

1 MEASUREMENT AND UNCERTAINTIES

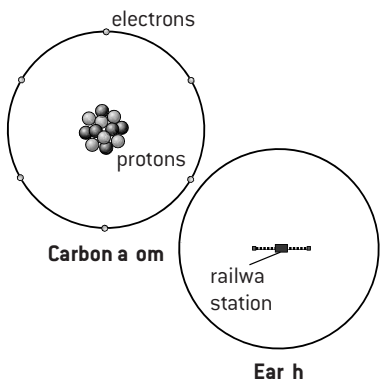
The realm of physics – range of magnitudes of quantities in our universe

ORDERS OF MAGNITUDE – INCLUDING THEIR RATIOS

Physics seeks to explain nothing less than the Universe itself. In attempting to do this, the range of the magnitudes of various quantities will be huge.

If the numbers involved are going to mean anything, it is important to get some feel for their relative sizes. To avoid 'getting lost among the numbers it is helpful to state them to the nearest **order of magnitude** or power of ten. The numbers are just rounded up or down as appropriate.

Comparisons can then be easily made because working out the ratio between two powers of ten is just a matter of adding or subtracting whole numbers. The diameter of an atom, 10^{-10} m, does not sound that much larger than the diameter of a proton in its nucleus, 10^{-15} m, but the ratio between them is 10^5 or 100,000 times bigger. This is the same ratio as between the size of a railway station (order of magnitude 10^2 m) and the diameter of the Earth (order of magnitude 10^7 m).



For example, you would probably feel very pleased with yourself if you designed a new, environmentally friendly source of energy that could produce 2.03×10^3 J from 0.72 kg of natural produce. But the meaning of these numbers is not clear – is this a lot or is it a little? In terms of orders of magnitudes, this new source produces 10^3 joules per kilogram of produce. This does not compare terribly well with the 10^3 joules provided by a slice of bread or the 10^8 joules released per kilogram of petrol.

You do NOT need to memorize all of the values shown in the tables, but you should try and develop a familiarity with them.

RANGE OF MASSES

Mass / kg	Quantity
10^{52}	total mass of observable Universe
10^{48}	
10^{44}	
10^{40}	mass of local galaxy (Milky Way)
10^{36}	
10^{32}	
10^{28}	mass of Sun
10^{24}	mass of Earth
10^{20}	total mass of oceans total mass of atmosphere
10^{16}	
10^{12}	
10^8	laden oil supertanker
10^4	elephant
10^0	human
10^{-4}	mouse
10^{-8}	grain of sand blood corpuscle
10^{-12}	
10^{-16}	bacterium
10^{-20}	
10^{-24}	haemoglobin molecule
10^{-28}	proton
10^{-32}	electron

RANGE OF LENGTHS

Size / m	Quantity
10^{26}	radius of observable Universe
10^{24}	
10^{22}	
10^{20}	radius of local galaxy (Milky Way)
10^{18}	
10^{16}	distance to nearest star
10^{14}	
10^{12}	
10^{10}	distance from Earth to Sun
10^8	distance from Earth to Moon
10^6	radius of the Earth deepest part of the ocean / highest mountain
10^4	tallest building
10^2	
10^0	
10^{-2}	length of fingernail
10^{-4}	thickness of piece of paper human blood corpuscle
10^{-6}	
10^{-8}	wavelength of light
10^{-10}	
10^{-12}	diameter of hydrogen atom
10^{-14}	wavelength of gamma ray
10^{-16}	diameter of proton

RANGE OF TIMES

Time / s	Quantity
10^{20}	
10^{18}	age of the Universe
10^{16}	age of the Earth
10^{14}	age of species <i>Homo sapiens</i>
10^{12}	
10^{10}	typical human lifespan
10^8	
10^6	1 year
10^4	1 day
10^2	
10^0	heartbeat
10^{-2}	
10^{-4}	period of high-frequency sound
10^{-6}	
10^{-8}	passage of light across a room
10^{-10}	
10^{-12}	vibration of an ion in a solid
10^{-14}	period of visible light
10^{-16}	
10^{-18}	
10^{-20}	passage of light across an atom
10^{-22}	
10^{-24}	passage of light across a nucleus

RANGE OF ENERGIES

Energy / J	Quantity
10^{44}	energy released in a supernova
10^{34}	
10^{30}	
10^{26}	energy radiated by Sun in 1 second
10^{22}	energy released in an earthquake
10^{18}	
10^{14}	energy released by annihilation of 1 kg of matter
10^{10}	energy in a lightning discharge
10^6	energy needed to charge a car battery
10^2	
10^{-2}	kinetic energy of a tennis ball during game
10^{-6}	energy in the beat of a fly's wing
10^{-10}	
10^{-14}	
10^{-18}	energy needed to remove electron from the surface of a metal
10^{-22}	
10^{-26}	

The SI system of fundamental and derived units

FUNDAMENTAL UNITS

Any measurement and every quantity can be thought of as being made up of two important parts:

1. the number and
2. the units.

Without **both** parts, the measurement does not make sense.

For example a person's age might be quoted as 'seventeen' but without the 'years' the situation is not clear. Are they 17 minutes, 17 months or 17 years old? In this case you would know if you saw them, but a statement like

$$\text{length} = 4.2$$

actually says nothing. Having said this, it is really surprising to see the number of candidates who forget to include the units in their answers to examination questions.

In order for the units to be understood, they need to be defined. There are many possible systems of measurement that have

been developed. In science we use the International System of units (SI). In SI, the **fundamental** or **base** units are as follows

Quantity	SI unit	SI symbol
Mass	kilogram	kg
Length	metre	m
Time	second	s
Electric current	ampere	A
Amount of substance	mole	mol
Temperature	kelvin	K
(Luminous intensity)	candela	cd

You do not need to know the precise definitions of any of these units in order to use them properly.

DERIVED UNITS

Having fixed the fundamental units, all other measurements can be expressed as different combinations of the fundamental units. In other words, all the other units are **derived units**. For example, the fundamental list of units does not contain a unit for the measurement of speed. The definition of speed can be used to work out the derived unit.

$$\text{Since speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Units of speed} = \frac{\text{units of distance}}{\text{units of time}}$$

$$= \frac{\text{metres}}{\text{seconds}} \quad (\text{pronounced 'metres per second'})$$

$$= \frac{\text{m}}{\text{s}}$$

$$= \text{m s}^{-1}$$

Of the many ways of writing this unit, the last way (m s^{-1}) is the best.

Sometimes particular combinations of fundamental units are so common that they are given a new derived name. For example, the unit of force is a derived unit – it turns out to be kg m s^{-2} . This unit is given a new name the newton (N) so that $1\text{N} = 1 \text{kg m s}^{-2}$.

The great thing about SI is that, so long as the numbers that are substituted into an equation are in SI units, then the answer will also come out in SI units. You can always 'play safe' by converting all the numbers into proper SI units. Sometimes, however, this would be a waste of time.

There are some situations where the use of SI becomes awkward. In astronomy, for example, the distances involved

are so large that the SI unit (the metre) always involves large orders of magnitudes. In these cases, the use of a different (but non SI) unit is very common. Astronomers can use the astronomical unit (AU), the light-year (ly) or the parsec (pc) as appropriate. Whatever the unit, the conversion to SI units is simple arithmetic.

$$1 \text{ AU} = 1.5 \times 10^{11} \text{ m}$$

$$1 \text{ ly} = 9.5 \times 10^{15} \text{ m}$$

$$1 \text{ pc} = 3.1 \times 10^{16} \text{ m}$$

There are also some units (for example the hour) which are so common that they are often used even though they do not form part of SI. Once again, before these numbers are substituted into equations they need to be converted. Some common unit conversions are given on page 3 of the IB data booklet.

The table below lists the SI derived units that you will meet.

SI derived unit	SI base unit	Alternative SI unit
newton (N)	kg m s^{-2}	–
pascal (Pa)	$\text{kg m}^{-1} \text{s}^{-2}$	N m^{-2}
hertz (Hz)	s^{-1}	–
joule (J)	$\text{kg m}^2 \text{s}^{-2}$	N m
watt (W)	$\text{kg m}^2 \text{s}^{-3}$	J s^{-1}
coulomb (C)	A s	–
volt (V)	$\text{kg m}^2 \text{s}^{-3} \text{A}^{-1}$	WA^{-1}
ohm (Ω)	$\text{kg m}^2 \text{s}^{-3} \text{A}^{-2}$	VA^{-1}
weber (Wb)	$\text{kg m}^2 \text{s}^{-2} \text{A}^{-1}$	V s
tesla (T)	$\text{kg s}^{-2} \text{A}^{-1}$	Wb m^{-2}
becquerel (Bq)	s^{-1}	–

PREFIXES

To avoid the repeated use of scientific notation, an alternative is to use one of the list of agreed prefixes given on page 2 in the IB data booklet. These can be very useful but they can also lead to errors in calculations. It is very easy to forget to include the conversion factor.

For example, $1 \text{ kW} = 1000 \text{ W}$. $1 \text{ mW} = 10^{-3} \text{ W}$ (in other words, $\frac{1\text{W}}{1000}$)

Estimation

ORDERS OF MAGNITUDE

It is important to develop a 'feeling' for some of the numbers that you use. When using a calculator, it is very easy to make a simple mistake (eg by entering the data incorrectly). A good way of checking the answer is to first make an estimate before resorting to the calculator. The multiple-choice paper (paper 1) does not allow the use of calculators.

Approximate values for each of the fundamental SI units are given below.

1 kg	A packet of sugar, 1 litre of water. A person would be about 50 kg or more
1 m	Distance between one's hands with arms outstretched
1 s	Duration of a heart beat (when resting – it can easily double with exercise)
1 amp	Current flowing from the mains electricity when a computer is connected. The maximum current to a domestic device would be about 10 A or so

1 kelvin 1K is a very low temperature. Water freezes at 273 K and boils at 373 K. Room temperature is about 300 K

1 mol 12 g of carbon-12. About the number of atoms of carbon in the 'lead' of a pencil

The same process can happen with some of the derived units.

1 m s⁻¹ Walking speed. A car moving at 30 m s⁻¹ would be fast

1 m s⁻² Quite a slow acceleration. The acceleration of gravity is 10 m s⁻²

1 N A small force – about the weight of an apple

1 V Batteries generally range from a few volts up to 20 or so, the mains is several hundred volts

1 Pa A very small pressure. Atmospheric pressure is about 10⁵ Pa

1 J A very small amount of energy – the work done lifting an apple off the ground

POSSIBLE REASONABLE ASSUMPTIONS

Everyday situations are very complex. In physics we often simplify a problem by making simple assumptions. Even if we know these assumptions are not absolutely true they allow us to gain an understanding of what is going on. At the end of the calculation it is often possible to go back and work out what would happen if our assumption turned out not to be true.

The table below lists some common assumptions. Be careful not to assume too much! Additionally we often have to assume that some quantity is constant even if we know that in reality it is varying slightly all the time.

Assumption	Example
Object treated as point particle	Mechanics: Linear motion and translational equilibrium
Friction is negligible	Many mechanics situations – but you need to be very careful
No thermal energy ("heat") loss	Almost all thermal situations
Mass of connecting string, etc. is negligible	Many mechanics situations
Resistance of ammeter is zero	Circuits
Resistance of voltmeter is infinite	Circuits
Internal resistance of battery is zero	Circuits
Material obeys Ohm's law	Circuits
Machine 100% efficient	Many situations
Gas is ideal	Thermodynamics
Collision is elastic	Only gas molecules have perfectly elastic collisions
Object radiates as a perfect black body	Thermal equilibrium, e.g. planets

SCIENTIFIC NOTATION

Numbers that are too big or too small for decimals are often written in **scientific notation**:

$$a \times 10^b$$

where a is a number between 1 and 10 and b is an integer.

e.g. $153.2 = 1.532 \times 10^2$; $0.00872 = 8.72 \times 10^{-3}$

SIGNIFICANT FIGURES

Any experimental measurement should be quoted with its uncertainty. This indicates the possible range of values for the quantity being measured. At the same time, the number of **significant figures** used will act as a guide to the amount of uncertainty. For example, a measurement of mass which is quoted as 23.456 g implies an uncertainty of ± 0.001 g (it has five significant figures), whereas one of 23.5 g implies an uncertainty of ± 0.1 g (it has three significant figures).

A simple rule for calculations (multiplication or division) is to quote the answer to the same number of significant digits as the LEAST precise value that is used.

For a more complete analysis of how to deal with uncertainties in calculated results, see page 5.

Uncertainties and error in experimental measurement

ERRORS – RANDOM AND SYSTEMATIC (PRECISION AND ACCURACY)

An experimental error just means that there is a difference between the recorded value and the 'perfect' or 'correct' value. Errors can be categorized as **random** or **systematic**.

Repeating readings does not reduce systematic errors.

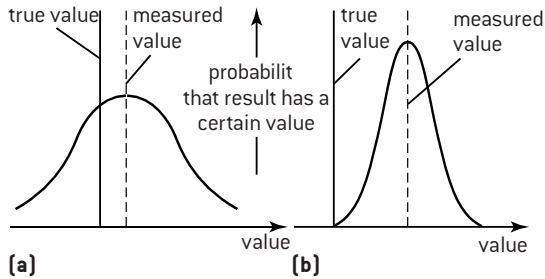
Sources of random errors include

- The readability of the instrument.
- The observer being less than perfect.
- The effects of a change in the surroundings.

Sources of systematic errors include

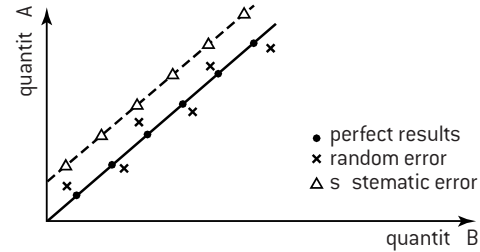
- An instrument with **zero error**. To correct for zero error the value should be subtracted from every reading.
- An instrument being wrongly **calibrated**.
- The observer being less than perfect in the same way every measurement.

An **accurate** experiment is one that has a small systematic error, whereas a **precise** experiment is one that has a small random error.



Two examples illustrating the nature of experimental results:
 (a) an accurate experiment of low precision
 (b) a less accurate but more precise experiment.

Systematic and random errors can often be recognized from a graph of the results.



Perfect results, random and systematic errors of two proportional quantities.

ESTIMATING THE UNCERTAINTY RANGE

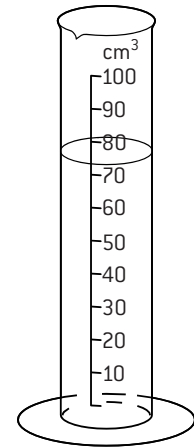
An **uncertainty range** applies to any experimental value. The idea is that, instead of just giving one value that implies perfection, we give the likely range for the measurement.

1. Estimating from first principles

All measurement involves a readability error. If we use a measuring cylinder to find the volume of a liquid, we might think that the best estimate is 73 cm³, but we know that it is not exactly this value (73.000000000000 cm³).

Uncertainty range is ± 5 cm³. We say volume = 73 ± 5 cm³.

Normally the uncertainty range due to readability is estimated as below.



Device	Example	Uncertainty
Analogue scale	Rulers, meters with moving pointers	\pm (half the smallest scale division)
Digital scale	Top-pan balances, digital meters	\pm (the smallest scale division)

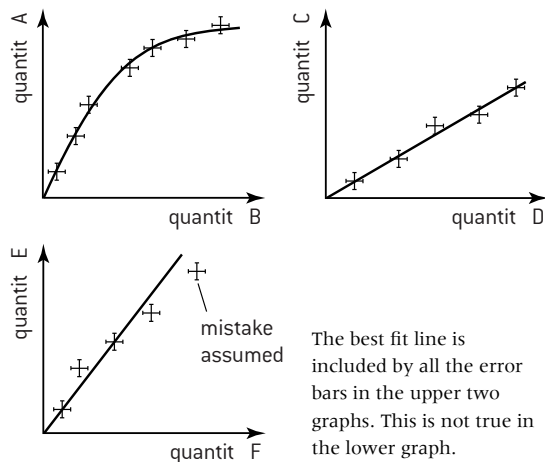
2. Estimating uncertainty range from several repeated measurements

If the time taken for a trolley to go down a slope is measured five times, the readings in seconds might be 2.01, 1.82, 1.97, 2.16 and 1.94. The average of these five readings is 1.98 s. The deviation of the largest and smallest readings can be calculated ($2.16 - 1.98 = 0.18$; $1.98 - 1.82 = 0.16$). The largest value is taken as the uncertainty range. In this example the time is 1.98 ± 0.18 s. It would also be appropriate to quote this as 2.0 ± 0.2 s.

GRAPHICAL REPRESENTATION OF UNCERTAINTY

In many situations the best method of presenting and analysing data is to use a graph. If this is the case, a neat way of representing the uncertainties is to use **error bars**. The graphs below explain their use.

Since the error bar represents the uncertainty range, the 'best-fit' line of the graph should pass through ALL of the rectangles created by the error bars.



SIGNIFICANT FIGURES IN UNCERTAINTIES

In order to be cautious when quoting uncertainties, final values from calculations are often rounded up to one significant figure, e.g. a calculation that finds the value of a force to be 4.264 N with an uncertainty of ± 0.362 N is quoted as 4.3 ± 0.4 N. This can be unnecessarily pessimistic and it is also acceptable to express uncertainties to two significant figures. For example, the charge on an electron is $1.602176565 \times 10^{-19}$ C $\pm 0.000000035 \times 10^{-19}$ C. In data booklets this is sometimes expressed as $1.602176565(35) \times 10^{-19}$ C.

Uncertainties in calculated results

MATHEMATICAL REPRESENTATION OF UNCERTAINTIES

For example if the mass of a block was measured as 10 ± 1 g and the volume was measured as 5.0 ± 0.2 cm³, then the full calculations for the density would be as follows.

$$\text{Best value or density} = \frac{\text{mass}}{\text{volume}} = \frac{10}{5} = 2.0 \text{ g cm}^{-3}$$

$$\text{The largest possible value of density} = \frac{11}{4.8} = 2.292 \text{ g cm}^{-3}$$

$$\text{The smallest possible value of density} = \frac{9}{5.2} = 1.731 \text{ g cm}^{-3}$$

$$\text{Rounding these values gives density} = 2.0 \pm 0.3 \text{ g cm}^{-3}$$

We can express this uncertainty in one of three ways – using **absolute, fractional or percentage uncertainties**.

If a quantity p is measured then the absolute uncertainty would be expressed as $\pm \Delta p$.

Then the fractional uncertainty is

$$\frac{\pm \Delta p}{p},$$

which makes the percentage uncertainty

$$\frac{\pm \Delta p}{p} \times 100\%.$$

In the example above, the fractional uncertainty of the density is ± 0.15 or $\pm 15\%$.

Thus equivalent ways of expressing this error are

$$\text{density} = 2.0 \pm 0.3 \text{ g cm}^{-3}$$

$$\text{OR density} = 2.0 \text{ g cm}^{-3} \pm 15\%$$

Working out the uncertainty range is very time consuming. There are some mathematical ‘short-cuts’ that can be used. These are introduced in the boxes below.

MULTIPLICATION, DIVISION OR POWERS

Whenever two or more quantities are multiplied or divided and they each have uncertainties, the overall uncertainty is approximately equal to the **addition** of the **percentage** (fractional) uncertainties.

Using the same numbers from above,

$$\Delta m = \pm 1 \text{ g}$$

$$\frac{\Delta m}{m} = \pm \left(\frac{1 \text{ g}}{10 \text{ g}} \right) = \pm 0.1 = \pm 10\%$$

$$\Delta V = \pm 0.2 \text{ cm}^3$$

$$\frac{\Delta V}{V} = \pm \left(\frac{0.2 \text{ cm}^3}{5 \text{ cm}^3} \right) = \pm 0.04 = \pm 4\%$$

$$\text{The total \% uncertainty in the result} = \pm (10 + 4)\% = \pm 14\%$$

$$14\% \text{ of } 2.0 \text{ g cm}^{-3} = 0.28 \text{ g cm}^{-3} \approx 0.3 \text{ g cm}^{-3}$$

So density = $2.0 \pm 0.3 \text{ g cm}^{-3}$ as before.

In symbols, if $y = \frac{ab}{c}$

$$\text{Then } \frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c} \quad [\text{note this is ALWAYS added}]$$

Power relationships are just a special case of this law.

$$\text{If } y = a^n$$

$$\text{Then } \frac{\Delta y}{y} = \left| n \frac{\Delta a}{a} \right| \quad (\text{always positive})$$

For example if a cube is measured to be 4.0 ± 0.1 cm in length along each side, then

$$\% \text{ Uncertainty in length} = \pm \frac{0.1}{4.0} = \pm 2.5\%$$

$$\text{Volume} = (\text{length})^3 = (4.0)^3 = 64 \text{ cm}^3$$

$$\begin{aligned} \% \text{ Uncertainty in [volume]} &= \% \text{ uncertainty in } [(\text{length})^3] \\ &= 3 \times (\% \text{ uncertainty in [length]}) \\ &= 3 \times (\pm 2.5\%) \\ &= \pm 7.5\% \end{aligned}$$

$$\begin{aligned} \text{Absolute uncertainty} &= 7.5\% \text{ of } 64 \text{ cm}^3 \\ &= 4.8 \text{ cm}^3 \approx 5 \text{ cm}^3 \end{aligned}$$

$$\text{Thus volume of cube} = 64 \pm 5 \text{ cm}^3$$

OTHER MATHEMATICAL OPERATIONS

If the calculation involves mathematical operations other than multiplication, division or raising to a power, then one has to find the highest and lowest possible values.

Addition or subtraction

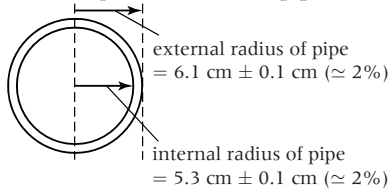
Whenever two or more quantities are added or subtracted and they each have uncertainties, the overall uncertainty is equal to the **addition** of the **absolute** uncertainties.

In symbols

$$\text{If } y = a \pm b$$

$$\Delta y = \Delta a + \Delta b \quad (\text{note ALWAYS added})$$

uncertainty of thickness in a pipe wall



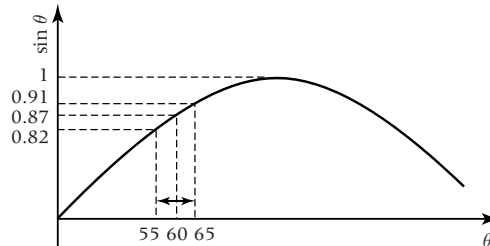
$$\begin{aligned} \text{thickness of pipe wall} &= 6.1 - 5.3 \text{ cm} \\ &= 0.8 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{uncertainty in thickness} &= \pm(0.1 + 0.1) \text{ cm} \\ &= 0.2 \text{ cm} \\ &= \pm 25\% \end{aligned}$$

Other functions

There are no ‘short-cuts’ possible. Find the highest and lowest values.

e.g. uncertainty of $\sin \theta$ if $\theta = 60^\circ \pm 5^\circ$



$$\text{if } \theta = 60^\circ \pm 5^\circ$$

$$\text{best value to } \sin \theta = 0.87$$

$$\text{max. } \sin \theta = 0.91$$

$$\text{min. } \sin \theta = 0.82$$

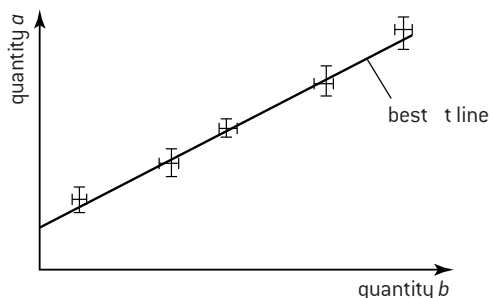
$$\therefore \sin \theta = 0.87 \pm 0.05$$

worst value used

Uncertainties in graphs

ERROR BARS

Plotting a graph allows one to visualize all the readings at one time. Ideally all of the points should be plotted with their error bars. In principle, the size of the error bar could well be different for every single point and so they should be individually worked out.



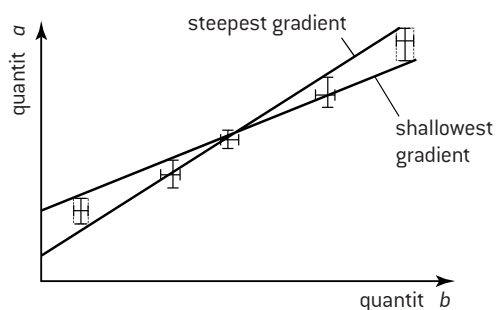
A full analysis in order to determine the uncertainties in the gradient of a best straight-line graph should **always make use of the error bars for all of the data points.**

In practice, it would often take too much time to add all the correct error bars, so some (or all) of the following short-cuts could be considered.

- Rather than working out error bars for each point – use the worst value and assume that all of the other error bars are the same.
- Only plot the error bar for the 'worst' point, i.e. the point that is furthest from the line of best fit. If the line of best fit is within the limits of this error bar, then it will probably be within the limits of all the error bars.
- Only plot the error bars for the first and the last points. These are often the most important points when considering the uncertainty ranges calculated for the gradient or the intercept (see right).
- Only include the error bars for the axis that has the worst uncertainty.

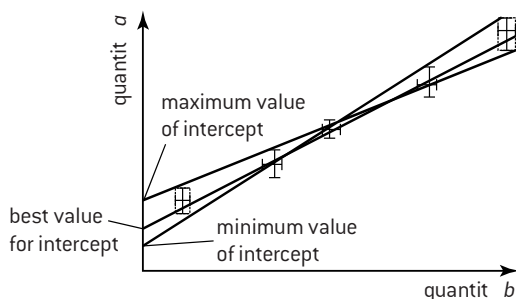
UNCERTAIN IN SLOPES

If the gradient of the graph has been used to calculate a quantity, then the uncertainties of the points will give rise to an uncertainty in the gradient. Using the steepest and the shallowest lines possible (i.e. the lines that are still consistent with the error bars) the uncertainty range for the gradient is obtained. This process is represented below.



UNCERTAIN IN INTERCEPTS

If the intercept of the graph has been used to calculate a quantity, then the uncertainties of the points will give rise to an uncertainty in the intercept. Using the steepest and the shallowest lines possible (i.e. the lines that are still consistent with the error bars) we can obtain the uncertainty in the result. This process is represented below.



Vectors and scalars

DIFFERENCE BETWEEN VECTORS AND SCALARS

If you measure any quantity, it must have a number AND a unit. Together they express the **magnitude** of the quantity. Some quantities also have a direction associated with them. A quantity that has magnitude and direction is called a **vector** quantity whereas one that has only magnitude is called a **scalar** quantity. For example, all forces are vectors.

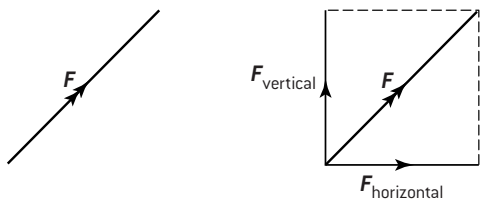
The table lists some common quantities. The first two quantities in the table are linked to one another by their definitions (see page 9). All the others are in no particular order.

Vectors	Scalars
Displacement \longleftrightarrow	Distance
Velocity \longleftrightarrow	Speed
Acceleration	Mass
Force	Energy (all forms)
Momentum	Temperature
Electric field strength	Potential or potential difference
Magnetic field strength	Density
Gravitational field strength	Area

Although the vectors used in many of the given examples are forces, the techniques can be applied to all vectors.

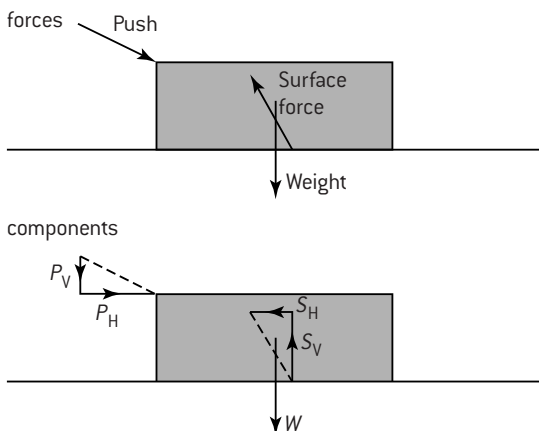
COMPONENTS OF VECTORS

It is also possible to 'split' one vector into two (or more) vectors. This process is called **resolving** and the vectors that we get are called the **components** of the original vector. This can be a very useful way of analysing a situation if we choose to resolve all the vectors into two directions that are at right angles to one another.



Splitting a vector into components

These 'mutually perpendicular' directions are totally independent of each other and can be analysed separately. If appropriate, both directions can then be combined at the end to work out the final resultant vector.



Pushing a block along a rough surface

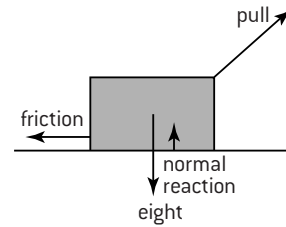
REPRESENTING VECTORS

In most books a bold letter is used to represent a vector whereas a normal letter represents a scalar. For example \mathbf{F} would be used to represent a force in magnitude AND direction. The list below shows some other recognized methods.

$$\vec{F}, \bar{F} \text{ or } \underline{F}$$

Vectors are best shown in diagrams using arrows:

- the relative magnitudes of the vectors involved are shown by the relative length of the arrows
- the direction of the vectors is shown by the direction of the arrows.

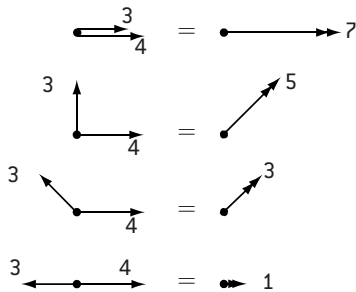


ADDITION / SUBTRACTION OF VECTORS

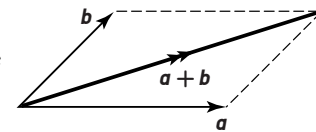
If we have a 3 N and a 4 N force, the overall force (resultant force) can be anything between 1 N and 7 N

depending on the directions involved.

The way to take the directions into account is to do a scale diagram and use the parallelogram law of vectors.



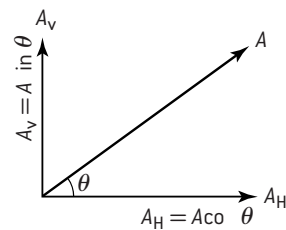
This process is the same as adding vectors in turn – the 'tail' of one vector is drawn starting from the head of the previous vector.



Parallelogram of vectors

TRIGONOMETRY

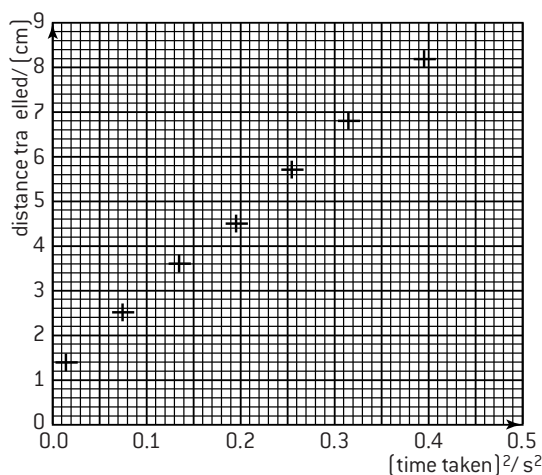
Vector problems can always be solved using scale diagrams, but this can be very time consuming. The mathematics of trigonometry often makes it much easier to use the mathematical functions of sine or cosine. This is particularly appropriate when resolving. The diagram below shows how to calculate the values of either of these components.



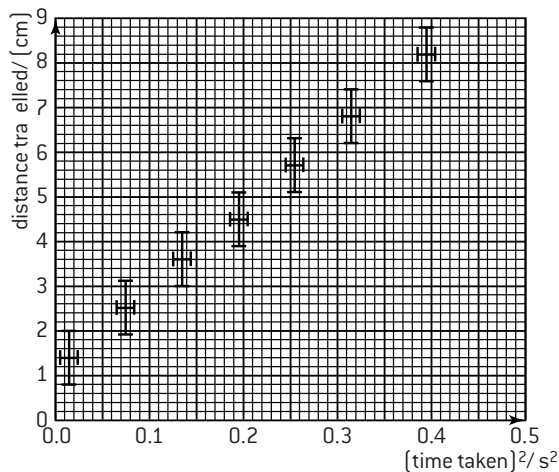
See page 14 for an example.

IB Questions – measurement and uncertainties

1. An object is rolled from rest down an inclined plane. The distance travelled by the object was measured at seven different times. A graph was then constructed of the distance travelled against the (time taken)² as shown below.

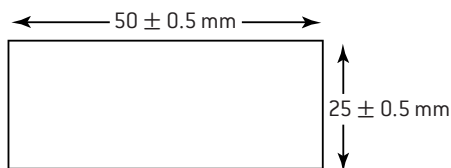


- a) (i) What quantity is given by the gradient of such a graph? [2]
 (ii) Explain why the graph suggests that the collected data is valid but includes a systematic error. [2]
 (iii) Do these results suggest that distance is proportional to (time taken)²? Explain your answer. [2]
 (iv) Making allowance for the systematic error, calculate the acceleration of the object. [2]
- b) The following graph shows that same data after the uncertainty ranges have been calculated and drawn as error bars.



Add two lines to show the range of the possible acceptable values for the gradient of the graph. [2]

2. The lengths of the sides of a rectangular plate are measured, and the diagram shows the measured values with their uncertainties.



Which one of the following would be the best estimate of the percentage uncertainty in the calculated area of the plate?

- A. $\pm 0.02\%$ C. $\pm 3\%$
 B. $\pm 1\%$ D. $\pm 5\%$

3. A stone is dropped down a well and hits the water 2.0 s after it is released. Using the equation $d = \frac{1}{2}gt^2$ and taking $g = 9.81 \text{ m s}^{-2}$, a calculator yields a value for the depth d of the well as 19.62 m. If the time is measured to $\pm 0.1 \text{ s}$ then the best estimate of the absolute error in d is

- A. $\pm 0.1 \text{ m}$ C. $\pm 1.0 \text{ m}$
 B. $\pm 0.2 \text{ m}$ D. $\pm 2.0 \text{ m}$

4. In order to determine the density of a certain type of wood, the following measurements were made on a **cube** of the wood.

Mass = 493 g
 Length of **each** side = 9.3 cm

The percentage uncertainty in the measurement of mass is $\pm 0.5\%$ and the percentage uncertainty in the measurement of length is $\pm 1.0\%$.

The best estimate for the uncertainty in the density is

- A. $\pm 0.5\%$ C. $\pm 3.0\%$
 B. $\pm 1.5\%$ D. $\pm 3.5\%$

5. Astronauts wish to determine the gravitational acceleration on Planet X by dropping stones from an overhanging cliff. Using a steel tape measure they measure the height of the cliff as $s = 7.64 \text{ m} \pm 0.01 \text{ m}$. They then drop three similar stones from the cliff, timing each fall using a hand-held electronic stopwatch which displays readings to one-hundredth of a second. The recorded times for three drops are 2.46 s, 2.31 s and 2.40 s.

- a) Explain why the time readings vary by more than a tenth of a second, although the stopwatch gives readings to one hundredth of a second. [1]
- b) Obtain the average time t to fall, and write it in the form (value \pm uncertainty), to the appropriate number of significant digits. [1]
- c) The astronauts then determine the gravitational acceleration a_g on the planet using the formula $a_g = \frac{2s}{t^2}$. Calculate a_g from the values of s and t , and determine the uncertainty in the calculated value. Express the result in the form $a_g = (\text{value} \pm \text{uncertainty})$, to the appropriate number of significant digits. [3]



6. This question is about finding the relationship between the forces between magnets and their separations.

In an experiment, two magnets were placed with their North-seeking poles facing one another. The force of repulsion, f , and the separation of the magnets, d , were measured and the results are shown in the table below.

Separation d/m	Force of repulsion f/N
0.04	4.00
0.05	1.98
0.07	0.74
0.09	0.32

- a) Plot a graph of $\log(\text{force})$ against $\log(\text{distance})$. [3]
 b) The law relating the force to the separation is of the form $f = kd^n$
 (i) Use the graph to find the value of n . [2]
 (ii) Calculate a value for k , giving its units. [3]