field.

Subtraction of vectors

Subtracting one vector from another is very simple – you just form the negative of the vector to be subtracted and add this to the other vector. The negative of a vector has the same magnitude but the opposite direction. Let's look at an example to see how this works:

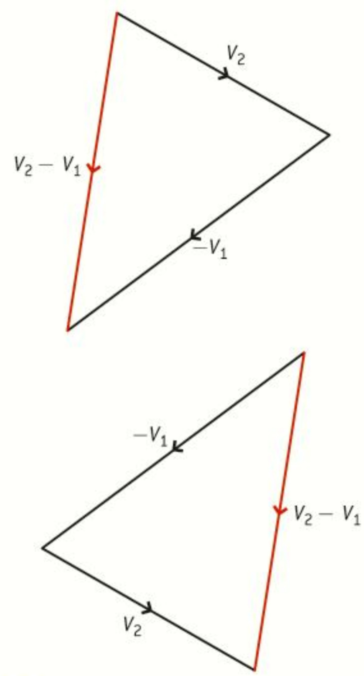
Suppose we wish to find the difference between two velocities v_1 and v_2 shown in figure 8.



▲ Figure 8 Positive and negative vectors.

In finding the difference between two values we subtract the first value from the second; so we need $-v_1$. We then add $-v_1$ to v_2 as shown in figure 9 to give the red resultant.

The order of combining the two vectors doesn't matter as can be seen from the two versions in figure 9. In each case the resultant is the same – it doesn't matter where the resultant is positioned as long as it has the same length and direction it is the same vector.



▲ Figure 9 Subtracting vectors.

Questions

1 Express the following units in terms of the SI fundamental units.

a) newton (N)
b) watt (W)
c) pascal (Pa)
d) coulomb (C)
e) volt (V)

(5 marks)

2 Express the following numbers to three significant figures.

a) 257.52
b) 0.002 347
c) 0.1783
d) 7873
e) 1.997

(5 marks)

3 Complete the following calculations and express your answers to the most appropriate number of significant figures.

a) 1.34×3.2
b) $\frac{1.34 \times 10^2}{2.1 \times 10^3}$
c) $1.87 \times 10^2 + 1.97 \times 10^3$
d) $(1.97 \times 10^5) \times (1.0 \times 10^4)$
e) $(9.47 \times 10^{-2}) \times (4.0 \times 10^3)$

(5 marks)

4 Use the appropriate metric multiplier instead of a power of ten in the following.

a) 1.1×10^4 V
b) 4.22×10^{-4} m
c) 8.5×10^{10} W
d) 4.22×10^{-7} m
e) 3.5×10^{-13} C

(5 marks)

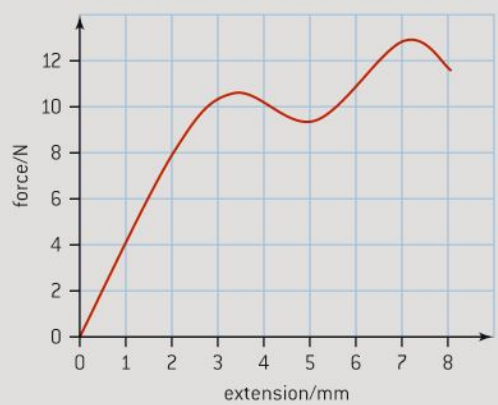
5 Write down the order of magnitude of the following (you may need to do some research).

a) the length of a human foot
b) the mass of a fly
c) the charge on a proton
d) the age of the universe
e) the speed of electromagnetic waves in a vacuum

(5 marks)

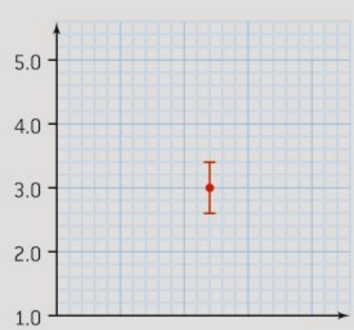
6 **a)** Without using a calculator estimate to one significant figure the value of $\frac{2\pi 4.9}{480}$.

b) When a wire is stretched, the area under the line of a graph of force against extension of the wire gives the elastic potential energy stored in the wire. Estimate the energy stored in the wire with the following characteristic:



(4 marks)

7 The grid below shows one data point and its associated error bar on a graph. The x-axis is not shown. State the y-value of the data point together with its absolute and percentage uncertainty.



(3 marks)

8 A ball falls freely from rest with an acceleration g. The variation with time t of its displacement s is given by $s = \frac{1}{2}gt^2$. The percentage uncertainty in the value of t is $\pm 3\%$ and that in the value of g is $\pm 2\%$. Calculate the percentage uncertainty in the value of s.

(2 marks)

9 The volume V of a cylinder of height h and radius r is given by the expression $V = \pi r^2 h$. In a particular experiment, r is to be determined from measurements of V and h. The percentage uncertainty in V is $\pm 5\%$ and that in h is $\pm 2\%$. Calculate the percentage uncertainty in r.

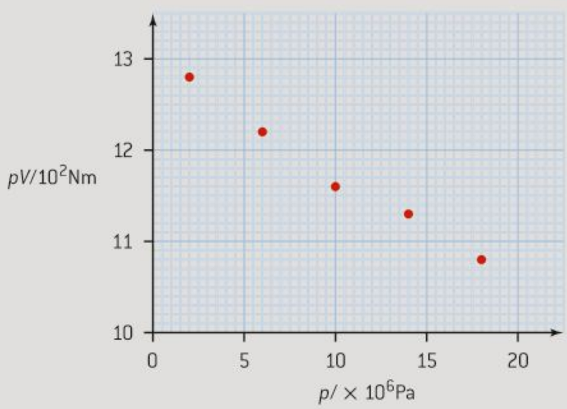
(3 marks)

10 (*IB*)

At high pressures, a real gas does not behave as an ideal gas. For a certain range of pressures, it is suggested that for one mole of a real gas at constant temperature the relation between the pressure p and volume V is given by the equation

$pV = A + Bp$ where A and B are constants.

In an experiment, 1 mole of nitrogen gas was compressed at a constant temperature of 150 K. The volume V of the gas was measured for different values of the pressure p. A graph of the product pV against p is shown in the diagram below.



a) Copy the graph and draw a line of best fit for the data points.

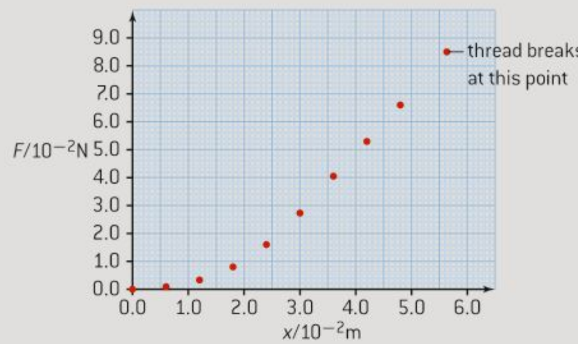
b) Use your graph to determine the values of the constants A and B in the equation $pV = A + Bp$

c) p was measured to an accuracy of 5% and V was measured to an accuracy of 2%. Determine the absolute error in the value of the constant A.

(6 marks)

11 (*IB*)

An experiment was carried out to measure the extension x of a thread of a spider's web when a load F is applied to it.



a) Copy the graph and draw a best-fit line for the data points.

b) The relationship between F and x is of the form

$F = kx^n$

State and explain the graph you would plot in order to determine the value n.

c) When a load is applied to a material, it is said to be *under stress*. The magnitude p of the stress is given by

$$p = \frac{F}{A}$$

where A is the cross-sectional area of the sample of the material.

Use the graph and the data below to deduce that the thread used in the experiment has a greater breaking stress than steel.

Breaking stress of steel $= 1.0 \times 10^9$ N m^{-2}

Radius of spider web thread $= 4.5 \times 10^{-6}$ m

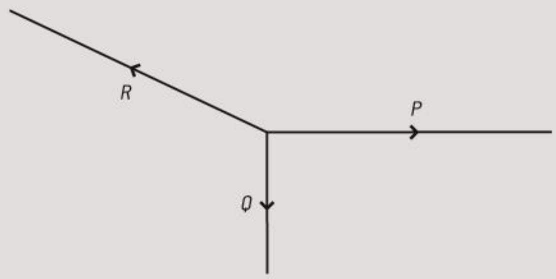
d) The uncertainty in the measurement of the radius of the thread is $\pm 0.1 \times 10^{-6}$ m. Determine the percentage uncertainty in the value of the area of the thread.

(9 marks)

12 A cyclist travels a distance of 1200 m due north before going 2000 m due east followed by 500 m south-west. Draw a scale diagram to calculate the cyclist's final displacement from her initial position.

(4 marks)

13 The diagram shows three forces *P*, *Q*, and *R* in equilibrium. *P* acts horizontally and *Q* vertically.



When $P = 5.0\,\text{N}$ and $Q = 3.0\,\text{N}$, calculate the magnitude and direction of *R*.

(3 marks)

14 A boat, starting on one bank of a river, heads due south with a speed of $1.5\ \text{m s}^{-1}$. The river flows due east at $0.8\ \text{m s}^{-1}$.

a) Calculate the resultant velocity of the boat relative to the bank of the river.

b) The river is 50 m wide. Calculate the displacement from its initial position when the boat reaches the opposite bank.

(7 marks)

15 A car of mass 850 kg rests on a slope at 25° to the horizontal. Calculate the magnitude of the component of the car's weight which acts parallel to the slope.

(3 marks)

2 MECHANICS

Introduction

Everything moves. The Earth revolves on its axis as it travels around the Sun. The Sun orbits within the Milky Way. Galaxies move apart.

Motion and its causes are important to a study of physics. We begin by defining the meaning of everyday terms such as distance, velocity, acceleration and go on to develop models for motion that will allow us to predict the future motion of an object.

2.1 Motion

Understanding

- → Distance and displacement
- → Speed and velocity
- → Acceleration
- → Graphs describing motion
- → Equations of motion for uniform acceleration
- → Projectile motion
- → Fluid resistance and terminal speed

Nature of science

An understanding of motion lies at the heart of physics. The areas of the subject are linked by the concepts of movement and the forces that produce motion. Links are used in a creative way by scientists to illuminate one part of the subject by reference to insights developed for other topics. The study of motion also relies on careful observation of the world so that accurate models of motion can be developed.

Applications and skills

- → Determining instantaneous and average values for velocity, speed, and acceleration
- → Solving problems using equations of motion for uniform acceleration
- → Sketching and interpreting motion graphs
- → Determining the acceleration of free-fall experimentally
- → Analysing projectile motion, including the resolution of vertical and horizontal components of acceleration, velocity, and displacement
- → Qualitatively describing the effect of fluid resistance on falling objects or projectiles, including reaching terminal speed

Equations

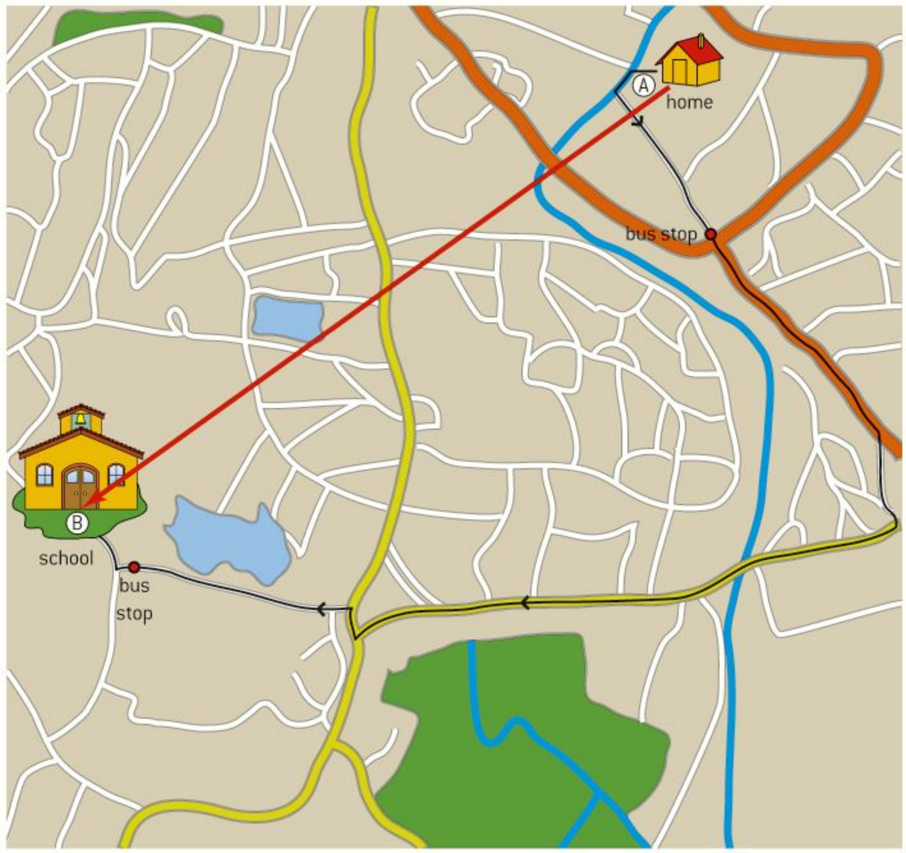
Kinematic equations of motion:

- → $v = u + at$
- → $s = ut + \frac{1}{2}at^2$
- → $v^2 = u^2 + 2as$
- → $s = \frac{(v + u)t}{2}$

Distance and displacement

Your journey to school is unlikely to take a completely straight line from home to classroom. How do we describe journeys when we turn a corner and change direction?

The map shows the journey a student makes to get to school together with the times of arrival at various points on the way. To keep life simple, the journey has no hills.



▲ Figure 1 Journey to school.

journey leg	time	distance for leg / m
leave home	08.10.00	0
walk to bus stop	08.20.15	800
bus arrives at stop	08.24.30	0
bus arrives near school	08.31.10	2400
walk from bus to school	08.34.00	200

The total length of this journey is 3.4 km including all the twists and turns. This is the distance travelled. As we saw in Topic 1, distance is a scalar quantity; it has no direction. If the student walks home by exactly the same route, then the distance travelled going home is the same as going to school.

Nature of science

How long is a piece of string?

Distance can be measured in any appropriate unit of length: metres, miles, and millimetres are all common. Some countries and some professions use alternatives. Surveyors use chains, the English-speaking world once used measures of length called rods, poles, and perches – all related to agricultural measurements. Astronomers use light years (a unit of length, not time) and a measure called simply the "astronomical unit". Sometimes in everyday life it is a question of using a convenient unit rather than the correct metric version. In your exam, however, lengths will be in multiples and sub-multiples of the metre or in a well-recognised scientific unit such as the light year.

Journeys can be defined by the starting point (A) and finishing point (B) without saying anything about the intermediate route. A vector is drawn that starts at A and finishes at B. This straight line from start to finish has, as a vector, *magnitude*, and *direction*. This vector measurement is known as the **displacement**.

As you saw in Sub-topic 1.3, when you give the details of a displacement you must always give two pieces of information: the magnitude (or size) of the quantity together with its unit, and also the direction of the vector. This direction can be written in a number of ways. One way is as a heading such as N35°E meaning that the finishing direction is at an angle of 35° clockwise from north. If you are a sailor or a keen orienteer, you may have other ways of measuring the direction of displacement.

The displacement of the student's journey to school (AB) is also shown on the diagram. The displacement is the vector that connects home to school. This time the journey from school to home is not the same as the trip to school. It has the same vector length, but the direction is the opposite to that of the outward route.

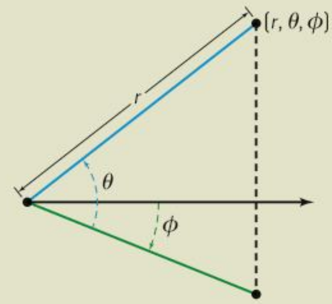
Nature of science

Moving in 3D

Displacement has been described here in terms of a journey in a flat landscape. Does a change in level alter things? Only one thing changes, and that is the number of pieces of information required to specify the final position relative to the start. Three pieces of information are now required: the magnitude plus its unit, the heading, and the *overall change in height* during the journey. Specifying motion in three dimensions requires three numbers or coordinates, you are already familiar with the idea of a coordinate from drawing and using graphs.

There is flexibility in how the three numbers can be chosen. You may have seen three-dimensional graphs with three axes each at 90° to the others, in this case coordinate numbers are given that relate to the distance along each axis. Another option is to use polar coordinates (figure 2): where a distance and two angles are required, one angle is the bearing from North, the other the angle up or down from the horizontal needed to look directly at the object above (or below) us.

In some circumstances, even the distance itself may not be required. Sailors use latitude and longitude when they are navigating. They stay on the surface of the sea and this is effectively a constant distance from the centre of the Earth.



▲ Figure 2 Polar coordinates.

Worked examples

1 A cyclist travels 16 km in 70 minutes. Calculate, in m s^{-1}, the speed of the cyclist.

Solution

70 minutes is $60 \times 70 =$ 4200 s, 16 km is 16 000 m. The quantities are now in the units required by the question.

The speed of the cyclist is $\frac{16\,000}{4200} = 3.8$ m s^{-1}.

2 The speed of light in a vacuum is 3.0×10^8 m s^{-1}. A star is 22 light years from Earth (1 light year is the distance travelled by light in one year). Calculate the distance of the star from Earth in kilometres.

Solution

Light travels 3.0×10^8 m in 1 s. So in a year it travels $3.0 \times 10^8 \times 365 \times 24 \times 60 \times 60 = 9.5 \times 10^{15}$ m. The distance of the star from the Earth is $22 \times 9.5 \times 10^{15} = 2.1 \times 10^{17}$ m.

The question asks for an answer in kilometres, so the distance is 2.1×10^{14} km.

Speed and velocity

In just the same way that there are scalar and vector measures of the length of a journey, so there are two ways of measuring how quickly we cover the ground.

The first of these is a scalar quantity **speed**, which is defined as $\frac{\text{distance travelled on the journey}}{\text{time taken for the journey}}$. Units for speed, familiar to you already, may include metre per second (m s^{-1}) and kilometres per hour (km h^{-1}), but any accepted distance unit can be combined with any time unit to specify speed.

Velocity is the vector term and, just as for displacement, a magnitude and direction are required. Examples might be "4.2 m s^{-1} due north" or "55 km h^{-1} at N22.5°E".

Nature of science

Measuring speed?

To measure speed it is necessary to measure the the distance travelled (using a "ruler") and time taken (using a "clock"). The trick is in a good choice of "ruler", "clock" and method for recording the measurements! A 30 cm ruler and a wrist watch will be fine for a biologist measuring the speed at which an earthworm moves. But if we want to measure the speed of a 100 m sprinter, then a measured distance on the ground, a good stop watch and a human observer is barely good enough. Even then the observer has to be careful to watch the smoke from the starting pistol and not to wait for the sound of the gun.

If you need to measure the speed of a soccer ball during a penalty kick, then stop watch-plus-human is no longer adequate to make a valid measurement. Perhaps a video camera that takes frames at a known rate (the clock) and a scale near the path of the ball visible on the picture (the ruler) is needed now?

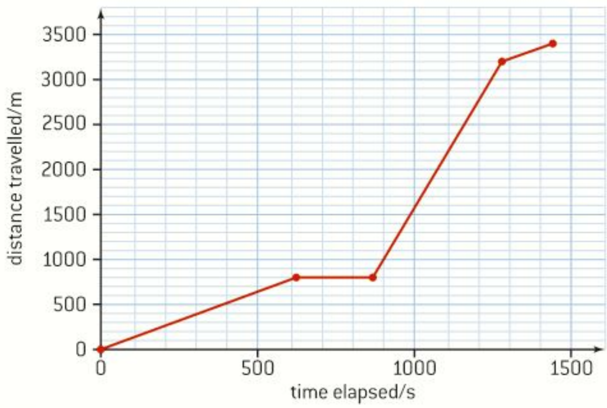
Move up to measuring the speed of a jet aircraft and the equipment needs to change again.

Choosing the right equipment for the task in hand is all part of the job of the working science student.

Describing motion with a graph – I

The use of two variables, distance and time, to calculate speeds and velocities means that things become more complicated. Figure 1 showed a map of the journey the student makes to school. Part of this journey is on foot, part is by bus. It is unlikely that the student will travel at the same speed all the time, as the bus will travel faster than the student walks.

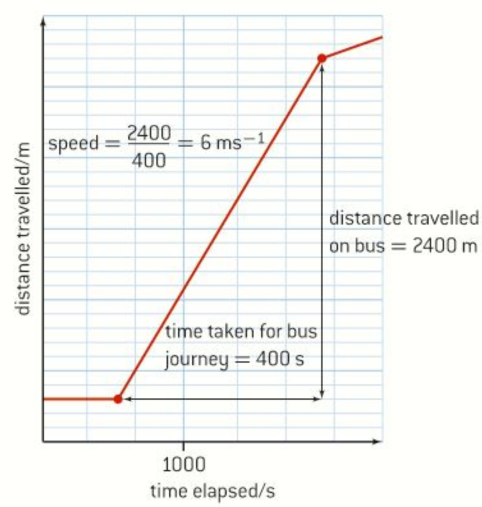
If we want to display data in a visual way, a graph of distance against time is one of the most common approaches.



▲ Figure 3 Distance–time graph.

The distance travelled by the student is plotted on the vertical axis while the time since the beginning of the journey is plotted horizontally (the clock times of the journey have been translated into elapsed times since the start of the journey for this graph). The different regions of the graph are identified and you should confirm that they are matched correctly. Notice how the gradient of the graph changes for the different parts of the journey: small values of the gradient for the walking sections of the journey, horizontal (zero gradient) for stationary at the bus stop, and steep for the bus journey. What will the graph for the journey home from school look like, assuming that the time for each segment of the journey is the same as in the morning?

Information can easily be extracted from this graph. The gradient of the graph is the speed. Add the overall direction to this speed and we have the velocity too.



▲ Figure 4 Distance–time graph from figure 3 with gradients.

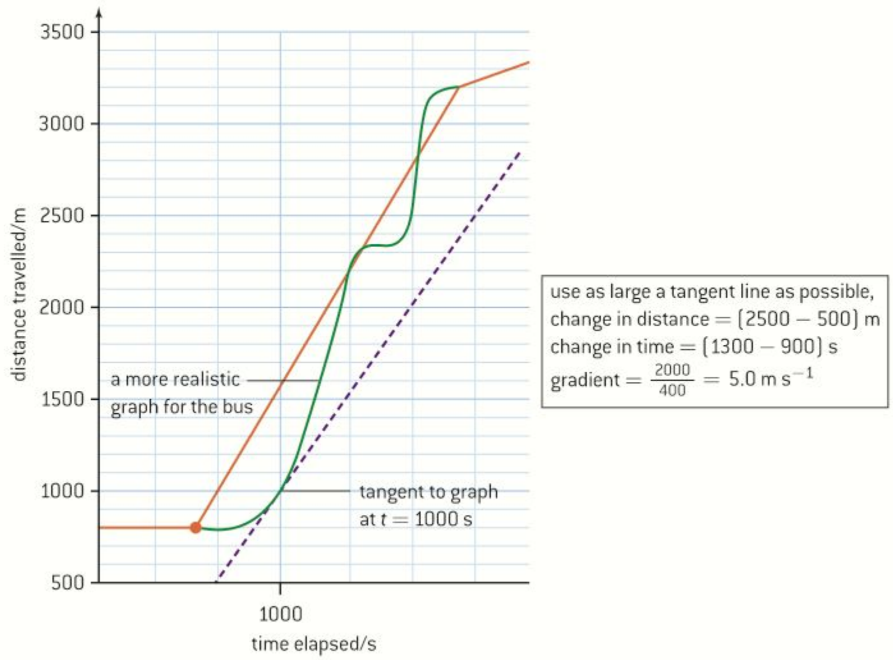
For the first walk to the bus stop the distance was 800 m and the time taken was 615 s. The constant walking speed was therefore $\frac{800}{615}$, which is 1.3 m s^{-1}.

The gradient of the bus journey is $\frac{2400}{400}$ (this is marked on the graph) and so the speed was 6.0 m s^{-1}.

Of course, even this journey with its changes is simplistic. Real journeys have few straight lines, so we must introduce some ways to handle rapidly varying speeds and velocities.

Instantaneous and average values

The bus driver knows how fast the bus in the student's journey is travelling because it is displayed on the speedometer. This is the **instantaneous speed**, as it gives the value of the speed at the moment in time at which the speed is determined.



▲ Figure 5 Instantaneous speed.

The instantaneous speed is also the gradient of the distance–time graph at the instant concerned. Figure 5 shows how this is calculated when 1000 s into the whole journey to school. The original red line for the bus from figure has been replaced by a green line that is more realistic for the motion of a real bus – the speed varies as the driver negotiates the traffic. You will frequently be asked to calculate gradients on physics graphs. Make sure that you can do this accurately by using a transparent ruler.

From a mathematical point of view we can describe the instantaneous speed as the **rate of change of position with respect to time**.
A mathematician will write this as $\frac{ds}{dt}$, where s is the distance and t is the time. You may also have seen $\frac{\Delta s}{\Delta t}$, where the symbol Δ means *change in*.
So $\frac{\Delta s}{\Delta t}$ is just shorthand for $\frac{\textit{change in distance}}{\textit{change in time}}$.

There is however another useful measure of speed. This is the **average speed** and is the speed calculated over the whole the journey without regard to variations in speed. So as an equation this is

$$\textit{average speed} = \frac{\text{distance travelled over whole journey}}{\text{time taken for whole journey}}$$

In terms of the distance–time graph, the average speed is equal to the gradient of the straight line that joins the beginning and the end of the time interval concerned. So, for the part of the student's journey up to the moment when the bus arrives at the stop, the distance travelled is 800 m, the time taken is 870 s (*including* the wait at the stop) so the average speed is 0.92 m s^{-1}.

Nature of science

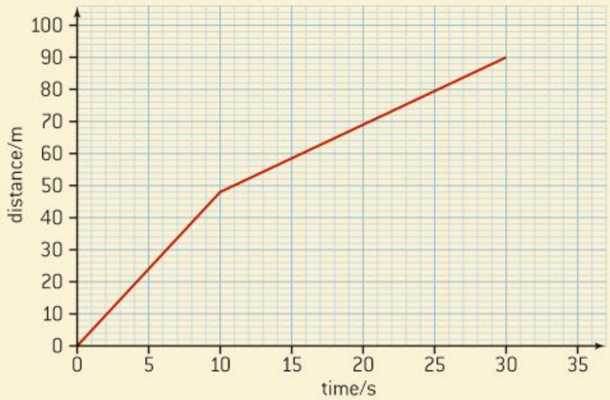
Calculating a gradient of a graph

Taking the gradient of a graph is actually a way of averaging results. As discussed in Topic 1, a straight-line graph drawn through points that have some scatter makes this obvious. The value of the gradient is the $\frac{\text{change in the value on the } y\text{-axis}}{\text{change in the value on the } x\text{-axis}}$. The unit associated with the gradient is the $\frac{y\text{-axis unit}}{x\text{-axis unit}}$. Don't forget to quote the unit every time you write down the value of the gradient.

Everything said here about average and instantaneous speeds can also refer to average and instantaneous velocities. Remember, of course, to include the directions when quoting these measurements.

Worked example

The graph shows how the distance run by a boy varies with the time since he began to run.



Calculate:

a) the instantaneous speed at
(i) 5.0 s (ii) 20 s

b) the average speed for the whole 30 s run.

Solution

a) (i) The boy runs at constant speed so the graph is straight from 0 – 10 s.

The gradient of the straight line is $\frac{48}{10} = 4.8\ \mathrm{m\ s^{-1}}$.

So the instantaneous speed at 5.0 s is $4.8\ \mathrm{m\ s^{-1}}$.

(ii) Again the boy is running at a constant speed, but this time slower than in the first 10 s.

The speed from 10 s to 30 s is:

$\frac{(90 - 48)}{(30 - 10)} = \frac{42}{20} = 2.1\ \mathrm{m\ s^{-1}}$.

The instantaneous speed at 20 s is $2.1\ \mathrm{m\ s^{-1}}$.

b) The total distance travelled in 30 s is 90 m, so the average speed is $\frac{90}{30} = 3.0\ \mathrm{m\ s^{-1}}$.

Acceleration

In real journeys, instantaneous speeds and velocities change frequently. Again we need to develop a mathematical language that will help us to understand the changes.

The quantity we use is **acceleration**. Acceleration is taken to be a vector, but sometimes we are not interested in the direction and then write the "magnitude of the acceleration" meaning the size of the acceleration ignoring direction.

The definition is: $\textit{acceleration} = \frac{\text{change in velocity}}{\text{time taken for the change}}$

and this means that the units of acceleration are $\frac{\mathrm{m\ s^{-1}}}{\mathrm{s}}$ which is usually written as $\mathrm{m\ s^{-2}}$ (or sometimes you will see $\mathrm{m/s^2}$).

It is important to understand what acceleration means, not just to be able to use it in an equation. If an object has an acceleration of $5\ \mathrm{m\ s^{-2}}$ then for every second it travels, its velocity increases in magnitude by $5\ \mathrm{m\ s^{-1}}$ in the direction of the acceleration vector.

As an example: the Japanese N700 train has a quoted acceleration of $0.72\ \mathrm{m\ s^{-2}}$. Assume that this is a constant value (very unlikely). One second after starting from rest, the speed of the train will be $0.72\ \mathrm{m\ s^{-1}}$. One second later (at 2 s from the start) the speed will be $0.72 + 0.72 = 1.44\ \mathrm{m\ s^{-1}}$. At 3 s it will be $2.16\ \mathrm{m\ s^{-1}}$ and so on. Each second the speed is $0.72\ \mathrm{m\ s^{-1}}$ more.

Worked example

How many seconds will it take the N700 to reach its maximum speed of $300\ \mathrm{km\ h^{-1}}$ on the Sanyo Shinkansen route?

Solution

$300\ \mathrm{km\ h^{-1}} \equiv = 83.3\ \mathrm{m\ s^{-1}}$

Time taken to reach the maximum speed:

$\frac{83.3}{0.72} = 116$ s,

just under 2 minutes.

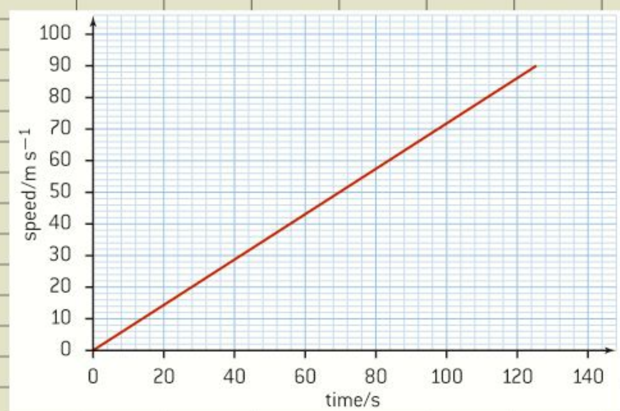
Nature of science

Spreadsheet models

One powerful way to think about acceleration (and other quantities that change in a predictable way) is to model them using a spreadsheet. The examples here use a version of Microsoft Excel© but any computer spreadsheet can be used for this.

This is a spreadsheet model for the N700 train. The value of the acceleration is in cell B1. Cells A4 to A29 give the time in increments of 5 s; the computed speed at each of these times is in cells B4 to B29. The speed is calculated by taking the *change* in time between the present cell and the one above it, and then multiplying by the acceleration (the acceleration is written as \$B\$1 so that the spreadsheet only uses this cell and does not drop down a cell every time the new speed is calculated). Finally, the spreadsheet plots speed against time showing that the graph is a straight line and that the acceleration is uniform.

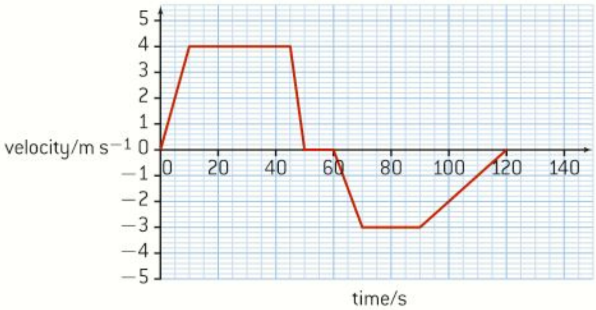
	A	B	C	D	E	F	G	H	I	J
1	acceleration/s	0.72								
2										
3	time/s	speed/ms^{-1}								
4	0	0								
5	5	3.6								
6	10	7.2								
7	15	10.8								
8	20	14.4								
9	25	18.0								
10	30	21.6								
11	35	25.2								
12	40	28.8								
13	45	32.4								
14	50	36.0								
15	55	39.6								
16	60	43.2								
17	65	46.8								
18	70	50.4								
19	75	54.0								
20	80	57.6								
21	85	61.2								
22	90	64.8								
23	95	68.4								
24	100	72.0								
25	105	75.6								
26	110	79.2								
27	115	82.8								
28	120	86.4								
29	125	90.0								



Describing motion with a graph – II

Distance–time plots lead to a convenient display of speed and velocity changes. Plots of speed (or velocity) against time also help to display and visualize acceleration.

The data table and the graph show a journey with various stages on a bicycle. From the start until 10 s has elapsed, the bicycle accelerates at a uniform rate to a velocity of +4 m s^{-1}. The positive sign means that the velocity is directed to the right.



▲ Figure 6 Velocity–time graph for the bicycle.

From 10 s to 45 s the cycle moves at a constant velocity of +4 m s^{-1} and at 45 s the cyclist applies the brakes so that the cycle stops in 5 s. The cycle is then stationary for 10 s.

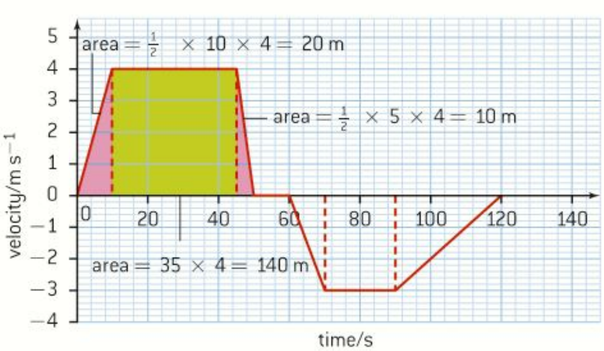
From then on the velocity is negative meaning that the cycle is travelling in the opposite direction. The pattern is similar, an acceleration to –3 m s^{-1}, a period of constant velocity and a deceleration to a stop at 120 s.

As before, the gradient of the graph has a meaning. The gradient of this velocity–time graph gives the magnitude (size) of the acceleration and its sign (direction) as well.

From 45 s to 50 s the velocity goes from 4 m s^{-1} to 0 and so the acceleration is $\frac{\text{final speed} - \text{initial speed}}{\text{time taken}}$ $\frac{(0–4)}{5} = –0.8$ m s^{-2}. From 90 s to 120 s the magnitude of the acceleration is $\frac{3}{30} = 0.1$ m s^{-2}. We need to take care with the sign of the acceleration here. Because the cycle is moving in the negative direction and is slowing down, the acceleration is positive (as is the gradient on the graph) – this simply means that there is a force acting to the right, that is, in the positive direction which is slowing the cycle down. We will discuss how force leads to acceleration later in this topic.

The area under a velocity–time graph gives yet more information. It tells us the total displacement of the moving object. The way to see this is to realize that the product of *velocity* × *time* is a displacement (and that product of *speed* × *time* is a distance). The units tell you this too: when the units are multiplied the seconds in $\frac{\text{metre}}{\cancel{\text{second}}} \times \cancel{\text{second}}$ cancel to leave metre only.

In the case of a graph with uniform acceleration, the areas, and hence the displacements (distances) are straightforward to calculate. Divide the graph into right-angled triangles and rectangles and then work out the areas for each individual part. This is shown in figure 7.



▲ Figure 7 Velocity-time graph broken down into areas.

The individual areas are shown on the diagram and for the motion up to a time of 60 s the area is 170 m; the area from the 60 s time to the end is −120 m. As usual, the negative sign indicates motion in the opposite direction to the original.

As discussed in Topic 1, when the velocity–time graph is curved, you will need to:

(i) estimate the number of squares

(ii) assess the area (distance) for one square, and finally

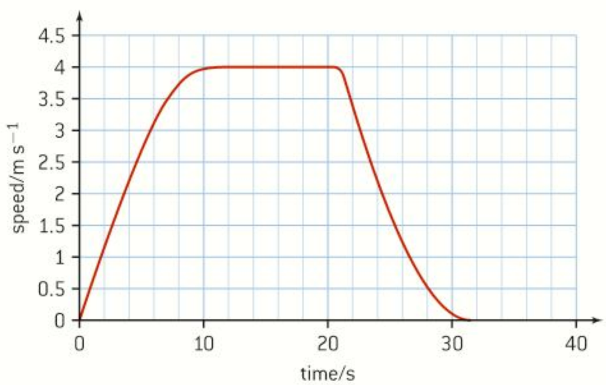
(iii) multiply the number of squares by the area of one square.

This will usually give you an estimate of the overall distance.

Figure 8 gives an example of how this is done.

There are about 85 squares between the *x*-axis and the line. (You may disagree slightly with this estimate, but that is fine – there is always an allowance made for this.)

Each of the squares is 2 s along the time axis and 0.5 m along the speed axis. So the area of one square is equivalent to 1.0 m of distance. The total distance travelled is 85 m (or, at least, somewhere between 80 and 90 m).



▲ Figure 8 A more difficult area to estimate.

The "*suvat*" equations of motion

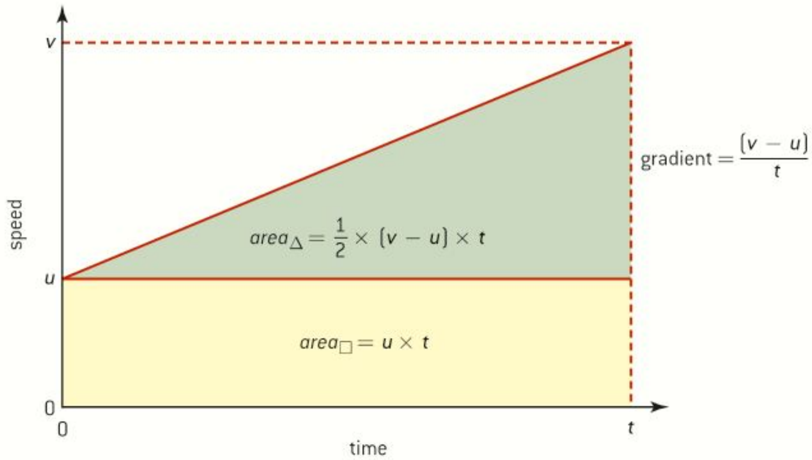
The graphs of distance–time and speed–time lead to a set of equations that can be used to predict the value of the parameters in motion. They also help you to understand the connection between the various quantities in your study of motion.

A note about symbols: from now on we will use a consistent set of symbols for the quantities. The table gives the list.

symbol	quantity
s	displacement/distance
u	initial (starting) velocity/speed
v	final velocity/speed
a	acceleration
t	time taken to travel the distance *s*

If you read down the list of symbols they spell out *suvat* and the equations are sometimes known by this name, another name for them is the *kinematic* equations of motion.

The derivation of the equations of uniformly accelerated motion begins from a simple graph of speed against time for a constant acceleration from velocity *u* to velocity *v* in a time *t*.



▲ Figure 9 Deriving the first two equations of motion.

The acceleration is the gradient of the graph:

$$\frac{\text{change in speed}}{\text{time taken for change}}$$

The change in speed is $v - u$, the time taken is t.

Therefore,

$$a = \frac{v - u}{t}$$

and re arranging gives

$v = u + at$	first equation of motion

The area under the speed–time graph from 0 to t is made up of two parts, the lower rectangle, $area_{\square}$ and the upper right-angled triangle, $area_{\Delta}$.

$area_{\Delta} = \frac{1}{2}\ base \times height = \frac{1}{2}t(v - u)$

$area_{\square} = base \times height = ut$

$s = \text{total area} = area_{\Delta} + area_{\square} = ut + \frac{1}{2}(v - u)t = ut + \frac{1}{2}(at)t$

So

$s = ut + \frac{1}{2}at^2$	second equation of motion

The first equation has no s in it; the second has no v. There are three more equations, one with a missing t and one with a missing a. There is one equation that has a missing u, but this is not often used.

To eliminate t from the first and second equations, re arrange the first in terms of t:

$$t = \frac{v - u}{a}$$

This can be substituted into the second equation:

$$s = u\,\frac{v - u}{a} + \frac{1}{2}\,a\left(\frac{v - u}{a}\right)^2$$

and

$$as = u(v - u) + \frac{1}{2}(v - u)^2 = uv - u^2 + \frac{1}{2}v^2 + \frac{1}{2}u^2 - \frac{1}{2}2uv$$

which gives

$$2as = v^2 - u^2$$

or

$v^2 = u^2 + 2as$	third equation of motion

The derivation of the final equation is left to you as an exercise:

$s = \left(\frac{v+u}{2}\right)t$	fourth equation of motion

Remember!

These equations only apply if the acceleration is *uniform*. In other words, acceleration must not change during the motion.

There are two ways to approach this proof. One way is to think about the meaning of the speed that corresponds to $\frac{v+u}{2}$ and then to recognize the time at which this speed occurs in the motion. The second way is to take the third equation and amalgamate it with the first.

You will not be expected to remember these proofs or the equations themselves (which appear in the data booklet), but they do illustrate how useful graphs and equations can be when solving problems in kinematics.

Worked examples

1 A driver of a car travelling at 25 m s^{-1} along a road applies the brakes. The car comes to a stop in 150 m with a uniform deceleration. Calculate **a)** the time the car takes to stop, and **b)** the deceleration of the car.

Solution

a) One way to answer kinematic equation questions is to begin by writing down what you do and don't know from the question.

$s = 150$ m; $u = 25$ m s^{-1}; $v = 0$; $a = ?$; $t = ?$

To work out t, the fourth equation is required: $s = \left(\frac{v+u}{2}\right)t$

which rearranges to $t = \left(\frac{2}{v+u}\right)s$

Substituting the values gives

$t = \left(\frac{2}{25}\right)150 = 2 \times 6 = 12$ s

b) To find a the equation $v^2 = u^2 + 2as$ is best.

Substituting:

$0 = 25^2 + 2 \times a \times 150$

$a = -\frac{25 \times 25}{300} = -\frac{25}{12} = -2.1$ m s^{-2}.

The minus sign shows that the car is decelerating rather than accelerating.

2 A cyclist slows uniformly from a speed of 7.5 m s^{-1} to a speed of 2.5 m s^{-1} in a time of 5.0 s.

Calculate **a)** the acceleration, and **b)** the distance moved in the 5.0 s.

Solution

a) $s = ?$; $u = 7.5$ m s^{-1}; $v = 2.5$ m s^{-1}; $a = ?$; $t = 5.0$ s

Use $v = u + at$ and therefore $2.5 = 7.5 + a \times 5.0$

so, $a = -\frac{5.0}{5.0} = -1.0$ m s^{-2}

The negative sign shows that this is a deceleration.

b) $s = ut + \frac{1}{2}at^2$ so
$s = 7.5 \times 5.0 - \frac{1}{2} \times 1.0 \times 5.0^2$
$= 37.5 - 12.5$
$= 25$ m

Projectile motion

Falling freely

When an object is released close to the Earth's surface, it accelerates downwards. We say that the force of gravity acts on the object, meaning that it is pulled towards the centre of the Earth. Equally the object pulls with the same force on the Earth in the opposite direction. Not surprisingly, with small objects, the effect of the force on the Earth is so small that we do not notice it.

In Topic 6, we shall look in more detail at the effects of gravity but for the moment we assume that there is a constant acceleration that acts on all bodies close to the surface of the Earth.

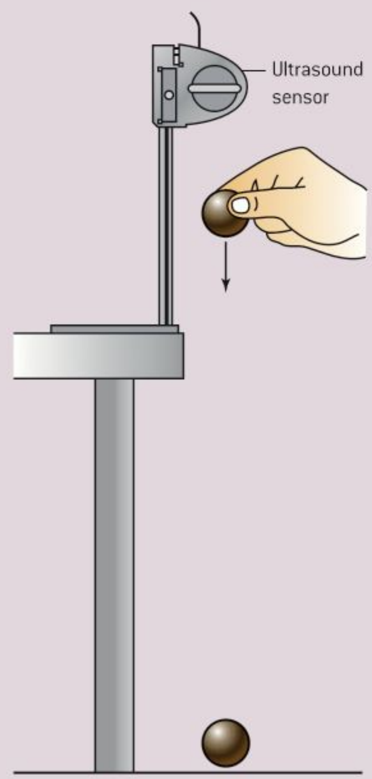
The acceleration due to gravity at the Earth's surface is given the symbol g. The accepted value varies from place to place on the surface, so that at Kuala Lumpur g is 9.776 m s^{-2} whereas at Stockholm it is 9.818 m s^{-2}. Reasons for the variation include variations in the shape of the Earth (it is not a perfect sphere, being slightly flattened at the poles) and effects that are due to the densities of the rocks in different locations. The different tangential speeds of the Earth at different latitudes also have an effect. It is better to buy gold by the newton at the equator and sell it at the North Pole than the other way round!

Investigate!

Measuring g

Alternative 1

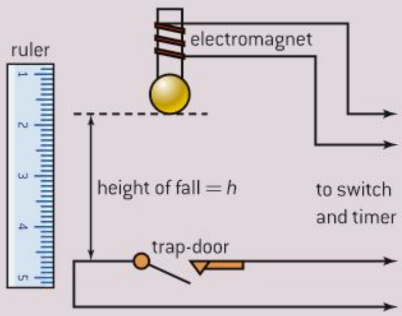
- There are a number of ways to measure g. This method uses a data logger to collect data. One of the problems with measuring g "by hand" is that the experiment happens quickly. Manual collection of the data is difficult.
- An ultrasound sensor is mounted so that it senses objects below it.
- Set the logging system up to measure the speed of the object over a time of about 1 s. Set a sensible interval between measurements.
- Switch on the logger system, and drop an object vertically that is large enough for the sensor to detect.
- The output from the system should be a speed–time graph that is a straight line; it may be that the logger's software can calculate the gradient for you.
- You could extend this experiment by testing objects of different mass but similar size and shape to confirm a suggestion by Galileo that such differences do not affect the drop.



▲ Figure 10 Ultrasound sensor.

Alternative 2

There are other options that do not involve a data logger.

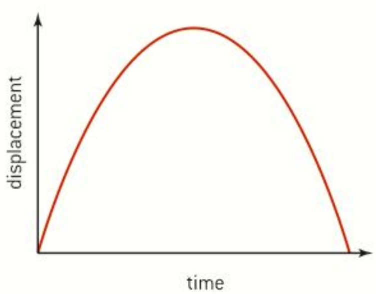


▲ Figure 11 Trap door method.

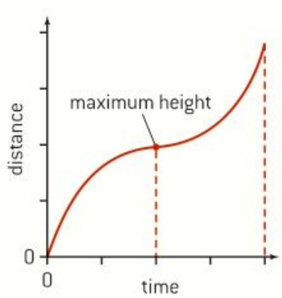
- A magnetic field holds a small steel sphere (such as a ball bearing) between two metal contacts. The magnetic field is produced by a coil of wire with an electric current in it. When the current is switched off, the field disappears and the sphere is released to fall vertically.
- As the sphere leaves the metal contacts, a clock starts. The clock stops when the sphere opens a small trapdoor and breaks the connection between the terminals of a timing clock or computer. (The exact details of these connections will depend on the equipment you have.)
- This system measures the time of flight t of the sphere from the contacts to the trapdoor.
- Measure the distance h from the bottom of the sphere to the top of the trapdoor (you might think about why these are the appropriate measurement points).
- A possible way to carry out the experiment is to measure t for one value of h – with, of course, a few repeat measurements for the same h. Then use $h = ut + \frac{1}{2}gt^2$ with $u = 0$ to calculate g.
- This is a one-off measurement that is prone to error. Can you think of some reasons why? One way to reduce the errors is to change the vertical distance h between the sphere and trapdoor and to plot a graph of h against t^2. The gradient of the graph is $\frac{g}{2}$. If you observe an intercept on the h-axis, what do you think it represents?

Alternative 3

- There is a further method that involves taking a video or a multiflash image of a falling object and analysing the images to measure g. You will see an example of such an analysis later in this sub-topic.



▲ Figure 12a Displacement–time for ball thrown vertically.



▲ Figure 12b Distance–time for ball thrown vertically.

What goes up must come down

Perhaps you have seen a toy rocket filled with water under pressure and fired vertically upwards? Or you may have thrown a ball vertically, high into the air?

After the ball has been released, as the pull of gravity takes effect, the ball slows down, eventually stopping at the top of its motion and then falling back to Earth. If there was no air resistance the displacement–time graph would look like figure 12a. Remember that this is a graph of vertical displacement against time, not the shape of the path the ball makes in the air – which is called the **trajectory**. The ball is going vertically up and then vertically down to land in the same spot from where it began.

A distance–time graph would look different (figure 12b), it gives similar information but without the direction part of the displacement and velocity vectors. Make sure that you understand the difference between these graphs.

The *suvat* equations introduced earlier can be used to analyse this motion. The initial vertical speed is u, the time to reach the highest point is t, the highest point is h and the acceleration of the rocket is $-g$. The sign of g is negative because upwards is the positive direction. As the acceleration due to gravity is downwards, g must have the opposite sign. The kinematic equations are printed again but with differences to reflect the vertical motion to the highest point:

$0 = u - gt$ which comes from $v = u + at$

$h = ut - \frac{1}{2}gt^2$ which comes from $s = ut + \frac{1}{2}at^2$

$0 = u^2 - 2gh$ which comes from $v^2 = u^2 + 2as$

If you want to find out the time for the entire motion (that is, up to the highest point and then back to Earth again), it is simply $2t$.

Reminder!

Try to remember this crucial point about the signs in the equations when you answer questions on vertical motion: upwards is +ve and downwards is −ve. Something else that students forget is that at the top of the motion the vertical speed of the rocket is zero.

Worked examples

1 A student drops a stone from rest at the top of a well. She hears the stone splash into the water at the bottom of the well 2.3 s after releasing the stone. Ignore the time taken for the sound to reach the student from the bottom of the well.

a) Calculate the depth of the well.

b) Calculate the speed at which the stone hits the water surface.

c) Explain why the time taken for the sound to reach the student can be ignored.

Solution

a) The acceleration due to gravity g is 9.8 m s^{-2}.

$u = 0$; $t = 2.3$ s.

$s = ut + \frac{1}{2}at^2$ and therefore

$s = 0 + \frac{1}{2} \times 9.8 \times 2.3^2$

$= 26$ m

b) $v = u + at$; $v = 0 + 9.8 \times 2.3 = 23$ m s^{-1}

c) The speed of sound is about 300 m s^{-1} and so the time to travel about 25 m is about 0.08 s. Only about 4% of the time taken for the stone to fall.

2 A hot-air balloon is rising vertically at a constant speed of 5.0 m s^{-1}. A small object is released from rest relative to the balloon when the balloon is 30 m above the ground.

a) Calculate the maximum height of the object above the ground.

b) Calculate the time taken to reach the maximum height.

c) Calculate the total time taken for the object to reach the ground.

Solution

a) The object is moving upwards at +5.0 m s^{-1} when it is released. The acceleration due to gravity is −9.8 m s^{-2}.

When the object is released it will continue to travel upwards but this upward speed will decrease under the influence of gravity. When it reaches its maximum height it will stop moving and then begin to fall.

$v^2 = u^2 + 2as$ and $s = \frac{0 - 5^2}{-2 \times 9.8} = +1.3$ m

This shows that the object rises a further 1.3 m above its release point, and is therefore 31.3 m above the ground at the maximum height.

b) $v = u + at$; $t = \frac{0 - 5}{-9.8} = +0.51$ s (the plus sign shows that this is 0.51 s after release)

c) After reaching the maximum height (at which point the speed is zero) the object falls with the acceleration due to gravity.

$s = 31.3$ m ; $u = 0$; $v = ?$; $a = -9.8$ m s^{-2}; $t = ?$

Using $s = ut + \frac{1}{2}at^2$, $-31.3 = 0 - 0.5 \times 9.8 \times t^2$.

(Notice that s is −31.3 m as it is in the opposite direction to the upwards + direction.)

This gives a value for t of ±2.53 s. The positive value is the one to use. Think about what the negative value stands for.

So the total time is the 0.51 s to get to the maximum height together with the 2.53 s to fall back to Earth.

This gives a total of 3.04 s which rounds to 3.0 s.

Notice that, in this example, if you carry the signs through consistently, they give you information about the motion of the object.

Moving horizontally

There is little that is new here. We are going to assume that the surface of the Earth is large enough for its surface to be considered flat (Topic 6 will go into more details of what happens in reality) and that there is no friction.

Gravity acts vertically and not in the horizontal direction. This will be important when we combine the horizontal and vertical motions later.

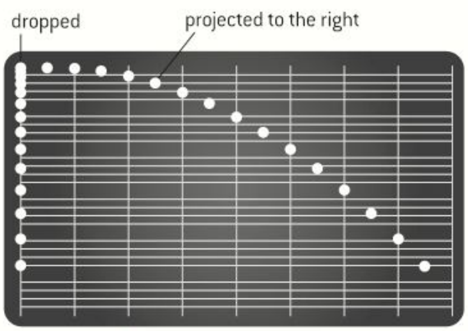
Because the horizontal acceleration is zero, the *suvat* equations are simple.

For horizontal motion:

- the horizontal velocity does not change
- the horizontal distance travelled is *horizontal speed* × *time for the motion*.

Putting it all together

A student throws a ball horizontally. Figure 13 shows multiple images of the ball every 0.10 s as it moves through the air. The picture also shows, for comparison, the image of a similar ball dropped vertically at the same moment as the ball is thrown. What do you notice about both images?

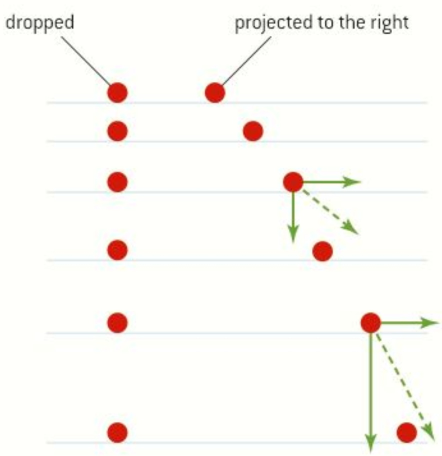


▲ Figure 13 Multi-flash images of two falling objects.

It is obvious which of the two balls was thrown horizontally. Careful examination of the images of this ball should convince you that the *horizontal* distance between them is constant. Knowing the time interval and the distance scale on the picture means that you can work out the initial (and unchanging) horizontal speed.

The images tell us about the vertical speeds too. The clue here is to concentrate on the ball that was dropped vertically. The distance between images (strictly, between the same point on the ball in each image) is increasing. The distance s travelled varies with time t from release as $s \propto t^2$. If t doubles then s should increase by factor of 4. Does it look as though this is what happens? Careful measurements from this figure followed by a plot of s against t^2 could help you to confirm this.

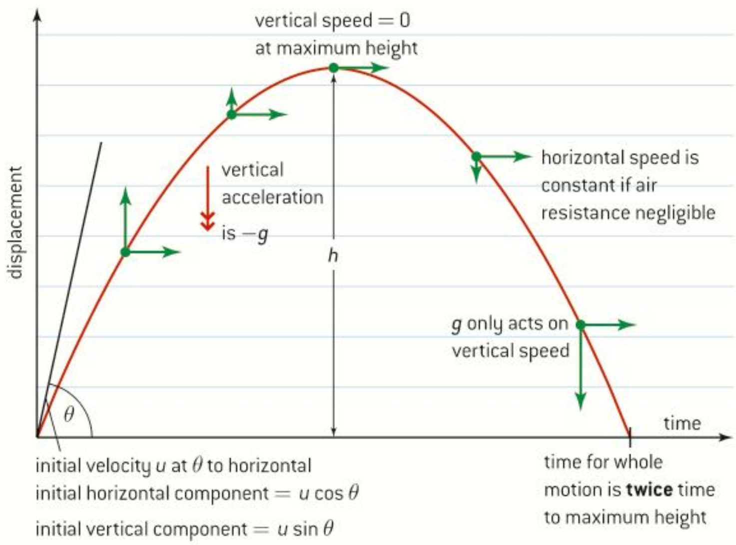
The two motions, horizontal and vertical, are completely independent of each other. The horizontal speed continues unchanged (we assumed no air resistance) while the vertical speed changes as gravity acts on the ball. This independence allows a straightforward analysis of the motion. The horizontal and the vertical parts of the motion can be split up and treated separately and then re-combined to answer questions about the velocity and the displacement for the whole of the motion.



▲ Figure 14 Horizontal and vertical speed components.

The position is summed up in figure 14. At two positions along the trajectory, the separate components of velocity are shown and the resultant (the actual velocity including its direction) is drawn.

Real-life situations do not always begin with horizontal motion. A well-aimed throw will project the ball upwards into the air to achieve the best range (overall distance travelled). But the general principles above still allow the situation to be analysed.



at maximum height:

$0 = u \sin\theta - gt$

$h = (u \sin\theta)t - \frac{gt^2}{2}$ and

$0 = u^2 \sin^2\theta - 2gh$

range:

$= 2t\,(u \cos\theta)$

t is the time to maximum height

▲ Figure 15 Projectile motion.

Study figure 15 carefully and apply the ideas in it to any projectile problems you need to solve.

Worked examples

1 An arrow is fired horizontally from the top of a tower 35 m above the ground. The initial horizontal speed is 30 m s^{-1}. Assume that air resistance is negligible.

Calculate:

a) the time for which the arrow is in the air

b) the distance from the foot of the tower at which the arrow strikes the ground

c) the velocity at which the arrow strikes the ground.

Solution

a) The time taken to reach the ground depends on the vertical motion of the arrow.

At the instant when the arrow is fired, the vertical speed is zero.

The time to reach the ground can be found using $s = ut + \frac{1}{2}at^2$

$t^2 = \frac{2 \times 35}{9.8}$, so t = 2.67 s or 2.7 s to 2 s.f.

b) The distance from the foot of the tower depends only on the horizontal speed.

$s = ut = 30 \times 2.67 = 80.1$ m = 80 m.

c) To calculate the velocity, the horizontal and vertical components are required. The horizontal component remains at 30 m s^{-1}. The vertical speed is calculated using $v = u + at$ and is $0 + 9.8 \times 2.67 = 26.2$ m s^{-1}.

The speed is $\sqrt{30^2 + 26.2^2} = 39.8$ m s^{-1} which rounds to 40 m s^{-1}.

The angle at which the arrow strikes the ground is $\tan\frac{26.2}{30} = 41°$

2 An object is thrown horizontally from a ship and strikes the sea 1.6 s later at a distance of 37 m from the ship.

Calculate:

a) the initial horizontal speed of the object

b) the height of the object above the sea when it was fired thrown.

Solution

a) The object travelled 37 m in 1.6 s and the horizontal speed was $\frac{37}{1.6} = 23$ m s^{-1}.

b) Use $s = ut + \frac{1}{2}at^2$ to calculate s above the sea.

$s = 0 + 0.5 \times 9.8 \times 1.6^2 = 12.5$ m