Mo ion

DEFINITIONS

These technical terms should not be con used with their 'everyday' use. In particular one should note that

- Vector quantities always have a direction associated with them.
- Generally, velocity and speed are NOT the same thing. This is particularly important i the object is not going in a straight line.
- The units o acceleration come rom its definition. (m s^{-1}) ÷ s = m s^{-2}.
- The definition o acceleration is precise. It is related to the change in **velocity** (not the same thing as the change in speed). Whenever the motion o an object changes, it is called acceleration. For this reason acceleration does not necessarily mean constantly increasing speed – it is possible to accelerate while at constant speed i the direction is changed.
- A deceleration means slowing down, i.e. negative acceleration i velocity is positive.

	Symbol	Definition	Example	SI Unit	Vector or scalar?
Displacement	*s*	The distance moved in a particular direction.	The displacement rom London to Rome is 1.43 × 10^{6} m southeast.	m	Vector
Velocity	or *u*	The rate o change o displacement. velocity = $\frac{\text{change o displacement}}{\text{time taken}}$	The average velocity during a flight rom London to Rome is 160 m s^{-1} southeast.	m s^{-1}	Vector
Speed	or *u*	The rate o change o distance. speed = $\frac{\text{distance gone}}{\text{time taken}}$	The average speed during a flight rom London to Rome is 160 m s^{-1}	m s^{-1}	Scalar
Acceleration	*a*	The rate o change o velocity. acceleration = $\frac{\text{change o velocity}}{\text{time taken}}$	The average acceleration o a plane on the runway during take-o is 3.5 m s^{-2} in a orwards direction. This means that on average, its velocity changes every second by 3.5 m s^{-1}	m s^{-2}	Vector

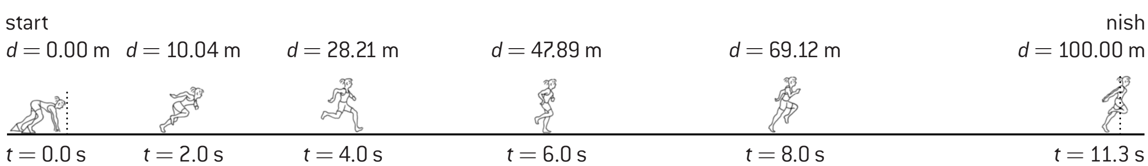
INSTANTANEOUS VS AVERAGE

It should be noticed that the average value (over a period o time) is very di erent to the instantaneous value (at one particular time).

In the example below, the positions o a sprinter are shown at di erent times a ter the start o a race.

The average speed over the whole race is easy to work out. It is the total distance (100 m) divided by the total time (11.3 s) giving 8.8 m s^{-1}.

But during the race, her instantaneous speed was changing all the time. At the end o the first 2.0 seconds, she had travelled 10.04 m. This means that her average speed over the first 2.0 seconds was 5.02 m s^{-1}. During these first two seconds, her instantaneous speed was increasing – she was accelerating. I she started at rest (speed = 0.00 m s^{-1}) and her **average** speed (over the whole two seconds) was 5.02 m s^{-1} then her instantaneous speed at 2 seconds must be more than this. In act the instantaneous speed or this sprinter was 9.23 m s^{-1}, but it would not be possible to work this out rom the in ormation given.

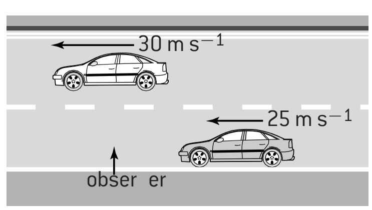


FRAMES OF REFERENCE

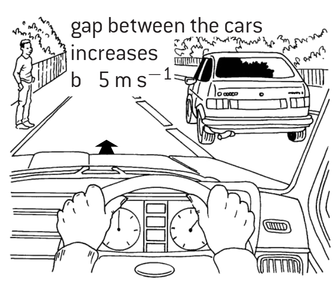
I two things are moving in the same straight line but are travelling at di erent speeds, then we can work out their **relative velocities** by simple addition or subtraction as appropriate. For example, imagine two cars travelling along a straight road at di erent speeds.

I one car (travelling at 30 m s^{-1}) overtakes the other car (travelling at 25 m s^{-1}), then according to the driver o the slow car, the relative velocity o the ast car is +5 m s^{-1}.

In technical terms what we are doing is moving rom one ** rame o re erence** into another. The velocities o 25 m s^{-1} and 30 m s^{-1} were measured according to a stationary observer on the side o the road. We moved rom this rame o re erence into the driver's rame o re erence.



one car overtaking another, as seen by an observer on the side o the road.



one car overtaking another, as seen by the driver o the slow car.

Graphical representation of motion

THE USE OF GRAPHS

Graphs are very use ul or representing the changes that happen when an object is in motion. There are three possible graphs that can provide use ul in ormation

- displacement–time or distance–time graphs
- velocity–time or speed–time graphs
- acceleration–time graphs.

There are two common methods o determining particular physical quantities rom these graphs. The particular physical quantity determined depends on what is being plotted on the graph.

1. Finding the gradient of the line.

To be a little more precise, one could find either the gradient o

- a straight-line section o the graph (this finds an average value), or
- the tangent to the graph at one point (this finds an instantaneous value).

2. Finding the area under the line.

To make things simple at the beginning, the graphs are normally introduced by considering objects that are just moving in one particular direction. I this is the case then there is not much di erence between the scalar versions (distance or speed) and the vector versions (displacement or velocity) as the directions are clear rom the situation. More complicated graphs can look at the component o a velocity in a particular direction.

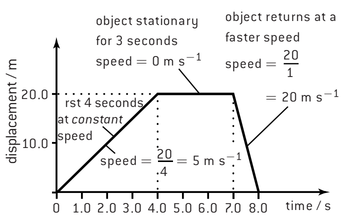
I the object moves orward then backward (or up then down), we distinguish the two velocities by choosing which direction to call positive. It does not matter which direction we choose, but it should be clearly labelled on the graph.

Many examination candidates get the three types o graph muddled up. For example a speed–time graph might be interpreted as a distance–time graph or even an acceleration–time graph. Always look at the axes o a graph very care ully.

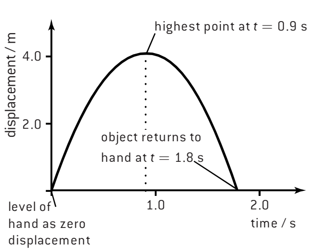
DISPLACEMENT–TIME GRAPHS

- The gradient o a displacement–time graph is the velocity
- The area under a displacement–time graph does not represent anything use ul

E amples



Object moves at constant speed, stops then returns.

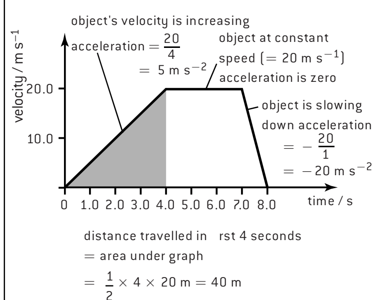


Object is thrown vertically upwards.

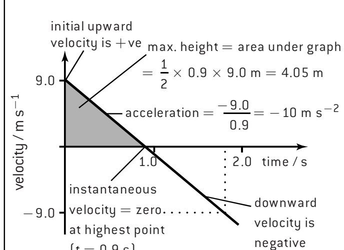
VELOCITY–TIME GRAPHS

- The gradient o a velocity–time graph is the acceleration
- The area under a velocity–time graph is the displacement

E amples



Object moves with constant acceleration, then constant velocity, then decelerates.

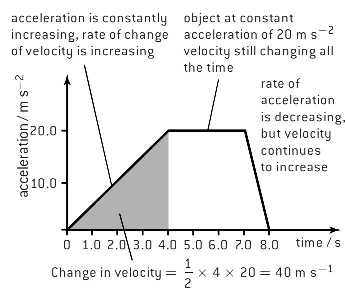


Object is thrown vertically upwards.

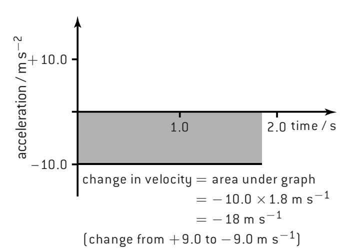
ACCELERATION–TIME GRAPHS

- The gradient o an acceleration–time graph is not o ten use ul (it is actually the rate o change o acceleration)
- The area under an acceleration–time graph is the change in velocity

E amples



Object moves with increasing, then constant, then decreasing acceleration.



Object is thrown vertically upwards.

EXAMPLE OF EQUATION OF UNIFORM MOTION

A car accelerates uni ormly rom rest. A ter 8 s it has travelled 120 m. Calculate: (i) its average acceleration (ii) its instantaneous speed a ter 8 s

(i) $s = ut + \frac{1}{2}at^2$

$\therefore\ 120 = 0 \times 8 + \frac{1}{2}a \times 8^2 = 32\,a$

$\underline{\underline{a = 3.75\ \text{m s}^{-2}}}$

(ii) $\;^2 = u^2 + 2\,as$

$= 0 + 2 \times 3.75 \times 120$

$= 900$

$\therefore\ \underline{\underline{= 30\ \text{m s}^{-1}}}$

Uniformly accelera ed mo ion

PRACTICAL CALCULATIONS

In order to determine how the velocity (or the acceleration) o an object varies in real situations, it is o ten necessary to record its motion. Possible laboratory methods include.

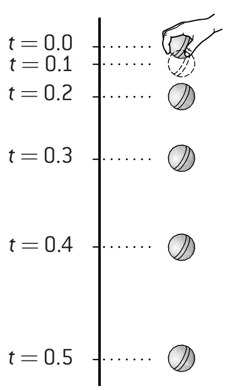
Ligh ga es

A light gate is a device that senses when an object cuts through a beam o light. The time or which the beam is broken is recorded. I the length o the object that breaks the beam is known, the average speed o the object through the gate can be calculated.

Alternatively, two light gates and a timer can be used to calculate the average velocity between the two gates. Several light gates and a computer can be joined together to make direct calculations o velocity or acceleration.

S robe pho ography

A strobe light gives out very brie flashes o light at fixed time intervals. I a camera is pointed at an object and the only source o light is the strobe light, then the developed picture will have captured an object's motion.



Ticker imer

A ticker timer can be arranged to make dots on a strip o paper at regular intervals o time (typically every fi tieth o a second). I the piece o paper is attached to an object, and the object is allowed to all, the dots on the strip will have recorded the distance moved by the object in a known time.

EQUATIONS OF UNIFORM MOTION

These equations can only be used when the acceleration is constant – don't orget to check i this is the case!

The list o variables to be considered (and their symbols) is as ollows

u	initial velocity
v	final velocity
a	acceleration (const)
t	time taken
s	distance travelled

The ollowing equations link these di erent quantities.

$$v = u + at$$

$$s = \left(\frac{u+v}{2}\right)t$$

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

The first equation is derived rom the definition o acceleration. In terms o these symbols, this definition would be

$$a = \frac{(v-u)}{t}$$

This can be rearranged to give the first equation.

$$v = u + at \qquad (1)$$

The second equation comes rom the definition o average velocity.

$$\text{average velocity} = \frac{s}{t}$$

Since the velocity is changing uni ormly we know that this average velocity must be given by

$$\text{average velocity} = \frac{(v+u)}{2}$$

or $\frac{s}{t} = \frac{(u+v)}{2}$

This can be rearranged to give

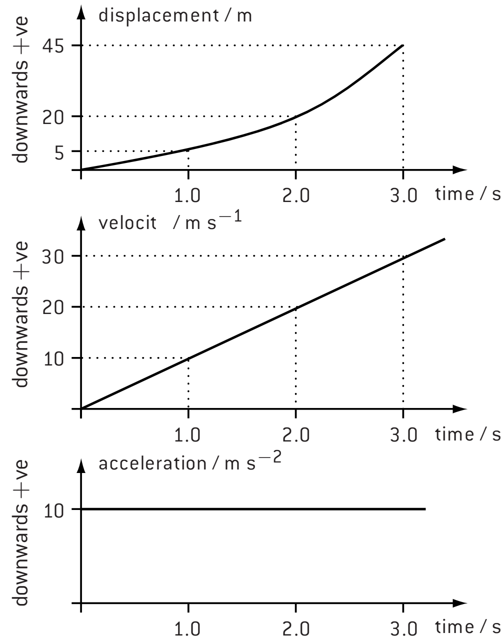
$$s = \frac{(u+v)t}{2} \qquad (2)$$

The other equations o motion can be derived by using these two equations and substituting or one o the variables (see previous page or an example o their use).

FALLING OBJECTS

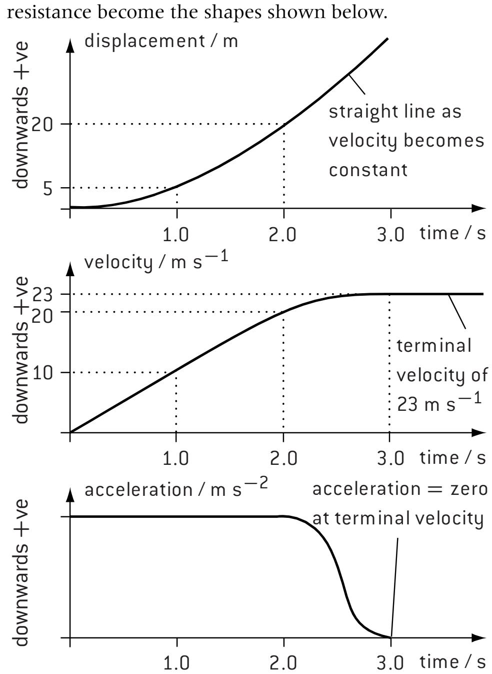
A very important example o uni ormly accelerated motion is the vertical motion o an object in a uni orm **gravitational field**. I we ignore the e ects o air resistance, this is known as being in **ree- all**.

Taking down as positive, the graphs o the motion o any object in ree- all are



In the absence o air resistance, all alling objects have the SAME acceleration o ree- all, INDEPENDENT o their mass.

Air resistance will (eventually) a ect the motion o all objects. Typically, the graphs o a alling object a ected by air resistance become the shapes shown below.

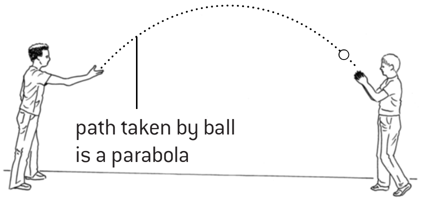


As the graphs show, the velocity does not keep on rising. It eventually reaches a maximum or **terminal velocity**. A piece o alling paper will reach its terminal velocity in a much shorter time than a alling book.

Projec ile mo ion

COMPONENTS OF PROJECTILE MOTION

If two children are throwing and catching a tennis ball between them, the path of the ball is always the same shape. This motion is known as **projectile motion** and the shape is called a **parabola**.

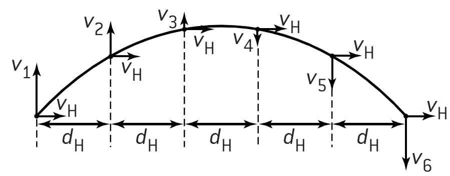


The only forces acting during its flight are gravity and friction. In many situations, air resistance can be ignored.

It is moving horizontally and vertically **at the same time** but the horizontal and vertical components of the motion are **independent** of one another. Assuming the gravitional force is constant, this is always true.

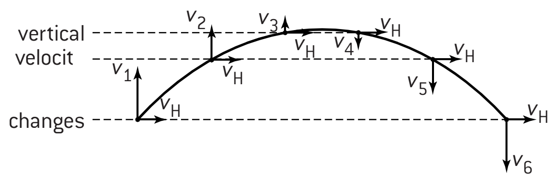
Horizon al componen

There are no forces in the horizontal direction, so there is no horizontal acceleration. This means that the horizontal velocity must be constant.



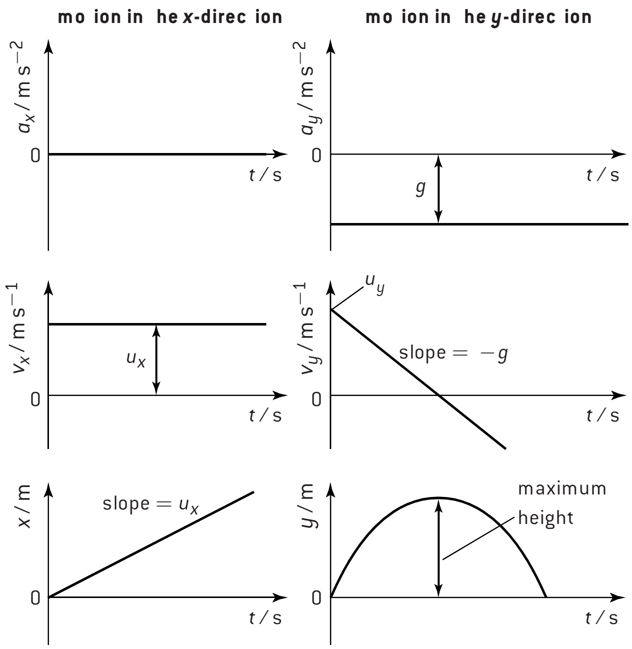
Ver ical componen

There is a constant vertical force acting down, so there is a constant vertical acceleration. The value of the vertical acceleration is 10 m s^{-2}, which is the acceleration due to gravity.



MATHEMATICS OF PARABOLIC MOTION

The graphs of the components of parabolic motion are shown below.



Once the components have been worked out, the actual velocities (or displacements) at any time can be worked out by vector addition.

The solution of any problem involving projectile motion is as follows:

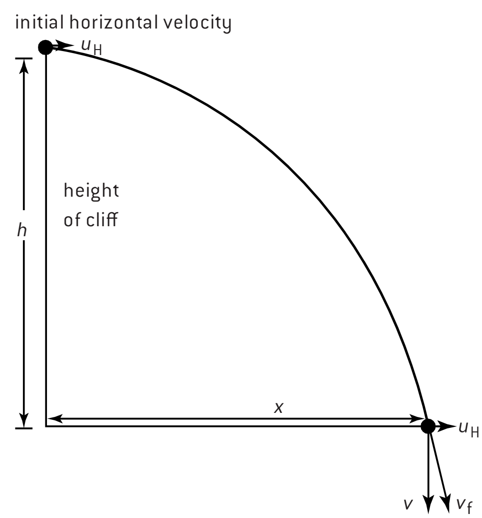
- use the angle of launch to resolve the initial velocity into components.
- the time of flight will be determined by the vertical component of velocity.
- the range will be determined by the horizontal component (and the time of flight).
- the velocity at any point can be found by vector addition.

Useful 'short-cuts' in calculations include the following facts:

- for a given speed, the greatest range is achieved if the launch angle is 45°.
- if two objects are released together, one with a horizontal velocity and one from rest, they will both hit the ground together.

EXAMPLE

A projectile is launched horizontally from the top of a cliff.



vertical motion	horizontal motion
$u = 0$	$u = u_H$
$v = ?$	$v = u_H$
$a = 10$ m s^{-2}	$a = 0$
$s = h$	$s = x$
$t = ?$	$t = ?$

$$s = ut + \frac{1}{2}at^2$$

so $h = 0 + \frac{1}{2} \times 10 \times t^2$

$\therefore$ $t^2 = \frac{2h}{10}$

$$t = \sqrt{\frac{2h}{10}} \text{ s}$$

Since $= u + at$

$= 0 + 10\sqrt{\frac{2h}{10}}$ m

$= \sqrt{20h}$ m s^{-1}

$x = u_H \times t$

$= u_H \times \sqrt{\frac{2h}{10}}$ m

The final velocity v_f is the vector addition of v and u_H.

Fluid resistance and free-fall

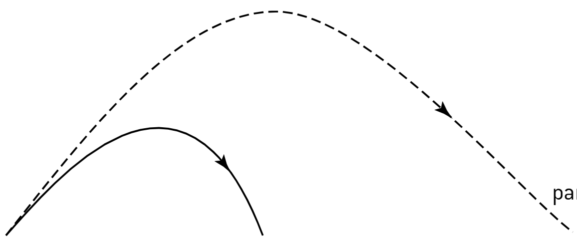
FLUID RESISTANCE

When an object moves through a fluid (a liquid or a gas), there will be a frictional fluid resistance that affects the object's motion. An example of this effect is the terminal velocity that is reached by a free-falling object, e.g. a spherical mass falling through a liquid or a parachutist falling towards the Earth. See page 11 for how the motion graphs will be altered in these situations.

Modelling the precise effect of fluid resistance on moving objects is complex but simple predictions are possible. The Engineering Physics option (see page 167) introduces a mathematical analysis of the frictional drag force that acts on a perfect sphere when it moves through a fluid. Key points to note are that:

- Viscous drag acts to oppose motion through a fluid
- The drag force is dependent on:
  - Relative velocity of the object with respect to the fluid
  - The shape and size of the object (whether the object is aerodynamic or not)
  - The fluid used (and a property called its viscosity).

For example page 12 shows how, in the absence of fluid resistance, an object that is in projectile motion will follow a parabolic path. When fluid resistance is taken into account, the vertical and the horizontal components of velocity will both be reduced. The effect will be a reduced range and, in the extreme, the horizontal velocity can be reduced to near zero.



EXPERIMENT TO DETERMINE FREE-FALL ACCELERATION

All experiments to determine the free-fall acceleration for an object are based on the use of a constant acceleration equation with recorded measurements of displacement and time. Some experimental set-ups will be more sophisticated and use more equipment than others. This increased use of technology potentially brings greater precision but can introduce more complications. Simple equipment often means that, with a limited time available for experimentation, it is easier for many repetitions to be attempted.

If an object free-falls a height, h, from rest in a time, t, the acceleration, g, can be calculated using $s = ut + \frac{1}{2}at^2$ which rearranges to give $= \frac{2h}{t^2}$. Rather than just calculating a single value, a more reliable value comes from taking a series of measurement of the different times of fall for different heights $h = \frac{1}{2}gt^2$. A graph of h on the -axis against t^2 on the x-axis will give a straight line graph that goes through the origin with a gradient equal to $\frac{1}{2}g$, making g twice the gradient.

Possible set-ups include:

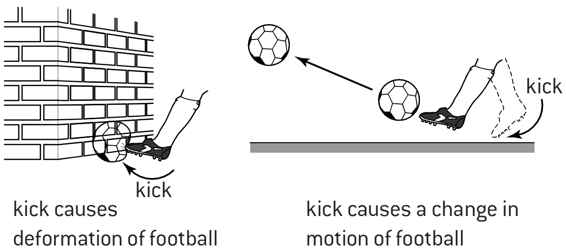
Set-up	Comments
Direct measurement of a falling object, e.g. ball bearing with a stop watch and a metre ruler	Very simple set-up meaning many repetitions easily achieved so random error can be eliminated. If height of fall is carefully controlled, great precision is possible even though equipment is standard. For a simple everyday object such as a ball bearing, the effect of air resistance will be negligible in the laboratory whereas the effect of air resistance on a Ping-Pong ball will be significant.
Electromagnet release and electronic timing version of the above	The increased precision of the timing can improve accuracy but set-up will take longer. Introduction of technology can mean that systematic errors are harder to identify.
Motion of falling object automatically recording on ticker-tape attached to falling object	Physical record allows detailed analysis of motion and thus allows the object's whole fall to be considered (not just the overall time taken) and for the data to be graphically analysed. Addition of moving paper tape introduces friction to the motion, however.
Distance sensor and data logger	All measurements can be automated and very precise. Software can be programmed to perform all the calculations and to plot appropriate graphs. Experimenter needs to understand how to operate the data logger and associated software.
Video analysis of falling object	Capturing a visual record of the object's fall against a known scale, allows detailed measurements to be taken. Timing information from the video recording needed, which often involves ICT.

Forces and free-bod diagrams

FORCES – WHAT THEY ARE AND WHAT THEY DO

In the examples below, a force (the kick) can cause deformation (the ball changes shape) or a change in motion (the ball gains a velocity). There are many different types of forces, but in general terms one can describe any force as 'the cause of a deformation or a velocity change'. The SI unit for the measurement of forces is the newton (N).

(a) deformation **(b) change in velocit**



Effect of a force on a football

- A (resultant) force causes a CHANGE in velocity. If the (resultant) force is zero then the velocity is constant. Remember a change in velocity is called an acceleration, so we can say that **a force causes an acceleration**. A (resultant) force is NOT needed for a constant velocity (see page 16).
- The fact that a force can cause deformation is also important, but the deformation of the ball was, in fact, not caused by just one force – there was another one from the wall.
- One force can act on only one object. To be absolutely precise the description of a force should include
  - its magnitude
  - its direction
  - the object on which it acts (or the part of a large object)
  - the object that exerts the force
  - the nature of the force

A description of the force shown in the example would thus be 'a 50 N push at 20° to the horizontal acting ON the football FROM the boot'.

DIFFERENT TYPES OF FORCES

The following words all describe the forces (the pushes or pulls) that exist in nature.

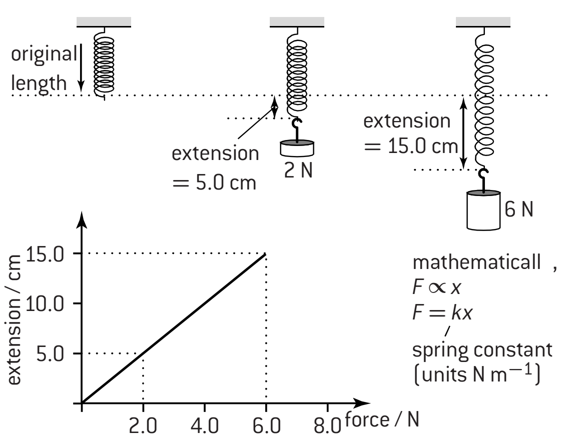
Gravitational force	**Normal reaction**	**Compression**
Electrostatic force	**Friction**	**Upthrust**
Magnetic force	**Tension**	**Lift**

One way of categorizing these forces is whether they result from the contact between two surfaces or whether the force exists even if a distance separates the objects.

The origin of all these everyday forces is either gravitational or electromagnetic. The vast majority of everyday effects that we observe are due to electromagnetic forces.

MEASURING FORCES

The simplest experimental method for measuring the size of a force is to use the **extension** of a spring. When a spring is in tension it increases in length. The difference between the natural length and stretched length is called the extension of a spring.



Hooke's law

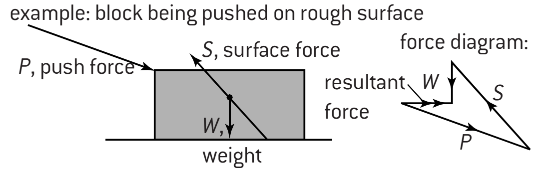
Hooke's law states that up to the elastic limit, the extension, *x*, of a spring is proportional to the tension force, *F*. The constant of proportionality *k* is called the **spring constant**. The SI units for the spring constant are N m^{-1}. Thus by measuring the extension, we can calculate the force.

FORCES AS VECTORS

Since forces are vectors, vector mathematics must be used to find the resultant force from two or more other forces. A force can also be split into its components. See page 7 for more details.

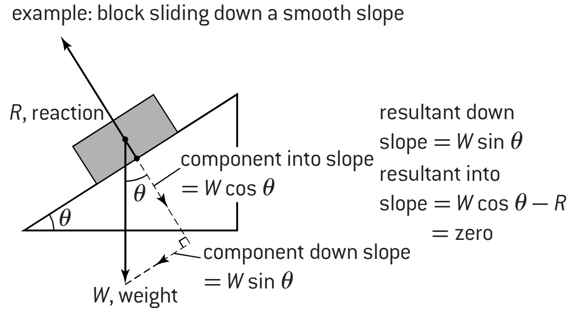
(a) b vector mathematics

example: block being pushed on rough surface



(b) b components

example: block sliding down a smooth slope



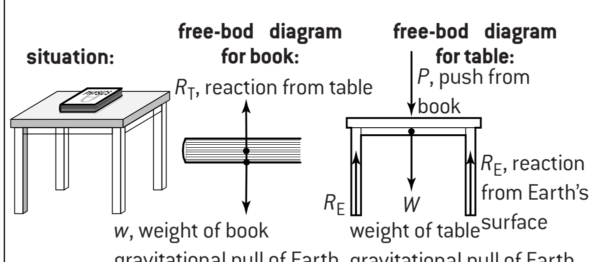
Vector addition

FREE-BODY DIAGRAMS

In a **free-body diagram**

- one object (and ONLY one object) is chosen
- all the forces on that object are shown and labelled.

For example, if we considered the simple situation of a book resting on a table, we can construct free-body diagrams for either the book or the table.

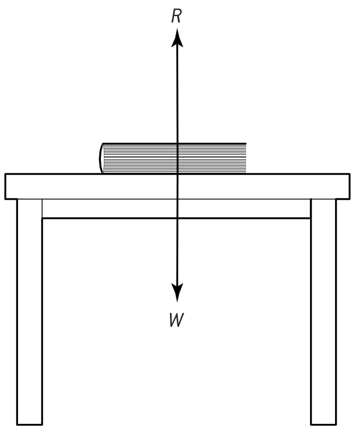


Ne ton's first la

NEWTON'S FIRST LAW

Newton's first law of motion states that 'an object continues in uniform motion in a straight line or at rest unless a resultant external force acts'. On first reading, this can sound complicated but it does not really add anything to the description of a force given on page 14. All it says is that a resultant force causes acceleration. No resultant force means no acceleration – i.e. 'uniform motion in a straight line'.

Book on a table at rest

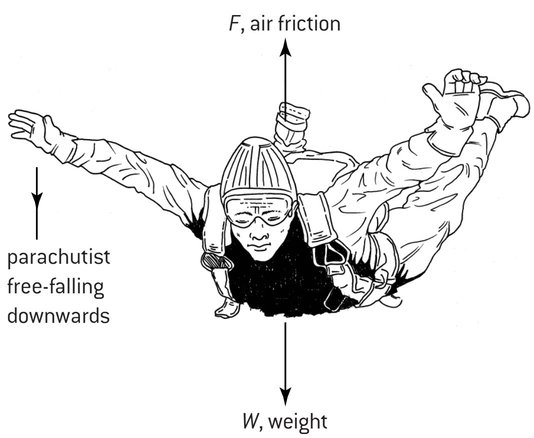


since acceleration $= $ zero

resultant force $=$ zero

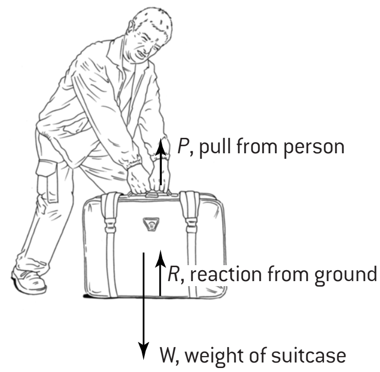
$\therefore R - W =$ zero

Parachutist in free fall



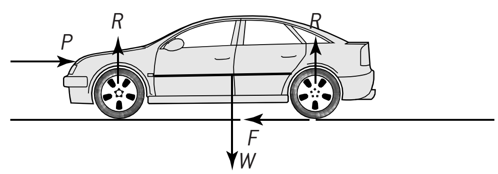
If $W > F$ the parachutist accelerates downwards.
As the parachutist gets faster, the air friction increases until $W = F$
The parachutist is at constant velocity
(the *acceleration* is zero).

Lifting a heavy suitcase



If the suitcase is too heavy to lift, it is not moving:
$\therefore$ acceleration $=$ zero
$\therefore P + R = W$

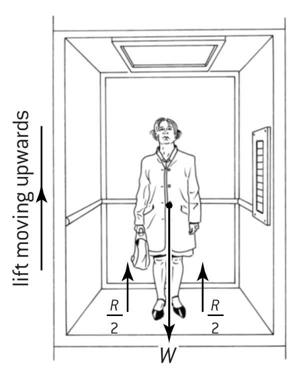
Car travelling in a straight line



F is force forwards, due to engine
P is force backwards due to air resistance

At all times force up ($2R$) $=$ force down (W).
If $F > P$ the car accelerates forwards.
If $F = P$ the car is at constant velocity (zero acceleration).
If $F < P$ the car decelerates (i.e. there is negative acceleration and the car slows down).

Person in a lift that is moving up ards



The total force up from the oor of the lift $= R$.
The total force down due to gravity $= W$.
If $R > W$ the person is accelerating upwards.
If $R = W$ the person is at constant velocity (acceleration $=$ zero).
If $R < W$ the person is decelerating (acceleration is negative).

Equilibrium

EQUILIBRIUM

If the resultant force on an object is zero then it is said to be in **translational equilibrium** (or just in equilibrium). Mathematically this is expressed as follows:

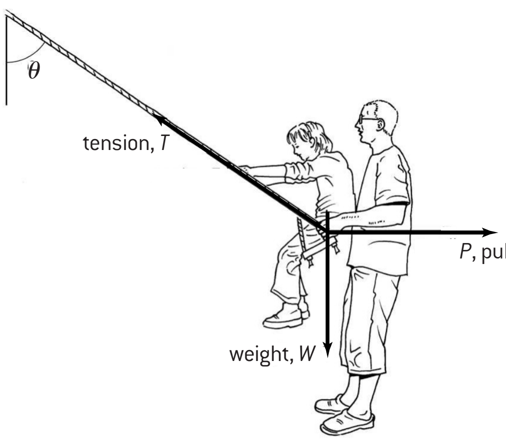
$\Sigma F = zero$

From Newton's first law, we know that the objects in the following situations must be in equilibrium.

1. An object that is constantly at rest.
2. An object that is moving with constant (uniform) velocity in a straight line.

Since forces are vector quantities, a zero resultant force means no force IN ANY DIRECTION.

For 2-dimensional problems it is sufficient to show that the forces balance in any two non-parallel directions. If this is the case then the object is in equilibrium.

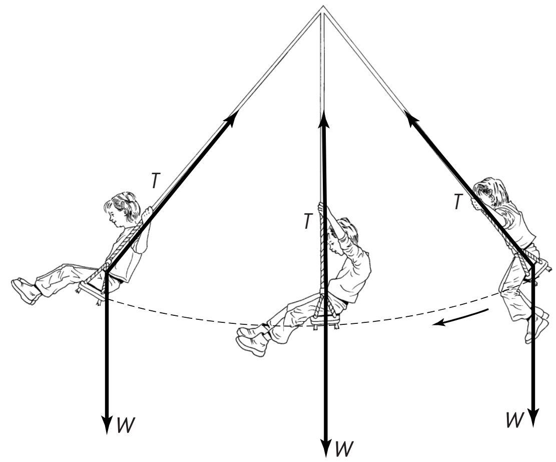


if in equilibrium:

$T \sin\theta = P$ (since no resultant horizontal force)

$T \cos\theta = W$ (since no resultant vertical force)

Translational equilibrium does NOT mean the same thing as being at rest. For example if the child in the previous example is allowed to swing back and forth, there are times when she is instantaneously at rest but he is never in equilibrium.



At the end of the swing the forces are not balanced but the child is instantaneousl at rest.

Forces are not balanced in the centre as the child is in circular motion and is accelerating (see page 65).

DIFFERENT TYPES OF FORCES

Name o orce	Description
Gravitational orce	The force between objects as a result of their masses. This is sometimes referred to as the **weight** of the object but this term is, unfortunately, ambiguous – see page 19.
Electrostatic orce	The force between objects as a result of their electric charges.
Magnetic orce	The force between magnets and/or electric currents.
Normal reaction	The force between two surfaces that acts at right angles to the surfaces. If two surfaces are smooth then this is the only force that acts between them.
Friction	The force that opposes the relative motion of two surfaces and acts along the surfaces. **Air resistance** or **drag** can be thought of as a frictional force – technically this is known as **fluid riction**.
Tension	When a string (or a spring) is stretched, it has equal and opposite forces on its ends pulling outwards. The tension force is the force that the end of the string applies to another object.
Compression	When a rod is compressed (squashed), it has equal and opposite forces on its ends pushing inwards. The compression force is the force that the ends of the rod applies to another object. This is the opposite of the tension force.
Upthrust	This is the upward force that acts on an object when it is submerged in a fluid. It is the buoyancy force that causes some objects to float in water (see page 164).
Li t	This force can be exerted on an object when a fluid flows over it in an asymmetrical way. The shape of the wing of an aircraft causes the aerodynamic lift that enables the aircraft to fly (see page 166).

Ne ton's second la

NEWTON'S SECOND LAW OF MOTION

Newton's first law states that a resultant orce causes an acceleration. His second law provides a means o calculating the value o this acceleration. The best way o stating the second law is use the concept o the **momentum** o an object. This concept is explained on page 23.

A correct statement o Newton's second law using momentum would be 'the resultant orce is proportional to the rate o change o momentum'. I we use SI units (and you always should) then the law is even easier to state – 'the resultant orce is equal to the rate o change o momentum'. In symbols, this is expressed as ollows

In SI units, $F = \frac{\Delta p}{\Delta t}$

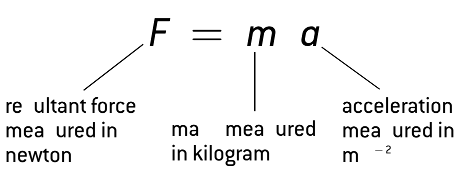
or, in ull calculus notation, $F = \frac{dp}{dt}$

p is the symbol or the momentum o a body.

Until you have studied what this means this will not make much sense, but this version o the law is given here or completeness.

An equivalent (but more common) way o stating Newton's second law applies when we consider the action o a orce on a single mass. I the amount o mass stays constant we can state the law as ollows. 'The resultant orce is proportional to the acceleration.' I we also use SI units then 'the resultant orce is equal to the product o the mass and the acceleration'.

In symbols, in SI units,



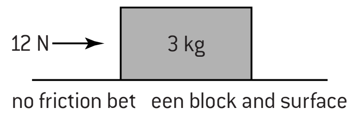
Note:

- The '$F = ma$' version o the law only applies i we use SI units – or the equation to work the mass must be in **kilograms** rather than in grams.
- F is the resultant orce. I there are several orces acting on an object (and this is usually true) then one needs to work out the resultant orce be ore applying the law.
- This is an experimental law.
- There are no exceptions – Newton's laws apply throughout the Universe. (To be absolutely precise, Einstein's theory o relativity takes over at very large values o speed and mass.)

The $F = ma$ version o the law can be used whenever the situation is simple – or example, a constant orce acting on a constant mass giving a constant acceleration. I the situation is more di ficult (e.g. a changing orce or a changing mass) then one needs to use the $F = \frac{dp}{dt}$ version.

EXAMPLES OF NEWTON'S SECOND LAW

1. Use o $F = ma$ in a simple situation

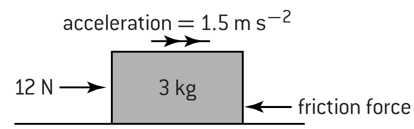


I a mass o 3 kg is accelerated in a straight line by a resultant orce o 12 N, the acceleration must be 4 m s⁻². Since

$F = ma,$

$a = \frac{F}{m} = \frac{12}{3} = 4 \text{ m s}^{-2}.$

2. Use o $F = ma$ in a slightly more complicated situation



I a mass o 3 kg is accelerated in a straight line by a orce o 12 N, and the resultant acceleration is 1.5 m s⁻², then we can work out the riction that must have been acting. Since

$F = ma$

resultant orce $= 3 \times 1.5$

$= 4.5$ N

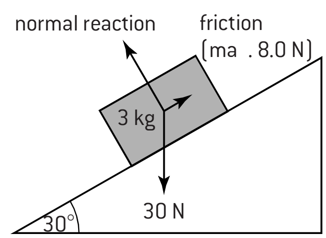
This resultant orce = orward orce − riction

there ore, riction = orward orce − resultant orce

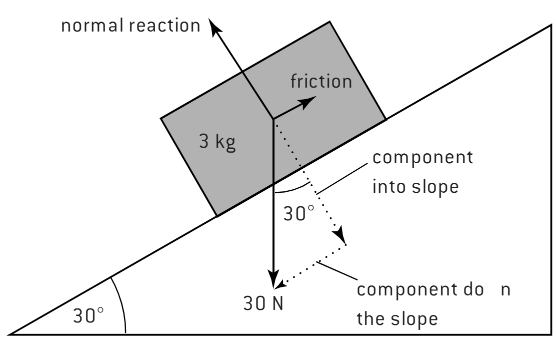
$= 12 - 4.5$ N

$= 7.5$ N

3. Use o $F = ma$ in a 2-dimensional situation



A mass o 3 kg eels a gravitational pull towards the Earth o 30 N. What will happen i it is placed on a 30 degree slope given that the maximum riction between the block and the slope is 8.0 N?



into slope: normal reaction = component into slope
The block does not accelerate into the slope.

do n the slope:

component down slope $= 30 \text{ N} \times \sin 30°$

$= 15$ N

maximum riction orce up slope $= 8$ N

resultant orce down slope $= 15 - 8$

$= 7$ N

$F = ma$

acceleration down slope $= \frac{F}{m}$

$= \frac{7}{3} = 2.3 \text{ m s}^{-2}$

Ne ton's third la

STATEMENT OF THE LAW

Newton s second law is an experimental law that allows us to calculate the effect that a force has. Newton s third law highlights the fact that forces always come in pairs. It provides a way of checking to see if we have remembered all the forces involved.

It is very easy to state. 'When two bodies A and B interact, the force that A exerts on B is equal and opposite to the force that B exerts on A . Another way of saying the same thing is that 'for every action on one object there is an equal but opposite reaction on another object .

In symbols,

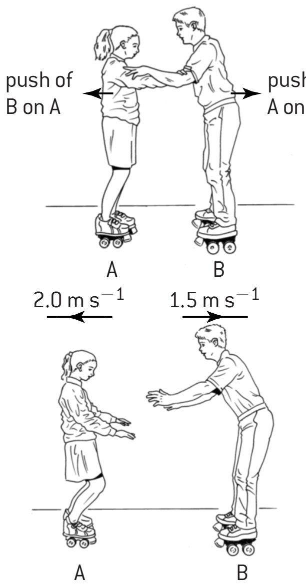
$$F_{AB} = -F_{BA}$$

Key points to notice include

- The two forces in the pair act on different objects – this means that equal and opposite forces that act on the same object are NOT Newton s third law pairs.
- Not only are the forces equal and opposite, but they must be of the same type. In other words, if the force that A exerts on B is a gravitational force, then the equal and opposite force exerted by B on A is also a gravitational force.

EXAMPLES OF THE LAW

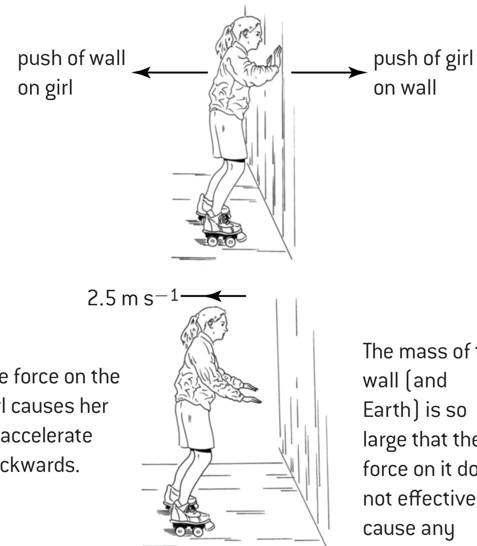
Forces bet een roller-skaters



If one roller-skater pushes another, they both feel a force. The forces must be equal and opposite, but the acceleration will be different (since they have different masses).

The person with the smaller mass will gain the greater velocity.

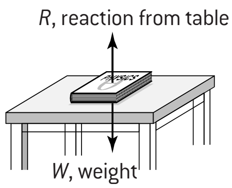
A roller-skater pushes off from a all



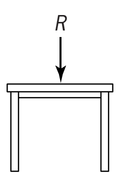
The force on the girl causes her to accelerate backwards.

The mass of the wall (and Earth) is so large that the force on it does not effectively cause any acceleration.

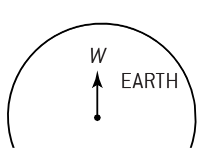
A book on a table – Ne ton's third la



These two forces are *not* third law pairs. There must be another force (on a different object) that pairs with each one:

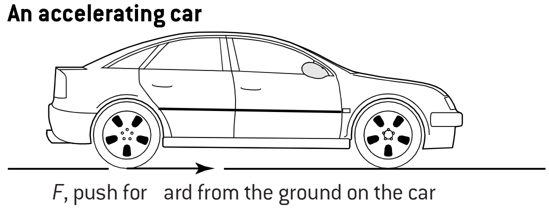


If the table pushes upwards on the book with force *R*, then the book must push down on the table with force *R*.



If the Earth pulls the book down with force *W*, then the book must pull the Earth up with force *W*.

An accelerating car



In order to accelerate, there must be a for ard force *on the car*. The engine makes the heels turn and the heels push on the ground.

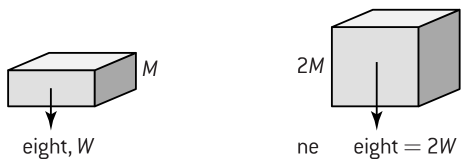
force from car on ground = – force from ground on car

Mass and eight

WEIGHT

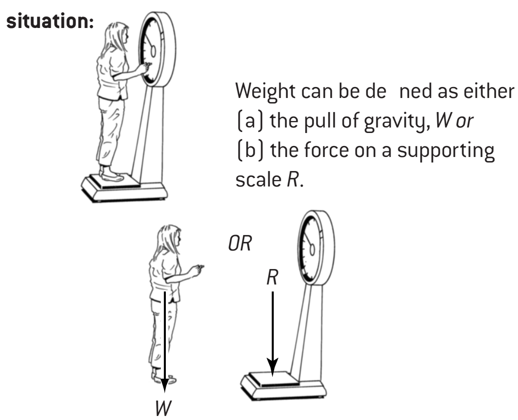
Mass and **eight** are two very different things. Unfortunately their meanings have become muddled in everyday language. Mass is the amount of matter contained in an object (measured in kg) whereas the weight of an object is a force (measured in N).

If an object is taken to the Moon, its mass would be the same, but its weight would be less (the gravitational forces on the Moon are less than on the Earth). On the Earth the two terms are often muddled because they are proportional. People talk about wanting to gain or lose weight – what they are actually worried about is gaining or losing mass.



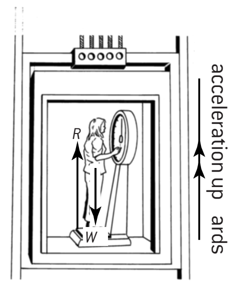
Double the mass means double the weight

To make things worse, the term 'weight' can be ambiguous even to physicists. Some people choose to define weight as the gravitational force on an object. Other people define it to be the reading on a supporting scale. Whichever definition you use, you weigh less at the top of a building compared with at the bottom – the pull of gravity is slightly less!



Two different definitions of 'weight'

Although these two definitions are the same if the object is in equilibrium, they are very different in non-equilibrium situations. For example, if both the object and the scale were put into a lift and the lift accelerated upwards then the definitions would give different values.



If the lift is accelerating up ards: $R > W$

The safe thing to do is to avoid using the term weight if at all possible! Stick to the phrase 'gravitational force' or force of gravity and you cannot go wrong.

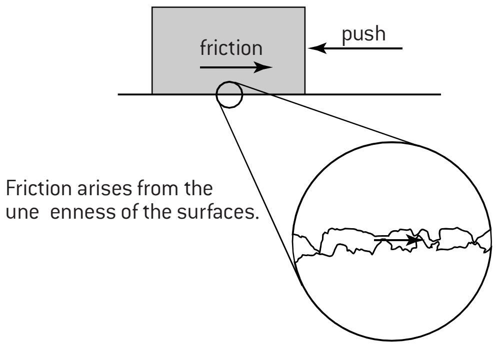
Gravitational force = $m\,g$

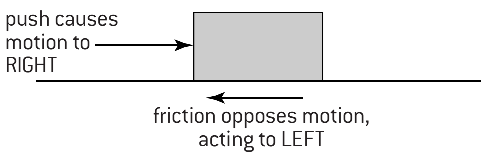
On the surface of the Earth, g is approximately 10 N kg^{-1}, whereas on the surface of the moon, $g \approx 1.6$ N kg^{-1}

Solid fric ion

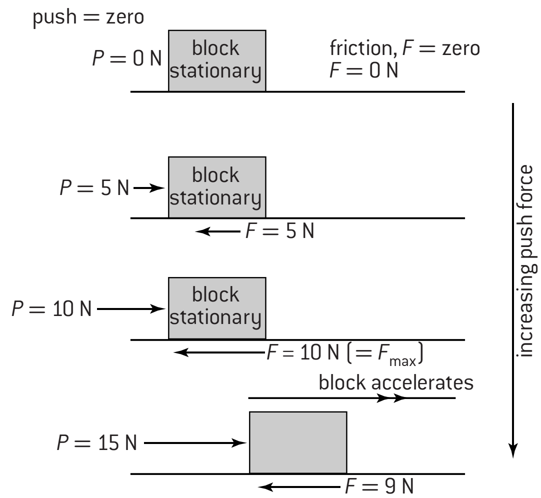
FACTORS AFFECTING FRICTION – STATIC AND DYNAMIC

Friction is the force that opposes the relative motion of two surfaces. It arises because the surfaces involved are not perfectly smooth on the microscopic scale. If the surfaces are prevented from relative motion (they are at rest) then this is an example of **static riction**. If the surfaces are moving, then it is called **dynamic riction** or **kinetic riction**.





A key experimental fact is that the value of static friction changes depending on the applied force. Up to a certain maximum force, F_{max}, the resultant force is zero. For example, if we try to get a heavy block to move, any value of pushing force below F_{max} would fail to get the block to accelerate.



The value of F_{max} depends upon

- the nature of the two surfaces in contact.
- the normal reaction force between the two surfaces. The maximum frictional force and the normal reaction force are proportional.

If the two surfaces are kept in contact by gravity, the value of F_{max} does NOT depend upon the area of contact

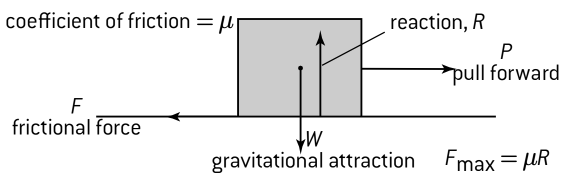
Once the object has started moving, the maximum value of friction slightly reduces. In other words,

$$F_k < F_{max}$$

For two surfaces moving over one another, the dynamic frictional force remains roughly constant even if the speed changes slightly.

COEFFICIENT OF FRICTION

Experimentally, the maximum frictional force and the normal reaction force are proportional. We use this to define the **coe ficient o riction,** μ.



The coefficient of friction is defined from the maximum value that friction can take

$$F_{max} = \mu R$$

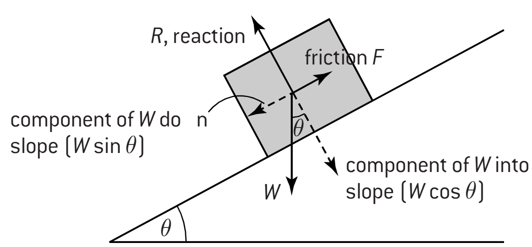
where R = normal reaction force

It should be noted that

- since the maximum value for dynamic friction is less than the maximum value for static friction, the values for the coefficients of friction will be different
  $\mu_d < \mu_s$
- the coefficient of friction is a ratio between two forces – it has no units.
- if the surfaces are smooth then the maximum friction is zero i.e. μ = 0.
- the coefficient of friction is less than 1 unless the surfaces are stuck together.
  $F_f \leq \mu_s R$ and $F_f = \mu_d R$

EXAMPLE

If a block is placed on a slope, the angle of the slope can be increased until the block just begins to slide down the slope. This turns out to be an easy experimental way to measure the coefficient of static friction.



If balanced,

$$F = W \sin\theta$$
$$R = W \cos\theta$$

θ is increased.
When block just starts moving,

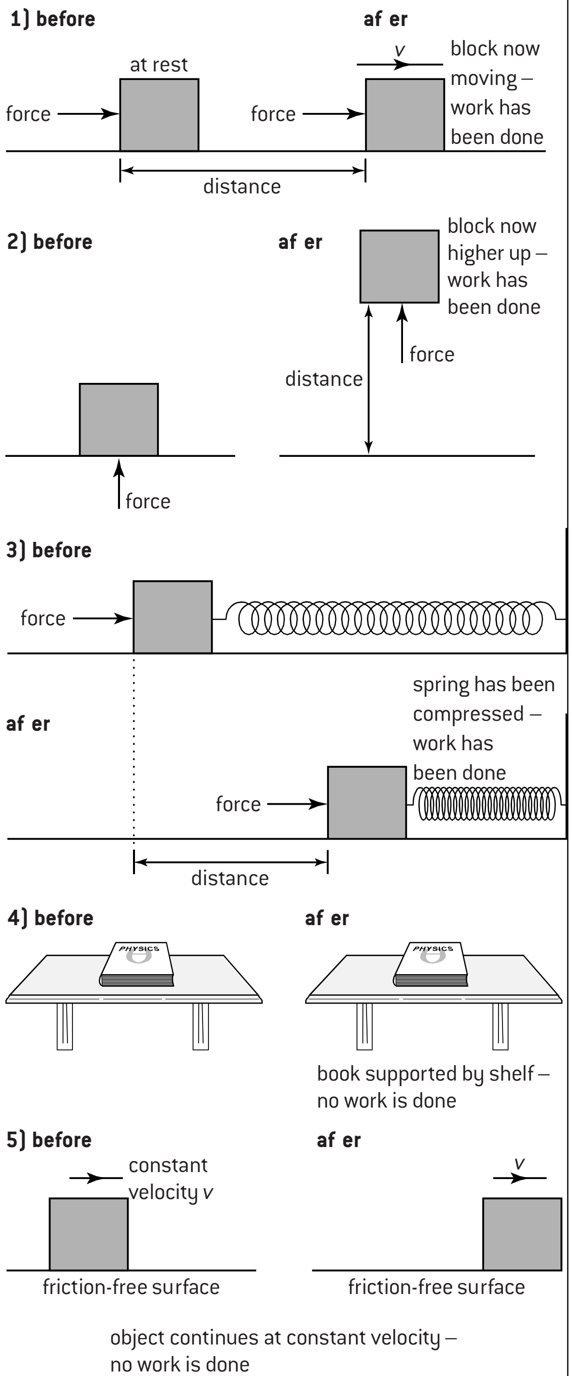
$$F = F_{ma}$$

$$\mu_{static} = \frac{F_{ma}}{R} = \frac{W \sin\theta}{W \cos\theta} = \tan\theta$$

Work

WHEN IS WORK DONE?

Work is done when a force moves its point of application in the direction of the force. If the force moves at right angles to the direction of the force, then no work has been done.

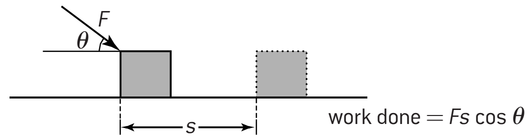


In the examples above the work done has had different results.

- In 1) the force has made the object move faster.
- In 2) the object has been lifted higher in the gravitational field.
- In 3) the spring has been compressed.
- In 4) and 5), NO work is done. Note that even though the object is moving in the last example, there is no force moving along its direction of action so no work is done.

DEFINITION OF WORK

Work is a scalar quantity. Its definition is as follows.



Wo k done = *F* *co* θ

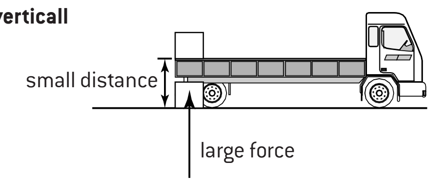
If the force and the displacement are in the same direction, this can be simplified to

'Work done = force × distance'

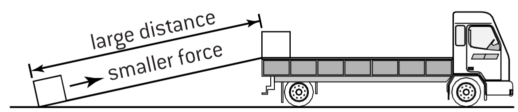
From this definition, the SI units for work done are N m. We define a new unit called the joule: 1 J = 1 N m.

EXAMPLES

(1) lifting verticall

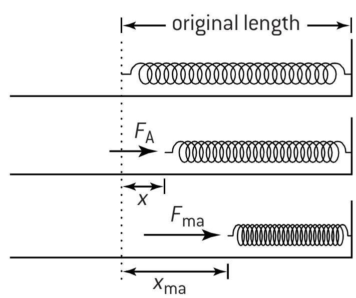


(2) pushing along a rough slope

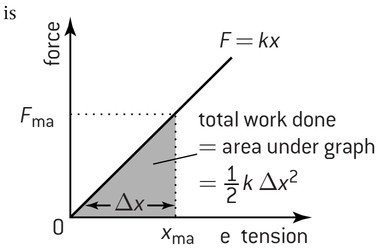


The task in the second case would be easier to perform (it involves less force) but overall it takes more work since work has to be done to overcome friction. In each case, the useful work is the same.

If the force doing work is not constant (for example, when a spring is compressed), then graphical techniques can be used.



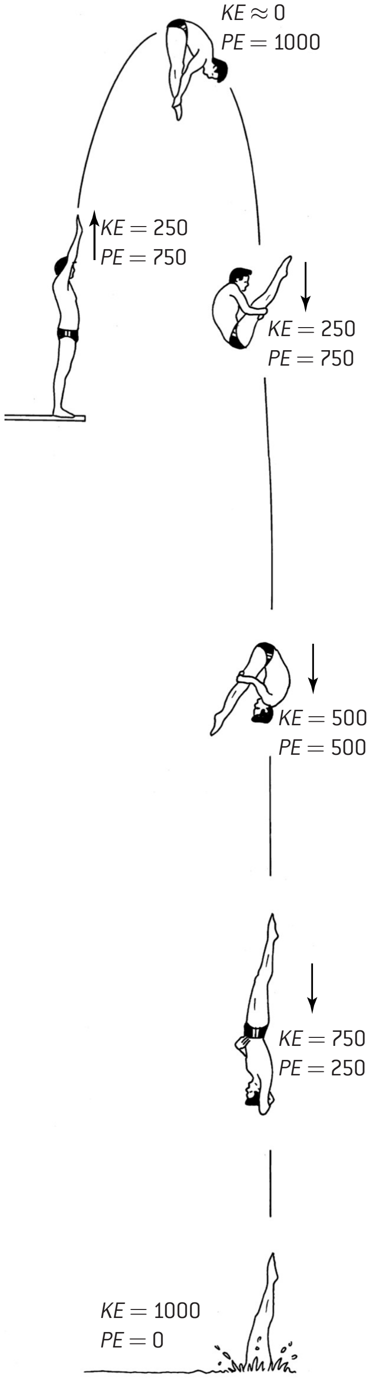
The total work done is the area under the force–displacement graph.



Useful equations for the work done include:

- work done when lifting something vertically = mgh
  where m represents mass (in kg)
  g represents the Earth's gravitational field strength (10 N kg⁻¹) h represents the height change (in m)
- work done in compressing or extending a spring = $\frac{1}{2}k\,\Delta x^2$

Energ and power



CONCEPTS OF ENERGY AND WORK

Energy and work are linked together. When you do work on an object, it gains energy and you lose energy. **The amount o energy trans erred is equal to the work done**. Energy is a measure o the amount o work done. This means that the units o energy must be the same as the units o work – joules.

ENERGY TRANSFORMATIONS – CONSERVATION OF ENERGY

In any situation, we must be able to account or the changes in energy. I it is 'lost' by one object, it must be gained by another. This is known as the **principle o conservation o energy**. There are several ways o stating this principle:

- Overall the total energy o any closed system must be constant.
- Energy is neither created nor destroyed, it just changes orm.
- There is no change in the total energy in the Universe.

ENERGY TYPES

Kinetic energy	**Gravitational potential**	**Elastic potential energy**
Radiant energy	**Electrostatic potential**	**Thermal energy**
Nuclear energy	**Solar energy**	**Chemical energy**
Electrical energy	**Internal energy**	**Light energy**

Equations or the first three types o energy are given below.

Kinetic energy = $\frac{1}{2}mv^2$ where m is the mass (in kg), v is the velocity (in m s^{-1})

$= \frac{p^2}{2m}$ where p is the momentum (see page 23) (in kg m s^{-1}), and m is the mass (in kg)

Gravitational potential energy = mgh where m represents mass (in kg), g represents the Earth's gravitational field (10 N kg^{-1}), h represents the height change (in m)

Elastic potential energy = $\frac{1}{2}k\,\Delta x^2$ where k is the spring constant (in N m^{-1}), Δx is the extension (in m)

POWER AND EFFICIENCY

1. Power

Power is defined as the RATE at which energy is trans erred. This is the same as the rate at which work is done.

$$\text{Power} = \frac{\text{energy trans erred}}{\text{time taken}}$$

$$\text{Power} = \frac{\text{work done}}{\text{time taken}}$$

The SI unit or power is the joule per second (J s^{-1}). Another unit or power is defined – the watt (W). 1 W = 1 J s^{-1}.

I something is moving at a constant velocity v against a constant rictional orce F, the power P needed is $P = F\,v$

2. E ficiency

Depending on the situation, we can categorize the energy trans erred (work done) as use ul or not. In a light bulb, the use ul energy would be light energy, the 'wasted' energy would be thermal energy (and non-visible orms o radiant energy).

We define e ficiency as the ratio o use ul energy to the total energy trans erred. Possible orms o the equation include:

$$\text{E ficiency} = \frac{\text{use ul work OUT}}{\text{total work IN}}$$

$$\text{E ficiency} = \frac{\text{use ul energy OUT}}{\text{total energy IN}}$$

$$\text{E ficiency} = \frac{\text{use ul power OUT}}{\text{total power IN}}$$

Since this is a ratio it does not have any units. O ten it is expressed as a percentage.

EXAMPLES

1. A grasshopper (mass 8 g) uses its hindlegs to push or 0.1s and as a result jumps 1.8 m high. Calculate (i) its take o speed, (ii) the power developed.

(i) PE gained = mgh

KE at start = $\frac{1}{2}mv^2$

$\frac{1}{2}mv^2 = mgh$ (conservation o energy)

$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 1.8}$

$= 6$ m s^{-1}

(ii) Power = $\frac{mgh}{t}$

$= \frac{0.008 \times 10 \times 1.8}{0.1}$

≈ 1.4 W

2. A 60W lightbulb has an e ficiency o 10%. How much energy is wasted every hour?

Power wasted = 90% o 60W

= 54W

Energy wasted = 54 × 60 × 60J

= 190 kJ

Momentum and impulse

DEFINITIONS – LINEAR MOMENTUM AND IMPULSE

Linear momentum (always given the symbol $\boldsymbol{p}$) is defined as the product o mass and velocity.

$$\text{Momentum} = \text{mass} \times \text{velocity}$$

$$\boldsymbol{p} = m\,\boldsymbol{v}$$

The SI units or momentum must be kg m s^{-1}. Alternative units o N s can also be used (see below). Since velocity is a vector, momentum must be a vector. In any situation, particularly i it happens quickly, the change o momentum $\Delta\boldsymbol{p}$ is called the **impulse** ($\Delta p = F\,\Delta t$).

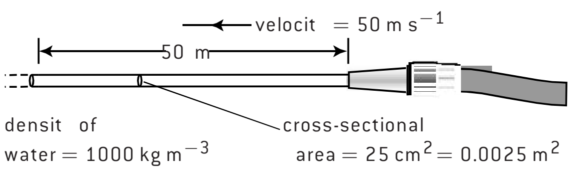
USE OF MOMENTUM IN NEWTON'S SECOND LAW

Newton's second law states that the resultant orce is proportional to the rate o change o momentum. Mathematically we can write this as

$$F = \frac{\textit{(final momentum} - \textit{initial momentum)}}{\textit{time taken}} = \frac{\Delta p}{\Delta t}$$

Example 1

A jet o water leaves a hose and hits a wall where its velocity is brought to rest. I the hose cross-sectional area is 25 cm^2, the velocity o the water is 50 m s^{-1} and the density o the water is 1000 kg m^{-3}, what is the orce acting on the wall?



In one second, a jet o water 50 m long hits the wall. So

volume o water hitting wall $= 0.0025 \times 50 = 0.125$ m^3 every second

mass o water hitting wall $= 0.125 \times 1000 = 125$ kg every second

momentum o water hitting wall $= 125 \times 50 = 6250$ kg m s^{-1} every second

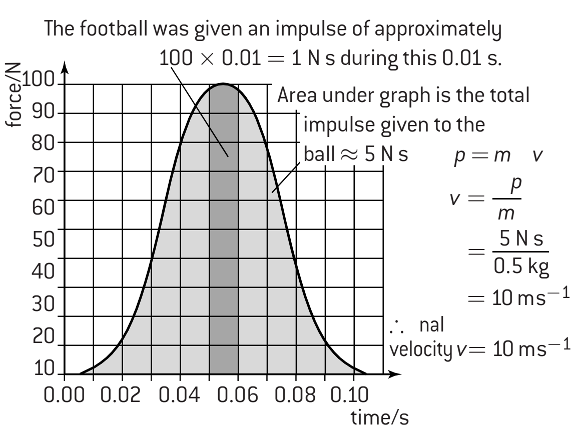
This water is all brought to rest,

$\therefore$ change in momentum, $\Delta p = 6250$ kg m s^{-1}

$\therefore$ orce $= \dfrac{\Delta p}{\Delta t} = \dfrac{6250}{1} = 6250$ N

Example 2

The graph below shows the variation with time o the orce on a ootball o mass 500 g. Calculate the final velocity o the ball.



CONSERVATION OF MOMENTUM

The law o conservation o linear momentum states that 'the total linear momentum o a system o interacting particles remains constant **provided there is no resultant external force**'.

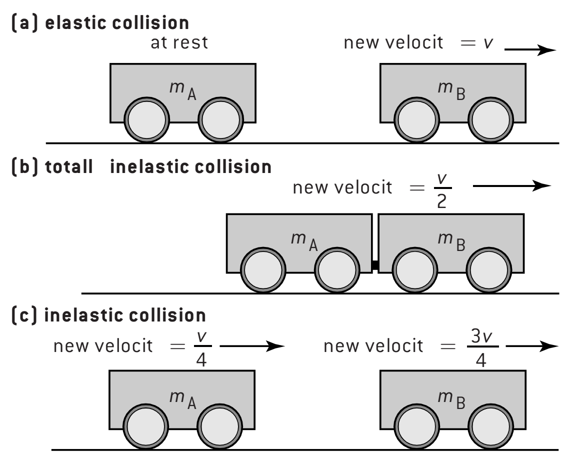
To see why, we start by imagining two isolated particles A and B that collide with one another.

- The orce rom A onto B, F_{AB} will cause B's momentum to change by a certain amount.
- I the time taken was Δt, then the momentum change (the impulse) given to B will be given by $\Delta p_B = F_{AB}\,\Delta t$
- By Newton's third law, the orce rom B onto A, F_{BA} will be equal and opposite to the orce rom A onto B, $F_{AB} = -F_{BA}$.
- Since the time o contact or A and B is the same, then the momentum change or A is equal and opposite to the momentum change or B, $\Delta p_A = -F_{AB}\,\Delta t$.
- This means that the total momentum (momentum o A plus the momentum o B) will remain the same. Total momentum is conserved.

This argument can be extended up to any number o interacting particles so long as the system o particles is still isolated. I this is the case, the momentum is still conserved.

ELASTIC AND INELASTIC COLLISIONS

The law o conservation o linear momentum is not enough to always predict the outcome a ter a collision (or an explosion). This depends on the nature o the colliding bodies. For example, a moving railway truck, m_A, velocity v, collides with an identical stationary truck m_B. Possible outcomes are:



In (a), the trucks would have to have elastic bumpers. I this were the case then no mechanical energy at all would be lost in the collision. A collision in which no mechanical energy is lost is called an **elastic collision**. In reality, collisions between everyday objects always lose some energy – the only real example o elastic collisions is the collision between molecules. For an elastic collision, the relative velocity o approach always equals the relative velocity o separation.

In (b), the railway trucks stick together during the collision (the relative velocity o separation is zero). This collision is what is known as a **totally inelastic collision**. A large amount o mechanical energy is lost (as heat and sound), but the total momentum is still conserved.

In energy terms, (c) is somewhere between (a) and (b). Some energy is lost, but the railway trucks do not join together. This is an example o an **inelastic collision**. Once again the total momentum is conserved.

Linear momentum is also conserved in explosions.

IB Quest ons – mechan cs

1. Two identical objects A and B fall from rest from different heights. If B takes twice as long as A to reach the ground, what is the ratio of the heights from which A and B fell? Neglect air resistance.

A. 1: $\sqrt{2}$ **B.** 1:2 **C.** 1:4 **D.** 1:8

2. A trolley is given an initial push along a horizontal floor to get it moving. The trolley then travels forward along the floor, gradually slowing. What is true of the horizontal force(s) on the trolley while it is slowing?

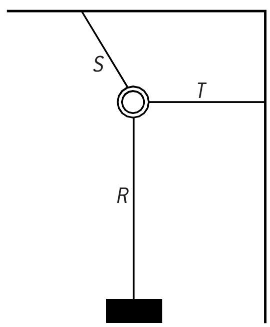
A. There is a forward force and a backward force, but the forward force is larger.

B. There is a forward force and a backward force, but the backward force is larger.

C. There is only a forward force, which diminishes with time.

D. There is only a backward force.

3. A mass is suspended by cord from a ring which is attached by two further cords to the ceiling and the wall as shown. The cord from the ceiling makes an angle of less than 45° with the vertical as shown. The tensions in the three cords are labelled *R*, *S* and *T* in the diagram.



How do the tensions *R*, *S* and *T* in the three cords compare in magnitude?

A. $R > T > S$ **B.** $S > R > T$

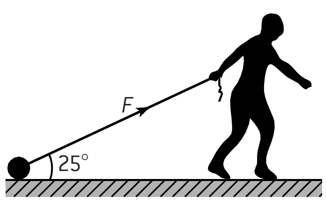
C. $R = S = T$ **D.** $R = S > T$

4. A 24 N force causes a 2.0 kg mass to accelerate at 8.0 m s^{-2} along a horizontal surface. The coefficient of dynamic friction is:

A. 0.0 **B.** 0.4

C. 0.6 **D.** 0.8

5. An athlete trains by dragging a heavy load across a rough horizontal surface.



The athlete exerts a force of magnitude F on the load at an angle of 25° to the horizontal.

a) Once the load is moving at a steady speed, the average horizontal frictional force acting on the load is 470 N.

Calculate the average value of F that will enable the load to move at constant speed. [2]

b) The load is moved a horizontal distance of 2.5 km in 1.2 hours.

Calculate

(i) the work done on the load by the force *F*. [2]

(ii) the minimum average power required to move the load. [2]

c) The athlete pulls the load uphill at the same speed as in part (a).

Explain, in terms of energy changes, why the minimum average power required is greater than in (b)(ii). [2]

6. A car and a truck are both travelling at the speed limit of 60 km h^{-1} but in opposite directions as shown. The truck has **t ice** the mass of the car.

The vehicles collide head-on and become entangled together.

a) During the collision, how does the force exerted by the car on the truck compare with the force exerted by the truck on the car? Explain. [2]

b) In what direction will the entangled vehicles move after collision, or will they be stationary? Support your answer, referring to a physics principle. [2]

c) Determine the speed (in km h^{-1}) of the combined wreck immediately after the collision. [3]

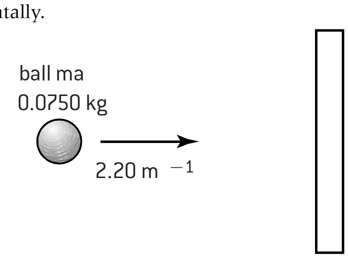
d) How does the acceleration of the car compare with the acceleration of the truck during the collision? Explain. [2]

e) Both the car and truck drivers are wearing seat belts. Which driver is likely to be more severely jolted in the collision? Explain. [2]

f) The total kinetic energy of the system decreases as a result of the collision. Is the principle of conservation of energy violated? Explain. [1]

7. a) A net force of magnitude *F* acts on a body. Define the *impulse I* of the force. [1]

b) A ball of mass 0.0750 kg is travelling horizontally with a speed of 2.20 m s^{-1}. It strikes a vertical wall and rebounds horizontally.



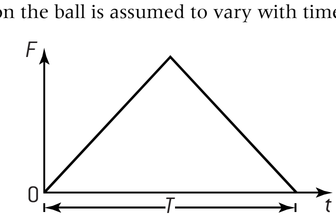
Due to the collision with the wall, 20 % of the ball's initial kinetic energy is dissipated.

(i) Show that the ball rebounds from the wall with a speed of 1.97 m s^{-1}. [2]

(ii) Show that the impulse given to the ball by the wall is 0.313 N s. [2]

c) The ball strikes the wall at time $t = 0$ and leaves the wall at time $t = T$.

The sketch graph shows how the force *F* that the wall exerts on the ball is assumed to vary with time *t*.



The time *T* is measured electronically to equal 0.0894 s.

Use the impulse given in (b)(ii) to estimate the average value of *F*. [4]