# HL Simple harmonic mo ion

## SIMPLE HARMONIC MOTION (SHM) EQUATION

SHM occurs when the orces on an object are such that the resultant acceleration, $a$, is directed towards, and is proportional to, its displacement, $x$, rom a fixed point.

$a \quad -x$ or $a = -(\text{constant}) \times x$

The mathematics o SHM is simplified i the constant o proportionality between $a$ and $x$ is identified as the square o another constant $\omega$ which is called the angular requency. Thus the general orm or the equation that defines SHM is:

$a = -\omega^2 x$

The solutions or this equation ollow below. The angular requency $\omega$ has the units o rad $s^{-1}$ and is related to the time period, $T$, o the oscillation by the ollowing equation.

$\omega = \frac{2\pi}{T}$

## IDENTIFICATION OF SHM

In order to analyse a situation to decide i SHM is taking place, the ollowing procedure should be ollowed.

- Identi y all the orces acting on an object when it is displaced an arbitrary distance $x$ rom its rest position using a ree-body diagram.
- Calculate the resultant orce using Newton's second law. I this orce is proportional to the displacement and always points back towards the mean position (i.e. $F \quad -x$) then the motion o the object must be SHM.
- Once SHM has been identified, the equation o motion must be in the ollowing orm:

  $a = -\left(\frac{\text{restoring orce per unit displacement, } k}{\text{oscillating mass, } m}\right) \times x$
- This identifies the angular requency $\omega$ as $\omega^2 = \left(\frac{k}{m}\right)$ or $\omega = \sqrt{\left(\frac{k}{m}\right)}$. Identification o $\omega$ allows quantitative equations to be applied.

## ACCELERATION, VELOCITY AND DISPLACEMENT DURING SHM

The variation with time o the acceleration, $a$, velocity, $v$, and displacement, $x$, o an object doing SHM depends on the angular requency $\omega$.

The precise ormat o the relationships depends on where the object is when the clock is started (time $t$ = zero). The le t hand set o equations correspond to an oscillation when the object is in the mean position when $t = 0$. The right hand set o equations correspond to an oscillation when the object is at maximum displacement when $t = 0$.

| | |
|---|---|
| $x = x_0 \sin \omega t$ | $x = x_0 \cos \omega t$ |
| $v = \omega x_0 \cos \omega t$ | $v = -\omega x_0 \sin \omega t$ |
| $a = -\omega^2 x_0 \sin \omega t$ | $a = -\omega^2 x_0 \cos \omega t$ |

The first two equations can be rearranged to produce the ollowing relationship:

$v = \pm\, \omega\sqrt{(x_0 - x^2)}$

$x_0$ is the amplitude o the oscillation measured in m

$t$ is the time taken measured in s

$\omega$ is the angular requency measured in rad $s^{-1}$

$\omega t$ is an angle that increases with time measured in radians. A ull oscillation is completed when $(\omega t) = 2\pi$ rad.

The angular requency is related to the time period $T$ by the ollowing equation.

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\text{m}}{\text{k}}}$

## TWO EXAMPLES OF SHM

Two common situations that approximate to SHM are:

1. Mass, $m$, on a vertical spring

   Provided that:
   - the mass o the spring is negligible compared to the mass o the load
   - riction (air riction) is negligible
   - the spring obeys Hooke's law with spring constant, $k$ at all times (i.e. elastic limit is not exceeded)
   - the gravitational field strength $g$ is constant
   - the fixed end o the spring cannot move.

   Then it can be shown that:

   $\omega^2 = \frac{k}{m}$

   Or $T = 2\pi\sqrt{\frac{m}{k}}$

2. The simple pendulum o length $l$ and mass $m$

   Provided that:
   - the mass o the string is negligible compared with the mass o the load
   - riction (air riction) is negligible
   - the maximum angle o swing is small ( 5° or 0.1 rad)
   - the gravitational field strength $g$ is constant
   - the length o the pendulum is constant.

   Then it can be shown that:

   $\omega^2 = \frac{g}{l}$

   Or $T = 2\pi\sqrt{\frac{l}{g}}$

   Note that the mass o the pendulum bob, $m$, is not in this equation and thus does not a ect the time period o the pendulum, $T$.

## EXAMPLE

A 600 g mass is attached to a light spring with spring constant 30 N $m^{-1}$.

(a) Show that the mass does SHM.

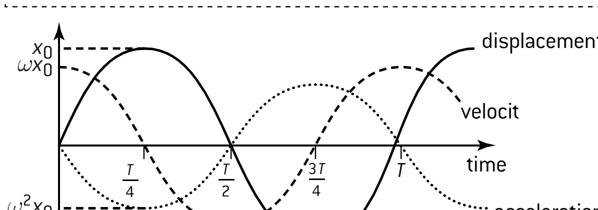
(b) Calculate the requency o its oscillation.

(a) Weight o mass = $mg$ = 6.0 N

Additional displacement $x$ down means that resultant orce on mass = $kx$ upwards. Since $F \quad -x$, the mass will oscillate with SHM.

(b) Since SHM, $T = 2\pi\sqrt{\left(\frac{m}{k}\right)} = 2\pi\sqrt{\left(\frac{0.6}{30}\right)} = 0.889\text{s}$

$f = \frac{1}{T} = \frac{1}{0.889} = 1.1$ Hz




- acceleration leads velocity by 90°
- velocity leads displacement by 90°
- acceleration and displacement are 180° out o phase
- displacement lags velocity by 90°
- velocity lags acceleration by 90°

# HL Energy c anges during simple armonic motion

During SHM, energy is interchanged between KE and PE. Providing there are no resistive forces which dissipate this energy, the total energy must remain constant. The oscillation is said to be **undamped**.

The kinetic energy can be calculated from

$$E_k = \frac{1}{2} mv^2 = \frac{1}{2} m\,\omega^2 (x_0 - x^2)$$

The potential energy can be calculated from

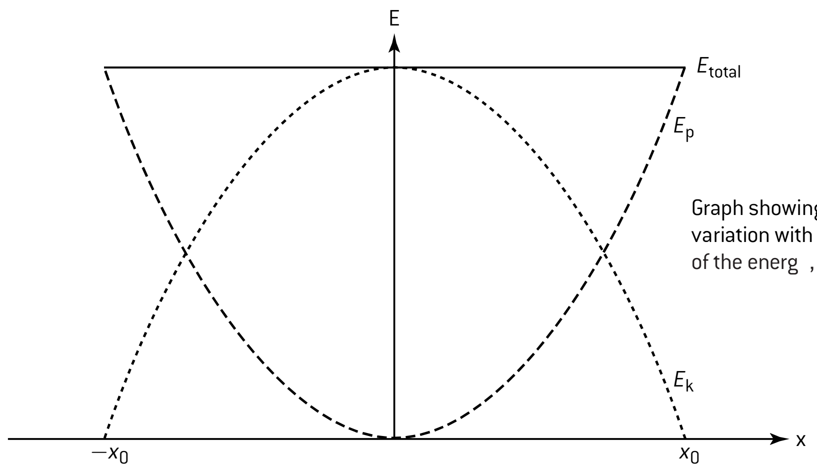
$$E_p = \frac{1}{2} m\,\omega^2 x^2$$

The total energy is

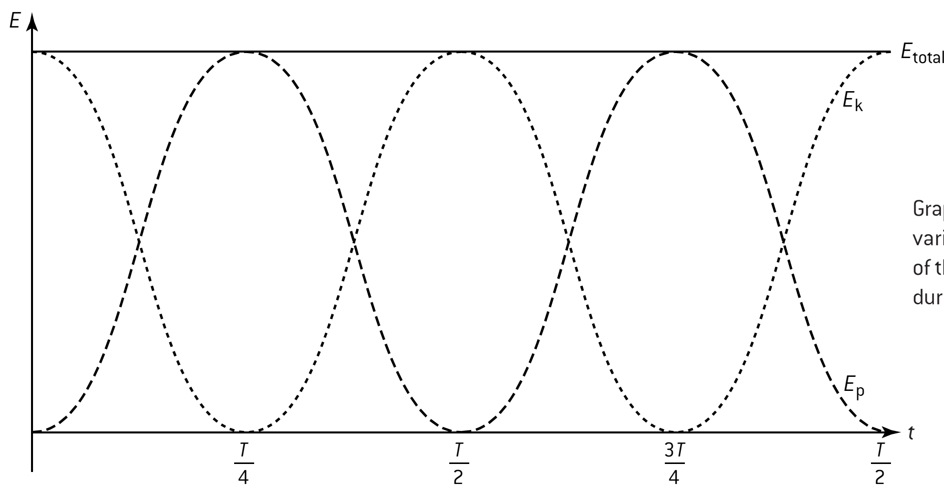
$$E = E_k + E_p = \frac{1}{2}\, m\,\omega^2 (x_0 - x^2) + \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} m\,\omega^2 x_0$$

Energy in SHM is proportional to:

- the mass $m$
- the (amplitude)$^2$
- the (frequency)$^2$




Graph showing the variation with distance, $x$ of the energ , $E$ during SHM

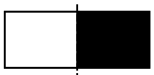
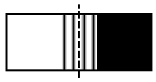
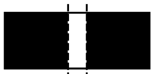
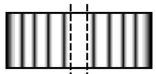
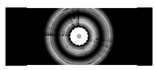
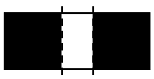
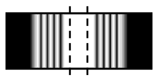



Graph showing the variation with time, $t$ of the energ , $E$ during SHM

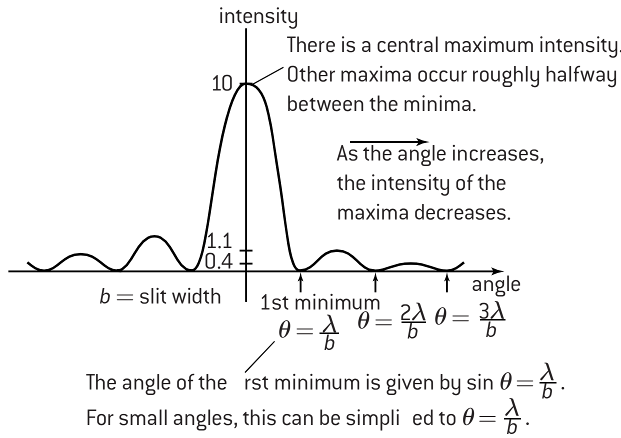
# HL Diffrac ion

## BASIC OBSERVATIONS

Diffraction is a wave effect. The objects involved (slits, apertures, etc.) have a size that is of the same order of magnitude as the wavelength of visible light.

| Na ure of obs acle | Geome rical shadow | Diffrac ion pa ern |
|---|---|---|
| **(a)** straight edge |  |  |
| **(b)** single long slit $b \sim 3\lambda$ |  |  |
| **(c)** circular aperture |  |  |
| **(d)** single long slit $b \sim 5\lambda$ |  |  |

The intensity plot for a single slit is:

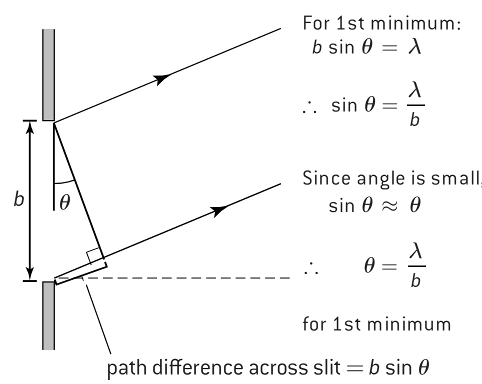



## EXPLANATION

The shape of the relative intensity versus angle plot can be derived by applying an idea called **Huygens' principle**. We can treat the slit as a series of secondary wave sources. In the forward direction ($\theta$ = zero) these are all in phase so they add up to give a maximum intensity. At any other angle, there is a path difference between the rays that depends on the angle.

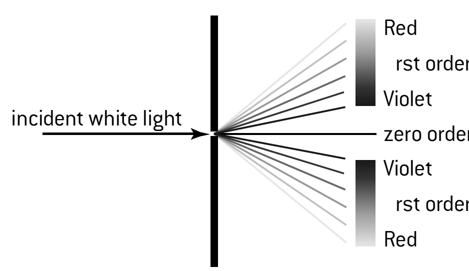
The overall result is the addition of all the sources. The condition for the first minimum is that the angle must make all of the sources across the slit cancel out.

The condition for the first maximum out from the centre is when the path difference across the whole slit is $\frac{3\lambda}{2}$. At this angle the slit can be analysed as being three equivalent sections each having a path difference of $\frac{\lambda}{2}$ across its length. Together, two of these sections will destructively interfere leaving the resulting amplitude to be $\frac{1}{3}$ of the maximum. Since intensity $\propto$ (amplitude)$^2$, the first maximum intensity out from the centre will be $\frac{1}{9}$ of the central maximum intensity. By a similar argument, the second maximum intensity out from the centre will have $\frac{1}{5}$ of the maximum amplitude and thus be $\frac{1}{25}$ of the central maximum intensity.




## SINGLE-SLIT DIFFRACTION WITH WHITE LIGHT

When a single slit is illuminated with white light, each component colour has a specific wavelength and so the associated maxima and minima for each wavelength will be located at a different angle. For a given slit width, colours with longer wavelengths (red, orange, etc.) will diffract more than colours with short wavelengths (blue, violet, etc.). The maxima for the resulting diffraction pattern will show all the colours of the rainbow with blue and violet nearer to the central position and red appearing at greater angles.



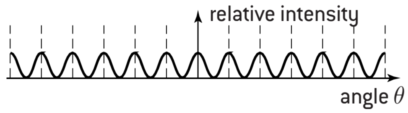
# Two-source interference of waves: Young's double-slit e  periment

## DOUBLE-SLIT INTERFERENCE

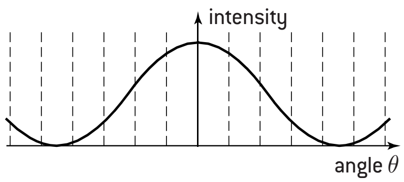
The double-slit interference pattern shown on page 47 was derived assuming that each slit was behaving like a perfect point source. This can only take place if the slits are infinitely small. In practice they have a finite width. The diffraction pattern of each slit needs to be taken into account when working out the overall double slit interference pattern as shown below.

Decreasing the slit width will mean that the observed pattern becomes more and more 'idealized'. Unfortunately, it will also mean that the total intensity of light will be decreased. The interference pattern will become harder to observe.

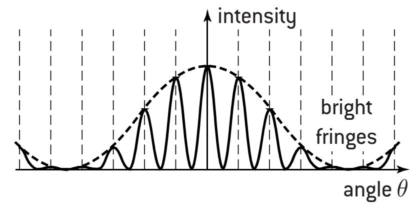
**(a) Young's fringes for in  nitely narrow slits**




**(b) diffraction pattern for a  nite-width slit**




**(c) Young's fringes for slits of  nite width**

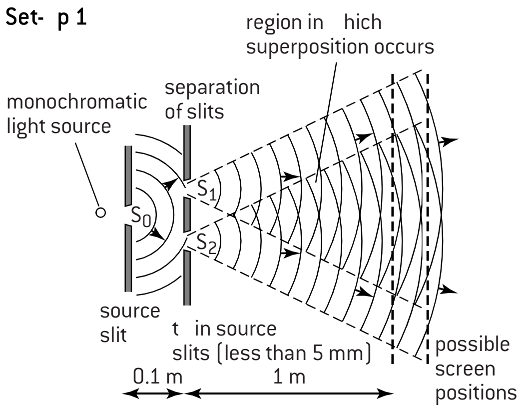



$s = \frac{\lambda D}{d}$ still applies but di  erent fringes will have di  erent intensities with it being possible for some fringes to be missing.

## INVESTIGATING YOUNG'S DOUBLE-SLIT EXPERIMENTALLY

Possible set-ups for the double-slit experiment are shown on page 47.

### Set-  p 1

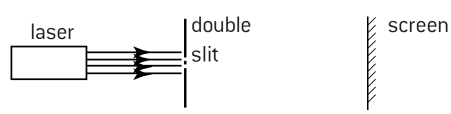



In the original set-up (set-up 1) light from the monochromatic source is diffracted at $S_0$ so as to ensure that $S_1$ and $S_2$ are receiving coherent light. Diffraction takes place providing $S_1$ and $S_2$ are narrow enough. The slit separations need to be approximately 1 mm (or less) thus the slit widths are of the order of 0.1 mm (or less). This would provide fringes that were separated by approximately 0.5mm on a screen (semi-transparent or translucent) situated 1m away. The laboratory will need to be darkened to allow the fringes to be visible and they can be viewed using a microscope.

The most accurate measurements for slit separation and fringe width are achieved using a **travelling microscope**. This is a microscope that is mounted on a frame so that it can be moved perpendicular to the direction in which it is pointing. The microscope is moved across ten or more fringes and the distance moved by the microscope can be read off from the scale. The precision of this measurement is often improved by utilizing a vernier scale.

In the simplified version (set-up 2) of the experiment, fringes can still be bright enough to be viewed several metres away from the slits and thus they can be projected onto an opaque screen (it is dangerous to look into a laser beam). Their separation can be then be directly measured with a ruler.

### Set-  p 2

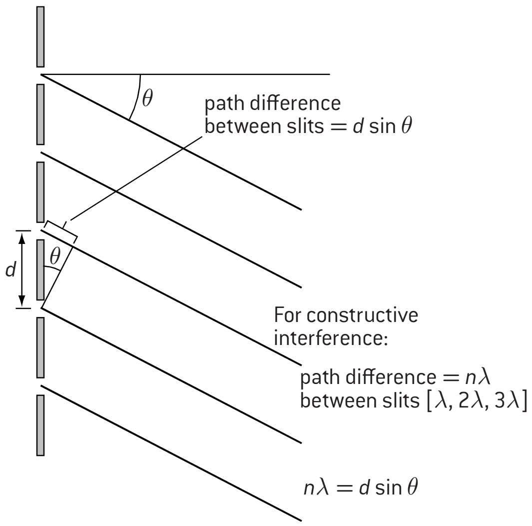


# Multiple-slit diffraction

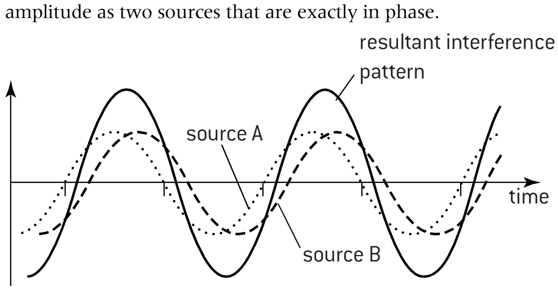
### THE DIFFRACTION GRATING

The diffraction that takes place at an individual slit affects the overall appearance of the fringes in Young's double-slit experiment (see page 98 for more details). This section considers the effect on the final interference pattern of adding further slits. A series of parallel slits (at a regular separation) is called a **diffraction grating**.

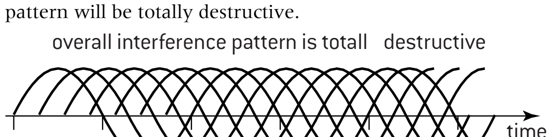
Additional slits at the same separation will not affect the condition for constructive interference. In other words, the angle at which the light from slits adds constructively will be unaffected by the number of slits. The situation is shown below.




This formula also applies to the Young's double-slit arrangement. The difference between the patterns is most noticeable at the angles where perfect constructive interference does not take place. If there are only two slits, the maxima will have a significant angular width. Two sources that are just out of phase interfere to give a resultant that is nearly the same amplitude as two sources that are exactly in phase.




The addition of more slits will mean that each new slit is just out of phase with its neighbour. The overall interference pattern will be totally destructive.




The addition of further slits at the same slit separation has the following effects:

- the principal maxima maintain the same separation
- the principal maxima become much sharper
- the overall amount of light being let through is increased, so the pattern increases in intensity.

**(a) 2 sli s**

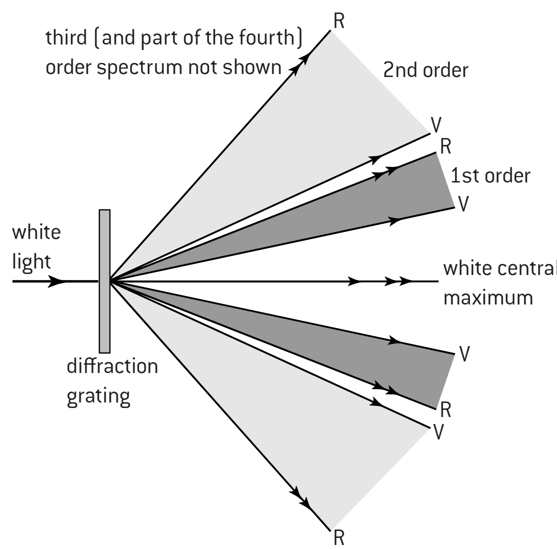
**(b) 4 sli s**

**(c) 50 sli s**

Grating patterns

### USES

One of the main uses of a diffraction grating is the accurate experimental measurement of the different wavelengths of light contained in a given spectrum. If white light is incident on a diffraction grating, the angle at which constructive interference takes place depends on wavelength. Different wavelengths can thus be observed at different angles. The accurate measurement of the angle provides the experimenter with an accurate measurement of the exact wavelength (and thus frequency) of the colour of light that is being considered. The apparatus that is used to achieve this accurate measurement is called a **spectrometer**.



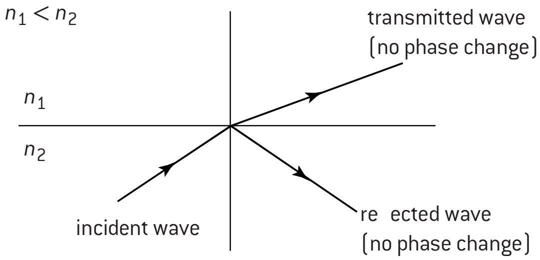
# HL Thin parallel films

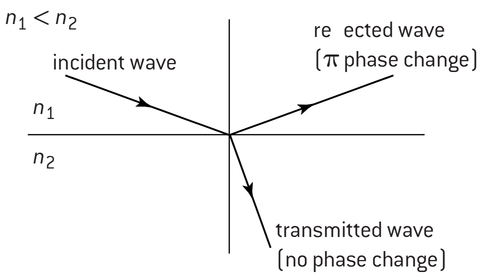
## PHASE CHANGES

There are many situations when interference can take place that also involve the reflection of light. When analysing in detail the conditions for constructive or destructive interference, one needs to take any **phase changes** into consideration. A phase change is the inversion of the wave that can take place at a reflection interface, but it does not always happen. It depends on the two media involved.

The technical term for the inversion of a wave is that it has 'undergone a phase change of $\pi$'.

- When light is reflected back from an optically denser medium there is a phase change of $\pi$.
- When light is reflected back from an optically less dense medium there is no phase change.



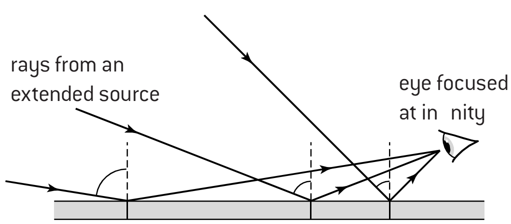




## EXAMPLE

The equations in the box on the right work out the angles for which constructive and destructive interference take place for a given wavelength. If the source of light is an extended source, the eye receives rays leaving the film over a range of values for $\theta$.

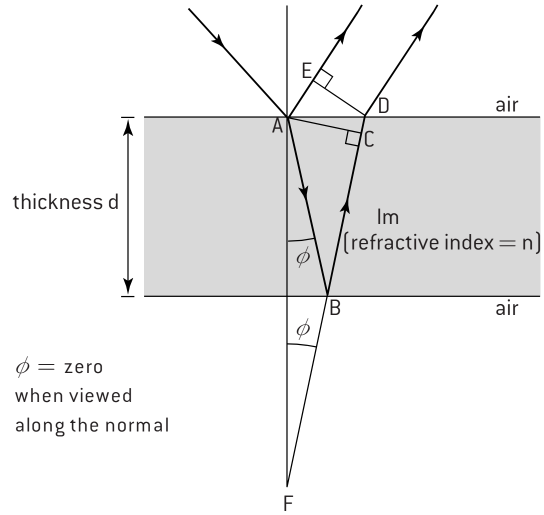
If white light is used then the situation becomes more complex. Provided the thickness of the film is small, then one or two colours may reinforce along a direction in which others cancel. The appearance of the film will be bright colours, such as can be seen when looking at

- an oil film on the surface of water or
- soap bubbles.




## CONDITIONS FOR INTERFERENCE PATTERNS

A parallel-sided film can produce interference as a result of the reflections that are taking place at both surfaces of the film.




From point A, there are two possible paths:

**1.** along path AE in air
**2.** along ABCD in the film of thickness d

These rays then interfere and we need to calculate the optical path difference.

The path AE in air is equivalent to CD in the film
So path difference = (AB + BC) in the film.

In addition, the phase change at A is equivalent to $\frac{\lambda}{2}$ path difference.

So total path difference $= (AB + BC)$ in film $+ \frac{\lambda}{2}$

$= (AB + BC) + \frac{\lambda}{2}$

By geometry:

$(AB + BC) = FC$

$= 2d \cos\phi$

$\therefore$ path difference $= 2d \cos\phi + \frac{\lambda}{2}$

if $2d \cos\phi = m\lambda$ : destructive

or when $\phi = 0$, $2d = m\lambda$: destructive

if $2d \cos\phi = \left(m + \frac{\lambda}{2}\right)\lambda$: constructive

or when $\phi = 0$, $2d = m\lambda$: constructive

$m = 0,1,2,3,4$

## APPLICATIONS

Applications of parallel thin films include:

- The design of non-reflecting radar coatings for military aircraft. If the thickness of the extra coating is designed so that radar signals destructively interfere when they reflect from both surfaces, then no signal will be reflected and an aircraft could go undetected.
- Measurements of thickness of oil slicks caused by spillage. Measurements of the wavelengths of electromagnetic signals that give constructive and destructive interference (at known angles) allow the thickness of the oil to be calculated.
- Design of non-reflecting surfaces for lenses (blooming), solar panels and solar cells. A strong reflection at any of these surfaces would reduce the amount of energy being usefully transmitted. A thin surface film can be added so that destructive interference takes place for a typical wavelength and thus maximum transmittance takes place at this wavelength.

# HL Resolution

## DIFFRACTION AND RESOLUTION

I two sources o light are very close in angle to one another, then they are seen as one single source o light. I the eye can tell the two sources apart, then the sources are said to be **resolved**. The di raction pattern that takes place at apertures a ects the eye's ability to resolve sources. The examples to the right show how the appearance o two line sources will depend on the di raction that takes place at a slit. The resulting appearance is the addition o the two overlapping di raction patterns. The graph o the resultant relative intensity o light at di erent angles is also shown.

These examples look at the situation o a line source o light and the di raction that takes place at a slit. A more common situation would be a point source o light, and the di raction that takes place at a circular aperture. The situation is exactly the same, but di raction takes place all the way around the aperture. As seen on page 97, the di raction pattern o the point source is thus concentric circles around the central position. The geometry o the situation results in a slightly di erent value or the first minimum o the di raction pattern.

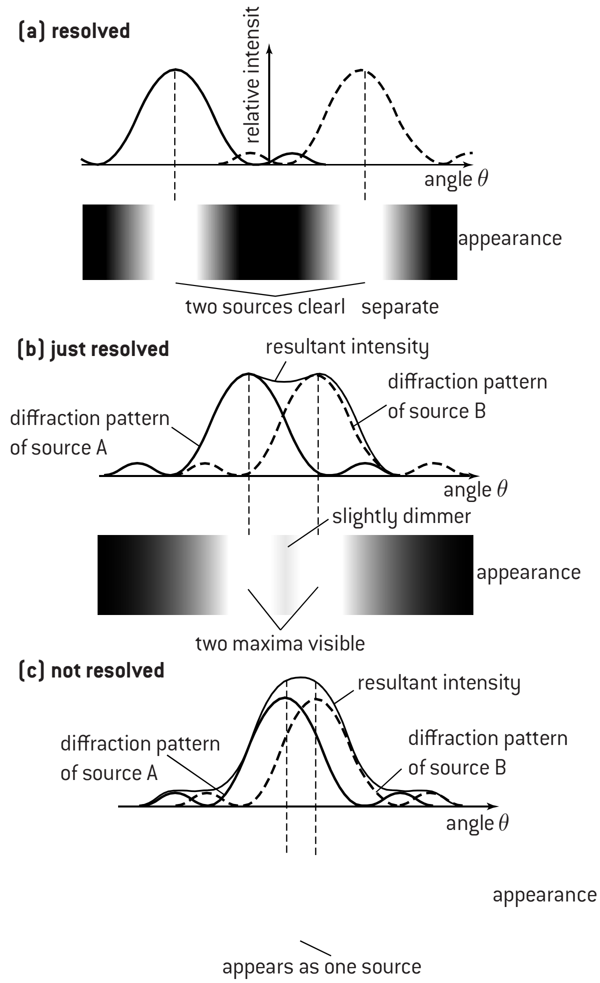
For a **slit**, the first minimum was at the angle

$$\theta = \frac{\lambda}{b}$$

For a **circular aperture**, the first minimum is at the angle

$$\theta = \frac{1.22\,\lambda}{b}$$

I two sources are just resolved, then the first minimum o one di raction pattern is located on top o the maximum o the other di raction pattern. This is known as the **Rayleigh criterion**.




## EXAMPLE

Late one night, a student was observing a car approaching rom a long distance away. She noticed that when she first observed the headlights o the car, they appeared to be one point o light. Later, when the car was closer, she became able to see two separate points o light. I the wavelength o the light can be taken as 500 nm and the diameter o her pupil is approximately 4 mm, calculate how ar away the car was when she could first distinguish two points o light. Take the distance between the headlights to be 1.8 m.

When just resolved

$$\theta = \frac{1.22 \times \lambda}{b}$$

$$= \frac{1.22 \times 5 \times 10^{-7}}{0.004}$$

$$= 1.525 \times 10^{-4}$$

Since $\theta$ small

$$\theta = \frac{1.8}{x} \quad [x \text{ is distance to car}]$$

$$\Rightarrow x = \frac{1.8}{1.525 \times 10^{-4}}$$

$$= 11.803$$

12 km

## RESOLVANCE OF DIFFRACTION GRATINGS

As a result o Rayleigh's criterion, there is a limit placed on a grating's ability to resolve di erent wavelengths. The resolvance, $R$, o a di raction grating is defined as the ratio between a wavelength being investigated, $\lambda$, and the smallest possible resolvable wavelength di erence, $\Delta\lambda$.

$$R = \frac{\lambda}{\Delta\lambda}$$

For any given grating, $R$ is dependent on the di raction order, $m$, being observed (first order: $m = 1$; second order: $m = 2$, etc.) and the total number o slits, $N$, on the grating that are being illuminated.

$$R = \frac{\lambda}{\Delta\lambda} = mN$$

Example:

In the sodium emission spectrum there are two wavelengths that are close to one another (the Na D-lines). These are 589.00 nm and 589.59 nm. In order or these to be resolved by a di raction grating, the resolvance must be

$$R = \frac{\lambda}{\Delta\lambda} = \frac{589.00}{0.59} = 1000$$

In the first order spectrum, at least 1000 slits must be illuminated whereas in the second order spectrum, the requirement drops to only 500 slits.

# The Doppler e ec

## DOPPLER EFFECT

The Doppler e ect is the name given to the change o requency o a wave as a result o the movement o the source or the movement o the observer.

When a source o sound is moving:

- Sound waves are emitted at a particular requency rom the source.
- The speed o the sound wave in air does not change, but the motion o the source means that the wave ronts are all 'bunched up' ahead o the source.
- This means that the stationary observer receives sound waves o reduced wavelength.
- Reduced wavelength corresponds to an increased requency o sound.

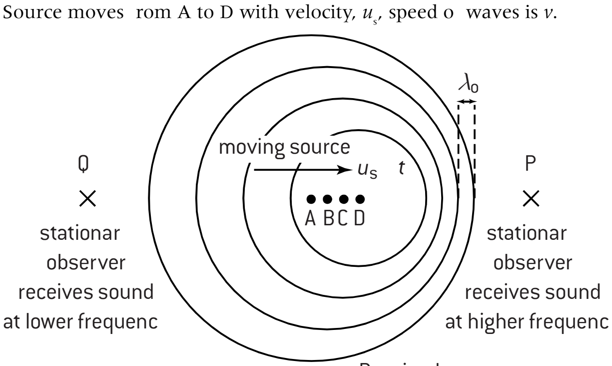
The overall e ect is that the observer will hear sound at a higher requency than it was emitted by the source. This applies when the source is moving towards the observer. A similar analysis quickly shows that i the source is moving away rom the observer, sound o a lower requency will be received.

A change o requency can also be detected i the source is stationary, but the observer is moving.

- When a police car or ambulance passes you on the road, you can hear the pitch o the sound change rom high to low requency. It is high when it is approaching and low when it is going away.
- Radar detectors can be used to measure the speed o a moving object. They do this by measuring the change in the requency o the reflected wave.
- For the Doppler e ect to be noticeable with light waves, the source (or the observer) needs to be moving at high speed. I a source o light o a particular requency is moving away rom an observer, the observer will receive light o a lower requency. Since the red part o the spectrum has lower requency than all the other colours, this is called a **red shift**.
- I the source o light is moving towards the observer, there will be a **blue shift**.

## MOVING SOURCE

Source moves rom A to D with velocity, $u_s$, speed o waves is $v$.




$f' = f\left(\frac{}{\pm u_s}\right)$

Received frequenc at P: $f' = f\left(\frac{}{- u_s}\right)$

Received frequenc at Q: $f' = f\left(\frac{}{+ u_s}\right)$

## MATHEMATICS OF THE DOPPLER EFFECT

Mathematical equations that apply to sound are stated on this page.

Un ortunately the same analysis does not apply to light – the velocities can not be worked out relative to the medium. It is, however, possible to derive an equation or light that turns out to be in exactly the same orm as the equation or sound as long as two conditions are met:

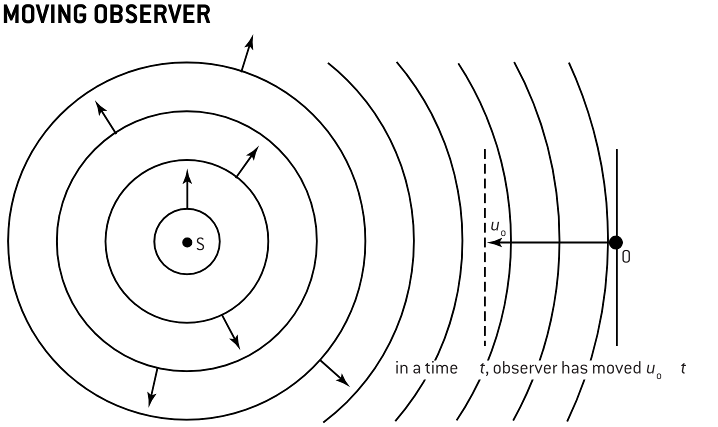
- the relative velocity o source and detector is used in the equations.
- this relative velocity is a lot less than the speed o light.

Providing $v << c$

$$\frac{\Delta f}{f} = \frac{\Delta \lambda}{\lambda} \approx \frac{v}{c}$$

- $\Delta f$: change in requency
- $f$: requency o source
- $\Delta\lambda$: change wavelength due to relative motion
- $\lambda$: wavelength when no relative motion
- $v$: relative speed o source and observer
- $c$: speed o light

## MOVING OBSERVER




$f' = f\left(\frac{\pm u_o}{}\right)$

If observer is moving awa from source:

$f' = f\left(\frac{- u_o}{}\right)$

If observer is moving towards source:

$f' = f\left(\frac{+ u_o}{}\right)$

## EXAMPLE

The requency o a car's horn is measured by a stationary observer as 200 Hz when the car is at rest. What requency will be heard i the car is approaching the observer at 30 m s⁻¹? (Speed o sound in air is 330 m s⁻¹.)

$f = 200$ Hz

$f' = ?$

$u_s = 30$ m s$^{-1}$

$v = 330$ m s$^{-1}$

$f' = 200\left(\frac{300}{300 - 30}\right)$

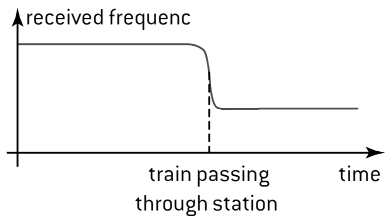
$= 200 \times 1.1$

$= 220$ Hz

# HL Examples and applications of t e Doppler effect

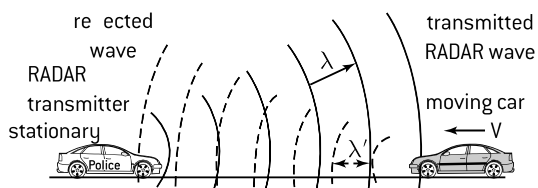
1. Train going through a station

   The sound emitted by a moving train's whistle is of constant frequency, but the sound received by a passenger standing on the platform will change. At any instant of time, it is the resolved component of the train's velocity towards the passenger that is used to calculate the frequency received.




2. Radars – speed measurement

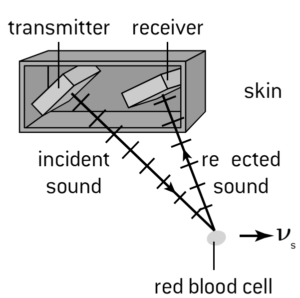
   In many countries the police use radar to measure speed of vehicles to see if they are breaking the speed limit.




   - Pulse of microwave radiation of known frequency emitted.
   - Pulse is reflected off moving car and received back at source.
   - Difference in emitted and received frequencies is used to calculate speed of car.
   - Double Doppler effect taking place:
     - ◊ Moving car receives a frequency that is higher than emitted as it is a moving observer.
     - ◊ Moving car acts as a moving source when sending signal back.

3. Medical physics – blood flow measurements

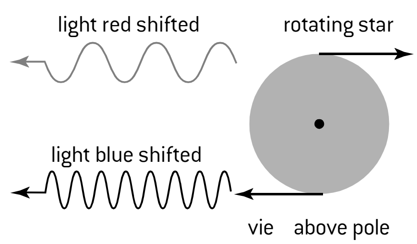
   Doctors can use a pulse of ultrasound to measure the speed of red blood cells in an analogous way that a pulse of microwaves is used to measure the speed of a moving car (above).




4. Receding galaxies – red shift
   - The relative intensities of the different wavelengths of light received from the stars in distant galaxies can be analysed.
   - The light shows a characteristic absorption spectrum.
   - The measured wavelengths are not the same as those associated with particular elements as measured in the laboratory.
   - For the vast majority of stars, all the received frequencies have been shifted towards the red end of the visible spectrum (i.e. to lower frequencies). The light shows a **red shift** (see page 202).
   - The magnitude of the red shift is used to calculate the recessional velocity and provides evidence for the Big Bang model of the creation of the Universe.

5. Rotating object

   The rotation of luminous objects (e.g. the Sun) can be measured by looking for a different Doppler shift on one side of the object compared with the other.




6. Broadening of spectral lines
   - Absorption and emission spectra provide evidence for discrete atomic energy levels (see page 69).
   - Precise measurements show that each individual level is actually equivalent to a small but defined wavelength range.
   - The gas molecules are moving so light from molecules will be subjected to Doppler shift.
   - Different molecules have a range of speeds so there will be a general **Doppler broadening** of the discrete wavelengths.
   - A higher temperature means a wider distribution of kinetic energies and hence more broadening to the spectral line.

# HL IB Questions – wave phenomena

1. When a train travels towards you sounding its whistle, the pitch o  the sound you hear is di  erent  rom when the train is at rest. This is because

   A. the sound waves are travelling  aster toward you.

   B. the wave  ronts o  the sound reaching you are spaced closer together.

   C. the wave  ronts o  the sound reaching you are spaced  urther apart.

   D. the sound  requency emitted by the whistle changes with the speed o  the train.

2. A car is travelling at constant speed towards a stationary observer whilst its horn is sounded. The  requency o  the note emitted by the horn is 660 Hz. The observer, however, hears a note o   requency 720 Hz.

   a) With the aid o  a diagram, explain why a higher  requency is heard. [2]

   b) I  the speed o  sound is 330 m $s^{-1}$, calculate the speed o  the car. [2]

3. This question is about using a di  raction grating to view the emission spectrum o  sodium.

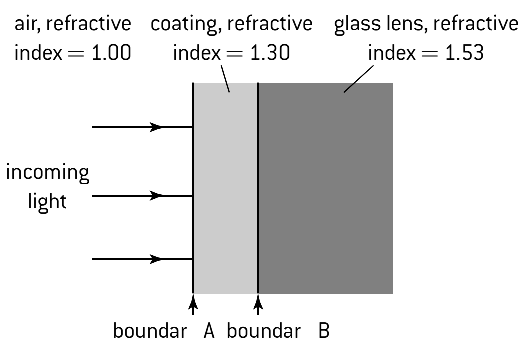
   Light  rom a sodium discharge tube is incident normally upon a di  raction grating having $8.00 \times 10^5$ lines per metre. The spectrum contains a double yellow line o  wavelengths 589 nm and 590 nm.

   a) Determine the angular separation o  the two lines when viewed in the second order spectrum. [4]

   b) State why it is more di ficult to observe the double yellow line when viewed in the first order spectrum. [1]

4. This question is about thin film inter erence.

   A transparent thin film is sometimes used to coat spectacle lenses as shown in the diagram below.




   a) State the phase change which occurs to light that

      (i) is transmitted at boundary A into the film. [1]

      (ii) is reflected at boundary B. [1]

      (iii) is transmitted at boundary A  rom the film into the air. [1]

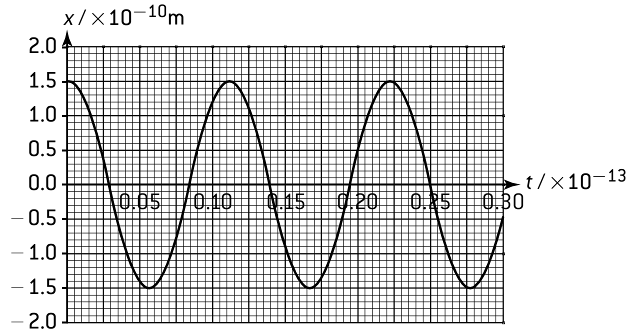
   b) Light o  wavelength 570 nm in air is incident on the coating. Determine the smallest thickness o  the coating required so that the reflection is minimized  or normal incidence. [2]

5. Simple harmonic motion and the greenhouse e  ect

   a) A body is displaced  rom equilibrium. State the **two** conditions necessary  or the body to execute simple harmonic motion. [2]

   b) In a simple model o  a methane molecule, a hydrogen atom and the carbon atom can be regarded as two masses attached by a spring. A hydrogen atom is much less massive than the carbon atom such that any displacement o  the carbon atom may be ignored.

   The graph below shows the variation with time *t* o  the displacement *x*  rom its equilibrium position o  a hydrogen atom in a molecule o  methane.




   The mass o  hydrogen atom is $1.7 \times 10^{-27}$ kg. Use data  rom the graph above

   (i) to determine its amplitude o  oscillation. [1]

   (ii) to show that the  requency o  its oscillation is $9.1 \times 10^{13}$ Hz. [2]

   (iii) to show that the maximum kinetic energy o  the hydrogen atom is $6.2 \times 10^{-18}$ J. [2]

   c) Sketch a graph to show the variation with time *t* o  the velocity *v* o  the hydrogen atom  or one period o  oscillation starting at $t = 0$. (There is no need to add values to the velocity axis.) [3]

   d) Assuming that the motion o  the hydrogen atom is simple harmonic, its  requency o  oscillation *f* is given by the expression

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_p}},$$

   where *k* is the  orce per unit displacement between a hydrogen atom and the carbon atom and $m_p$ is the mass o  a proton.

   (i) Show that the value o  *k* is approximately 560 N $m^{-1}$. [1]

   (ii) Estimate, using your answer to (d)(i), the maximum acceleration o  the hydrogen atom. [2]

   e) Methane is classified as a greenhouse gas.

   (i) Describe what is meant by a greenhouse gas. [2]

   (ii) Electromagnetic radiation o   requency $9.1 \times 10^{13}$ Hz is in the in rared region o  the electromagnetic spectrum. Suggest, based on the in ormation given in (b)(ii), why methane is classified as a greenhouse gas. [2]