HL Pho oelec ric effec

PHOTOELECTRIC EFFECT

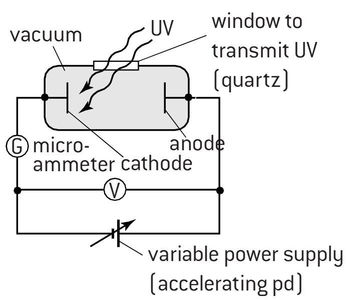
Under certain conditions, when light (ultra-violet) is shone onto a metal sur ace (such as zinc), electrons are emitted rom the sur ace.

More detailed experiments (see below) showed that:

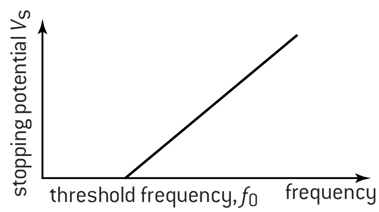
- Below a certain **threshold frequency**, f_0, no photoelectrons are emitted, no matter how long one waits.
- Above the threshold requency, the maximum kinetic energy o these electrons depends on the requency o the incident light.
- The number o electrons emitted depends on the intensity o the light and does not depend on the requency.
- There is no noticeable delay between the arrival o the light and the emission o electrons.

These observations cannot be reconciled with the view that light is a wave. A wave o any requency should eventually bring enough energy to the metal plate.

STOPPING POTENTIAL EXPERIMENT



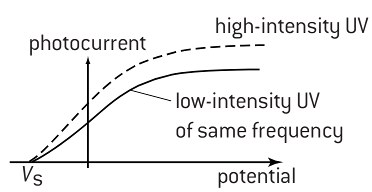
The stopping potential depends on the requency o UV light in the linear way shown in the graph below.



In the apparatus above, photoelectrons are emitted by the cathode. They are then accelerated across to the anode by the potential di erence.

The potential between cathode and anode can also be reversed.

In this situation, the electrons are decelerated. At a certain value o potential, the stopping potential, V_s, no more photocurrent is observed. The photoelectrons have been brought to rest be ore arriving at the anode.



The stopping potential is a measure o the maximum kinetic energy o the electrons.

Max KE o electrons = $V_s e$

[since pd = $\frac{\text{energy}}{\text{charge}}$

and e = charge on an electron]

$$\frac{1}{2} mv^2 = V_s e \qquad v = \sqrt{\frac{2V_s e}{m}}$$

EXAMPLE

What is the maximum velocity o electrons emitted rom a zinc sur ace ($\phi = 4.2$ eV) when illuminated by EM radiation o wavelength 200 nm?

$\phi = 4.2 \text{ eV} = 4.2 \times 1.6 \times 10^{-19} \text{ J} = 6.72 \times 10^{-19} \text{ J}$

Energy o photon $= h\frac{c}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-7}}$

$= 9.945 \times 10^{-19}$ J

KE o electron $= (9.945 - 6.72) \times 10^{-19}$ J

$= 3.225 \times 10^{-19}$ J

$$v = \sqrt{\frac{2\,KE}{m}}$$

$$= \sqrt{\frac{2 \times 3.225 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$= 8.4 \times 10^5 \text{ m s}^{-1}$

EINSTEIN MODEL

Einstein introduced the idea o thinking o light as being made up o particles.

His explanation was:

- Electrons at the sur ace need a certain minimum energy in order to escape rom the sur ace. This minimum energy is called the **work function** o the metal and given the symbol ϕ.
- The UV light energy arrives in lots o little packets o energy – the packets are called photons.
- The energy in each packet is fixed by the requency o UV light that is being used, whereas the number o packets arriving per second is fixed by the intensity o the source.
- The energy carried by a photon is given by

$$E = hf$$

Planck's constant 6.63×10^{-34} J s

energy in joules

 requency o light in Hz

- Di erent electrons absorb di erent photons. I the energy o the photon is large enough, it gives the electron enough energy to leave the sur ace o the metal.
- Any 'extra' energy would be retained by the electron as kinetic energy.
- I the energy o the photon is too small, the electron will still gain this amount o energy but it will soon share it with other electrons.

Above the threshold requency, incoming energy o photons = energy needed to leave the sur ace + kinetic energy.

In symbols,

$E_{max} = hf - \phi$

$hf = \phi + E_{max}$ or $hf = \phi + V_s e$

This means that a graph o   requency against stopping potential should be a straight line o gradient $\frac{e}{h}$.

HL Matter waves

WAVE–PARTICLE DUALITY

The photoelectric e ect o light waves clearly demonstrates that light can behave like particles, but its wave nature can also be demonstrated – it reflects, re racts, di racts and inter eres just like all waves. So what exactly is it? It seems reasonable to ask two questions.

1. *Is light a   ave or is it a particle?*

The correct answer to this question is 'yes'! At the most undamental and even philosophical level, light is just light. Physics tries to understand and explain what it is. We do this by imagining models o its behaviour. Sometimes it helps to think o it as a wave and sometimes it helps to think o it as a particle, but neither model is complete. Light is just light. This dual nature o light is called **wave particle duality**.

2. *If light   aves can sho   particle properties, can particles such as electrons sho   ave properties?*

Again the correct answer is 'yes'. Most people imagine moving electrons as little particles having a definite size, shape, position and speed. This model does not explain why electrons can be di racted through small gaps. In order to di ract they must have a wave nature. Once again they have a dual nature. See the experiment below.

DE BROGLIE HYPOTHESIS

I matter can have wave properties and waves can have matter properties, there should be a link between the two models. The de Broglie hypothesis is that all moving particles have a 'matter wave' associated with them. This matter wave can be thought o as a probability unction associated with the moving particle. The (amplitude)^2 o the wave at any given point is a measure o the probability o finding the particle at that point. The wavelength o this matter wave is given by the de Broglie equation:

$$\lambda = \frac{hc}{pc} = \frac{hc}{E} \text{ or photons}$$

λ is the wavelength in m

h is Plank's constant = 6.63×10^{-34} J s

c is the speed o light = 3.0×10^{8} m s^{-1}

p is the momentum o the particle

The higher the energy, the lower the de Broglie wavelength. This equation was introduced on page 69 as the method o calculating a photon's wavelength rom its energy, E. In order or the wave nature o particles to be observable in experiments, the particles o ten have very high velocities. In these situations the proper calculations are relativistic but simplifications are possible.

1. At very high energies: $pc = E$

In these situations, the rest energy o the particles can be negligible compared with their energy o motion.

For example, the rest energy o an electron (0.511 MeV) is negligible i it has been accelerated through an e ective potential di erence o 420 MV to have kinetic energy o 420 MeV. In these circumstances the total energy o an electron is e ectively 420 MeV. The de Broglie wavelength o 420 MeV electrons is:

$$\lambda = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^{8}}{420 \times 10^{6} \times 1.6 \times 10^{-19}} = 2.9 \times 10^{-15}\text{ m}$$

2. At low energies

In these situations the relationship can be restated in terms o the momentum p o the particle measured in kg m s^{-1} (in non-relativistic mechanics, P = mass × velocity):

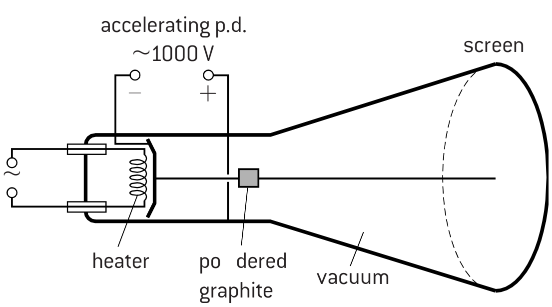
$$\lambda = \frac{h}{p}$$

For example, electrons accelerated through 1 kV would gain a KE o 1.6×10^{-16} J. Since KE and non-relativistic momentum are related by $E_K = \frac{p^2}{2m}$, this gives $p = 1.7 \times 10^{-23}$ kg m s^{-1}

$$\lambda = \frac{6.6 \times 10^{-34}}{1.7 \times 10^{-23}} = 3.9 \times 10^{-11}\text{ m}$$

ELECTRON DIFFRACTION EXPERIMENT

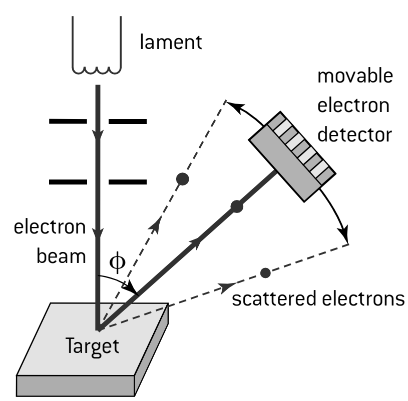
In order to show di raction, an electron 'wave' must travel through a gap o the same order as its wavelength. The atomic spacing in crystal atoms provides such gaps. I a beam o electrons impinges upon powdered carbon then the electrons will be di racted according to the wavelength.



The circles correspond to the angles where constructive inter erence takes place. They are circles because the powdered carbon provides every possible orientation o gap. A higher accelerating potential or the electrons would result in a higher momentum or each electron. According to the de Broglie relationship, the wavelength o the electrons would thus decrease. This would mean that the size o the gaps is now proportionally bigger than the wavelength so there would be less di raction. The circles would move in to smaller angles. The predicted angles o constructive inter erence are accurately verified experimentally.

DAVISSON AND GERMER EXPERIMENT (1927)

The diagram below shows the principle behind the Davisson and Germer electron di raction experiment.



A beam o electrons strikes a target nickel crystal. The electrons are scattered rom the sur ace. The intensity o these scattered electrons depends on the speed o the electrons (as determined by their accelerating potential di erence) and the angle.

A maximum scattered intensity was recorded at an angle that quantitatively agrees with the constructive inter erence condition rom adjacent atoms on the sur ace.

Atomic spectra and atomic energ   states

INTRODUCTION

As we have already seen, atomic spectra (emission and absorption) provide evidence or the quantization o the electron energy levels. See page 69 or the laboratory set-up. Di erent atomic models have attempted to explain these energy levels. The first quantum model o matter was the Bohr model or hydrogen: modern models describe the electrons by using wave unctions (see page 125).

HYDROGEN SPECTRUM

The emission spectrum o atomic hydrogen consists o particular wavelengths. In 1885 a Swiss schoolteacher called Johann Jakob Balmer ound that the visible wavelengths fitted a mathematical ormula.

These wavelengths, known as the **Balmer series**, were later shown to be just one o several similar series o possible wavelengths that all had similar ormulae. These can be expressed in one overall ormula called the **Rydberg formula**.

$$\frac{1}{\lambda} = R_H\left(\frac{1}{\ ^2} - \frac{1}{m^2}\right)$$

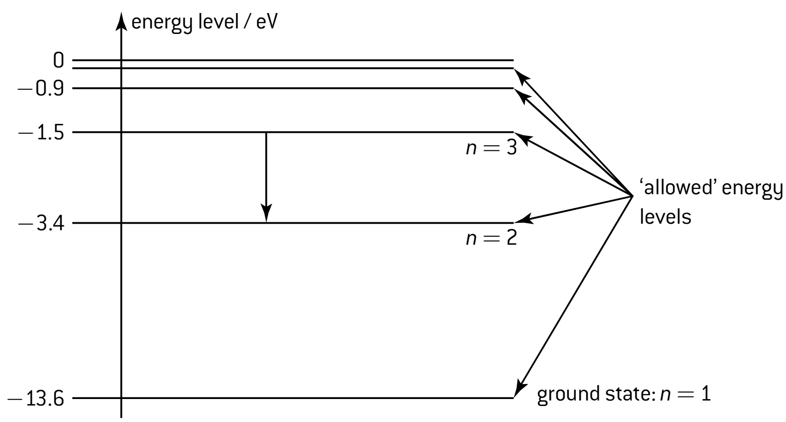
λ – the wavelength

m – a whole number larger than 2 i.e. 3, 4, 5 etc

For the **Lyman series** o lines (in the ultra-violet range)   = 1. For the **Balmer series**   = 2. The other series are the **Paschen** (= 3), **Brackett** (= 4), and the **Pfund** (= 5) series. In each case the constant R_H, called the **Rydberg constant**, has the one unique value, $1.097 \times 10^7\ m^{-1}$.

EXAMPLE

The diagram below represents some o the electron energy levels in the hydrogen atom. Calculate the wavelength o the photon emitted when an electron alls rom   = 3 to   = 2.



Energy di erence in levels = 3.4 – 1.5 = 1.9 eV = $1.9 \times 1.6 \times 10^{-19}$ J = 3.04×10^{-19} J

Frequency o photon $f = \frac{E}{h} = \frac{3.04 \times 10^{-19}}{6.63 \times 10^{-34}} = 4.59 \times 10^{14}$ Hz

Wavelength o photon $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8}{4.59 \times 10^{14}} = 6.54 \times 10^{-7}$ m = 654 nm

This is in the visible part o the electromagnetic spectrum and one wavelength in the Balmer series.

PAIR PRODUCTION AND PAIR ANNIHILATION

Matter and radiation interactions are not restricted to the absorption or emission o radiation by matter (such as takes place in absorption or emission spectra, above). As introduced on page 73, or every 'normal' matter particle that exists, there will be a corresponding antimatter particle which has the same mass but every other property is opposite. For example:

- The antiparticle o an electron, e^- (or β^-) is a positron, e^+ (or β^+)
- The antiparticle or a proton, p^+ is the antiproton, p^-
- The antiparticle or a neutrino, ν is an antineutrino, $\bar{\nu}$

When a particle and its corresponding antiparticle meet they **annihilate** one another and the mass is converted into radiation. As seen on page 78, these annihilations must obey certain conservations and in particular the conservation o energy, momentum and charge.

When an electron e^- and a positron e^+ annihilate typically they create two photons. Each photon has a momentum and i combined momentum o the electron–positron pair was initially zero, then the two photons will be travelling in opposite directions. The reverse process is also possible – photons o su ficient energy can convert into a pair o particles (one matter and one antimatter). Much o the energy goes into the rest masses o the particles with any excess going into the kinetic energy o the particles that have been created. Typically or pair production to take place, the photon needs to interact with a nucleus. The nucleus is not changed in the interaction but is involved in the overall conservation o momentum and energy that must take place. Without its ability to 'absorb' some o the momentum, the interaction could not occur.

HL Bohr model of he a om

BOHR MODEL

Niels Bohr took the standard 'planetary' model o the hydrogen atom and filled in the mathematical details. Unlike planetary orbits, there are only a limited number o 'allowed' orbits or the electron. Bohr suggested that these orbits had fixed multiples o angular momentum. The orbits were quantized in terms o angular momentum. The energy levels predicted by this quantization were in exact agreement with the discrete wavelengths o the hydrogen spectrum. Although this agreement with experiment is impressive, the model has some problems associated with it.

Bohr postulated that:

- An electron does not radiate energy when in a stable orbit. The only stable orbits possible or the electron are ones where the **angular momentum** o the orbit is an integral multiple o $\frac{h}{2\pi}$ where h is a fixed number (6.6×10^{-34} J s) called **Planck's constant**. Mathematically

  $$m_e vr = \frac{nh}{2\pi}$$

  [angular momentum is equal to $m_e vr$]

- When electrons move between stable orbits they radiate (or absorb) energy.

$$F_{\text{electrostatic}} = \text{centripetal orce}$$

$$\therefore \frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{m_e v^2}{r}$$

but $v = \frac{nh}{2\pi m_e r}$ [rom 1st postulate]

$$\therefore r = \frac{\varepsilon_0 n^2 h^2}{\pi m_e e^2} \text{ (by substitution)}$$

Total energy o electron $= KE + PE$

where $KE = \frac{1}{2} m_e v^2 = \frac{1}{2} \frac{e^2}{(4\pi\varepsilon_0 r)}$

and $PE = -\frac{e^2}{4\pi\varepsilon_0 r}$ [electrostatic PE]

so total energy $E_n = -\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 r} = -\frac{m_e e^4}{8\varepsilon_0^2 n^2 h^2}$

This final equation shows that:

- the electron is bound to (= 'trapped by') the proton because overall it has negative energy.
- the energy o an orbit is proportional to $-\frac{1}{n^2}$. In electronvolts

  $$E_n = -\frac{13.6}{n^2}$$

The second postulate can be used (with the ull equation) to predict the wavelength o radiation emitted when an electron makes a transition between stable orbits.

$$hf = E_2 - E_1$$

$$= \frac{m_e e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

but $f = \frac{c}{\lambda}$

$$\therefore \frac{1}{\lambda} = \frac{m_e e^4}{8\varepsilon_0^2 c h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

It should be noted that:

- this equation is o the same orm as the Rydberg ormula.
- the values predicted by this equation are in very good agreement with experimental measurement.
- the Rydberg constant can be calculated rom other (known) constants. Again the agreement with experimental data is good.

The limitations to this model are:

- i the same approach is used to predict the emission spectra o other elements, it ails to predict the correct values or atoms or ions with more than one electron.
- the first postulate (about angular momentum) has no theoretical justification.
- theory predicts that electrons should, in act, not be stable in circular orbits around a nucleus. Any accelerated electron should radiate energy. An electron in a circular orbit is accelerating so it should radiate energy and thus spiral in to the nucleus.
- it is unable to account or relative intensity o the di erent lines.
- it is unable to account or the fine structure o the spectral lines.

NUCLEAR RADII AND NUCLEAR DENSITIES

Not surprisingly, more massive nuclei have larger radii. Detailed analysis o the data implies that the nuclei have a spherical distribution o positive charge with an essentially constant density. The results are consistent with a model in which the protons and neutrons can be imagined to be hard spheres that are bonded tightly together in a sphere o constant density. A nucleus that is twice the size o a smaller nucleus will have roughly 8 ($=2^3$) times the mass.

The nuclear radius R o element with atomic mass number A can be modelled by the relationship:

$$R = R_0 A^{\frac{1}{3}}$$

Where R_0 is a constant roughly equal to 10^{-15} m (or 1 m).
$R_0 = 1.2 \times 10^{-15}$ m = 1.2 m.

e.g. The radius o a uranium-238 nucleus is predicted to be

$$R = 1.2 \times 10^{-15} \times (238)^{\frac{1}{3}} \text{ m} = 7.4 \text{ m}$$

The volume o a nucleus, V, o radius, R is given by:

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi A R_0^3$$

Where the mass number A is equal to the number o nucleons

The number o nucleons per unit volume $= \frac{A}{V} = \frac{3A}{4\pi A R_0^3}$

$$= \frac{3}{4\pi R_0^3}$$

The mass o a nucleon is m ($\approx 1.7 \times 10^{-27}$ kg), so the nuclear density ρ is:

$$\rho = \frac{3m}{4\pi R_0^3} = \frac{3 \times 1.7 \times 10^{-27}}{4\pi(1.2 \times 10^{-15})^3} = 2 \times 10^{17} \text{ kg m}^{-3}$$

This is a vast density (a teaspoon o matter o this density has a mass $\approx 10^{12}$ kg). The only macroscopic objects with the same density as nuclei are neutron stars (see page 200).

The Schr dinger model of the atom

SCHRÖDINGER MODEL

Erwin Schrödinger (1887–1961) built on the concept of matter waves and proposed an alternative model of the hydrogen atom using wave mechanics. The **Copenhagen interpretation** is a way to give a physical meaning to the mathematics of wave mechanics.

- The description of particles (matter and/or radiation) in quantum mechanics is in terms of a **wavefunction** ψ. This wavefunction has no physical meaning but the square of the wavefunction does.
- ψ is a **complex number**.
- At any instant of time, the wavefunction has different values at different points in space.
- The mathematics of how this wavefunction develops with time and interacts with other wavefunctions is like the mathematics of a travelling wave.
- The probability of finding the particle (electron or photon, etc.) at any point in space within the atom is given by the square of the amplitude of the wavefunction at that point.
- The square of the absolute value of ψ, $|\psi|^2$, is a real number corresponding to the probability density of finding the particle in a given place.
- When an observation is made the wavefunction is said to **collapse**, and the complete physical particle (electron or photon, etc.) will be observed to be at one location.

The standing waves on a string have a fixed wavelength but for energy reasons the same is not true for the electron wavefunctions. As an electron moves away from the nucleus it must lose kinetic energy because they have opposite charges. Lower kinetic energy means that it would be travelling with a lower momentum and the de Broglie relationship predicts a longer wavelength. This means that the possible wavefunctions that fit the boundary conditions have particular shapes.

The wavefunction provides a way of working out the probability of finding an electron at that particular radius. $|\psi|^2$ at any given point is a measure of the probability of finding the electron at that distance away from the nucleus – in any direction.

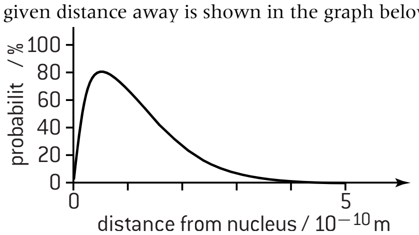
$$p(r) = |\psi|^2 \Delta V$$

probability of detecting the electrons in a small volume of space, ΔV

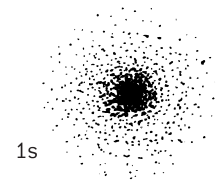
The wavefunction exists in all three dimensions, which makes it hard to visualize. Often the electron orbital is pictured as a cloud. The exact position of the electron is not known but we know where it is more likely to be.

In Schrödinger's model there are different wavefunctions depending on the total energy of the electron. Only a few particular energies result in wavefunctions that fit the boundary conditions – electrons can only have these particular energies within an atom. An electron in the ground state has a total energy of –13.6 eV, but its position at any given time is undefined in this model. The wavefunction for an electron of this energy can be used to calculate the probability of finding it at a given distance away from the nucleus.

- The resulting **orbital** for the electron can be described in terms of the probability of finding the electron at a certain distance away. The probability of finding the electron at a given distance away is shown in the graph below.

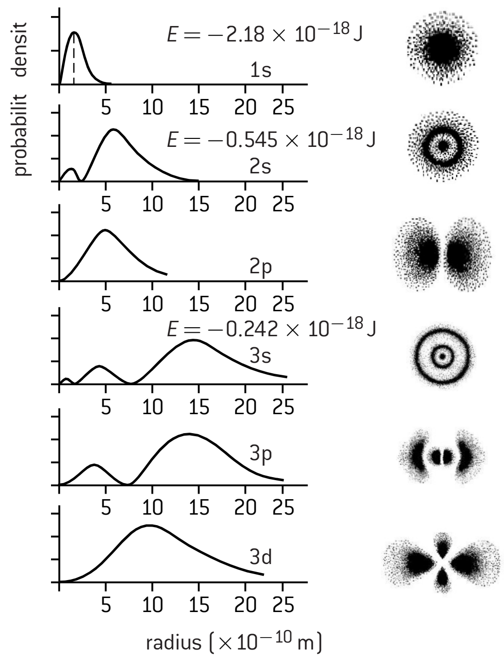


- The electron in this orbital can be visualized as a 'cloud' of varying electron density. It is more likely to be in some places than other places, but its actual position in space is undefined.



Electron cloud for the 1s orbital in hydrogen

There are other fixed total energies for the electron that result in different possible orbitals. In general as the energy of the electron is increased it is more likely to be found at a further distance away from the nucleus.

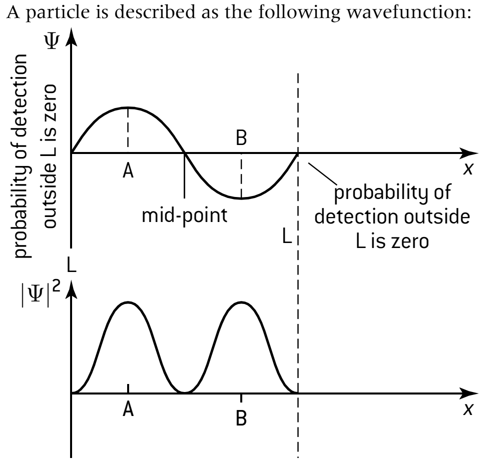


Probability density functions for some orbitals in the hydrogen atom. The scale on the vertical axis is different from graph to graph.

The wavefunction is central to quantum mechanics and, in principle, should be applied to all particles.

Example:

A particle is described as the following wavefunction:



The particle will not be detected at the mid point and the probability of detection at A = probability of detection at B.

HL The Heisenberg uncertaint principle and the loss of determinism

HEISENBERG UNCERTAINTY PRINCIPLE

The **Heisenberg uncertainty principle** identifies a undamental limit to the possible accuracy o any physical measurement. This limit arises because o the nature o quantum mechanics and not as a result o the ability (or otherwise) o any given experimenter. He showed that it was impossible to measure exactly the position **and** the momentum o a particle simultaneously. The more precisely the position is determined, the less precisely the momentum is known in this instant, and vice versa. They are linked variables and are called **conjugate quantities**.

There is a mathematical relationship linking these uncertainties.

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

Δx The uncertainty in the measurement o position

Δp The uncertainty in the measurement o momentum

Measurements o energy and time are also linked variables.

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

ΔE The uncertainty in the measurement o energy

Δt The uncertainty in the measurement o time

The implications o this lack o precision are pro ound. Be ore quantum theory was introduced, the physical world was best described by deterministic theories – e.g. Newton's laws. A deterministic theory allows us (in principle) to make absolute predictions about the uture.

Quantum mechanics is not deterministic. It cannot ever predict exactly the results o a single experiment. It only gives us the probabilities o the various possible outcomes. The uncertainty principle takes this even urther. Since we cannot know the precise position and momentum o a particle at any given time, its uture can never be determined precisely. The best we can do is to work out a range o possibilities or its uture.

It has been suggested that science would allow us to calculate the uture so long as we know the present exactly. As Heisenberg himsel said, it is not the conclusion o this suggestion that is wrong but the premise.

ESTIMATES FROM THE UNCERTAINTY PRINCIPLE

Example calculation: The position o a proton is measured with an accuracy o $\pm 1.0 \times 10^{-11}$ m. What is the minimum uncertainty in the proton's position 1.0 s later?

$$\Delta x \Delta p \geq \frac{h}{4\pi} \qquad \therefore \quad \Delta x \times m\Delta v \geq \frac{h}{4\pi}$$

$$\Delta v \geq \frac{h}{4\pi m \Delta x} = \frac{6.63\times 10^{-34}}{4\pi \times 1.67\times 10^{-27}\times 1.0\times 10^{-11}}$$

$$= 3200 \text{ m s}^{-1}$$

Thus uncertainty in position a ter 1.0 s = 3200 m = 3.2 km

The uncertainty principle can also be applied to illuminate some general principles but, to quote Richard Feynman (Feynman lectures on Physics, volume III, 1963), '[the application] *must not be taken too seriously; the idea is right but the analysis is not very accurate*'.

1. Estimate o the energy o an electron in an atom.

   When an electron is known to be confined within an atom, then the uncertainty in its position Δx must be less than the size o the atom, a. I we equate the two, this means the uncertainty or its momentum can be estimated as:

   $$\Delta p \approx \frac{h}{4\pi \Delta x} \approx \frac{h}{4\pi a}$$

   I we take this uncertainty in the momentum as a value or the momentum o the electron ($\Delta p \approx p$), the equations o classical mechanics can estimate the kinetic energy o the electron:

   $$E_K = \frac{p^2}{2m} \approx \frac{h^2}{32\pi^2 m a^2}$$

   The diameter o a hydrogen atom is approximately 10^{-10} m, so the estimation o the kinetic energy is:

   $$E_K \approx \frac{(6.6 \times 10^{-34})^2}{32\pi^2 \times 9.3\times 10^{-31} \times (10^{-10})^2} = 1.5 \times 10^{-19} \text{ J}$$

   $$\approx 1 \text{ eV}$$

   This calculation is a very rough estimate but correctly predicts the right order o magnitude or the electron's kinetic energy (ground state o electron in H atom is −13.6 eV).

2. Impossibility o an electron existing within a nucleus o an atom.

   The above calculation can be repeated imagining an electron being trapped inside the nucleus o size 10^{-14} m. I confined to a space this small, the electron's kinetic energy would be estimated to be a actor o 10^8 times bigger. An electron with an energy o the order o 100 MeV cannot be bound to a nucleus and thus it would have enough energy to escape.

3. Estimate o li etime o an electron in an excited energy state.

   The **spectral linewidth** associated with an atom's emission spectrum is usually taken to be very small – only discrete wavelengths are observed. As a result o the uncertainty principle, the linewidth is, however, not zero. Practically, there will be a very limited range o wavelengths associated with any given transition and thus the uncertainty associated with the energy di erence between the two levels involved is very small. An estimate o the li etime o an electron in the excited state can be made using the uncertainty principle as the uncertainty in energy, ΔE, o a transition is inversely proportional to the average li etime, Δt, in the excited state:

   $$\Delta E \Delta t \approx \frac{h}{4\pi}$$

   I $\Delta E = 5 \times 10^{-7}$ eV

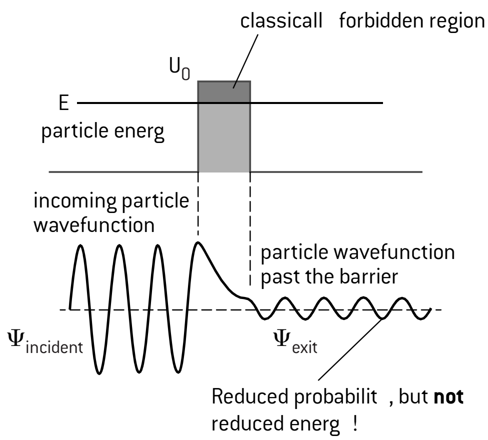
   $$\Delta t \approx \frac{6.6 \times 10^{-34}}{4\pi \times 5 \times 10^{-7} \times 1.6 \times 10^{-19}}$$

   $$= 6.6 \times 10^{-10} \approx 1 \text{ ns}$$

Tunnelling, po en ial barrier and fac ors affec ing unnelling probabili y

Heisenberg's uncertainty relationship can be used to explain the quantum phenomenon o **tunnelling**. The situation being considered is a particle that is trapped because its energy E is less than the energy it needs to escape (U_0). In classical physics, i a particle does not have enough energy to escape rom the **potential barrier** then it will always remain trapped inside the system. An example would be a 500 g tennis ball with a total energy o 4 J bouncing up and down between two walls that are 1.2 m high. In order to get over one o the walls, the tennis ball needs to have a potential energy o $mgh = 0.5 \times 10 \times 1.2 = 6$ J. Since it only has 4 J it must remain trapped by the walls.

In an equivalent microscopic situation (e.g. an electron trapped inside an atom with energy E which is less than the energy U_0 needed to escape), the rules o quantum physics mean that it is now possible or the particle to escape! The particle's wave unction is continuous and does not drop immediately to zero when it meets the sides o the potential well but the amplitude decreases exponentially. This means that i the barrier has a finite width then the wave unction does continue on the other side o the barrier (with reduced amplitude). There ore there is a probability that particle will be able to escape despite not having enough energy to do so. Escaping the potential well does not use up any o the particle's total energy.



An explanation can be o ered in terms o the uncertainty principle. In order or the particle to escape it would need a greater total energy ($E + \Delta E = U_0$). The particle can 'disobey' the law o conservation o energy by 'borrowing' an amount o energy ΔE provided it 'pays it back' in a time Δt such that the uncertainty principle applies:

$$\Delta E \Delta t \approx \frac{h}{4\pi}$$

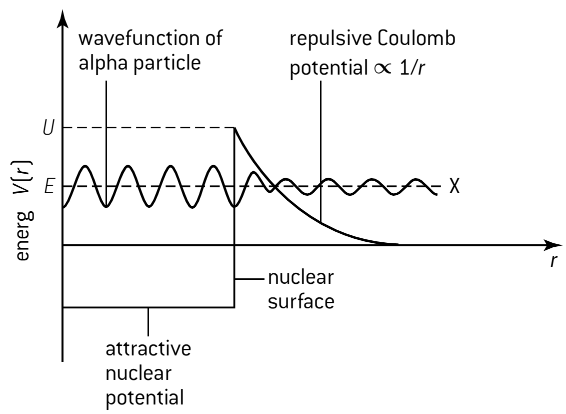
The longer the barrier, the more time it takes the particle to tunnel. Increased tunnelling time will reduce the maximum possible uncertainty in the energy.

Example 1 – alpha decay

The protons and neutrons that orm alpha particles already exist within nuclei and when emitted overall there is a release o energy. For example uranium-238 has a hal -li e o about 4.5 billion years. It decays by emitting an alpha particle:

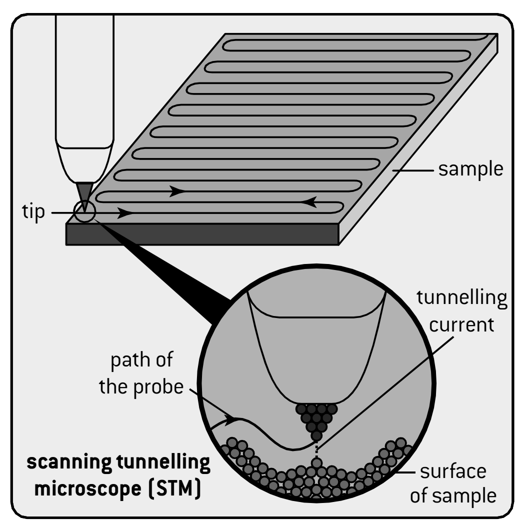
$$^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\alpha$$

The energy o the emitted alpha particle is 4.25 MeV which is less than the total potential energy needed to escape the strong orce within the nucleus. I we imagine an alpha particle being ormed inside the uranium nucleus, it can only escape by tunnelling through the potential barrier. In this example, the very long hal -li e must mean that the probability o the tunnelling process taking place (given by $|\psi|^2$) must be very low.



Example 2 – tunnelling electron microscope

In a **scanning tunnelling microscope**, a very fine metal tip is scanned close to, but not touching (separated by a ew nm), a sample metal sur ace. There is a potential di erence between the probe and the sur ace but the electrons in the sur ace do not have enough energy to escape the potential energy barrier as represented by the work unction ϕ. Quantum tunnelling can, however, take place and a tunnelling current will flow as the wave unction o an electron at the sur ace will extend beyond the metal sur ace. Some electrons will tunnel the gap and electrical current will be measurable. The value o the current depends on the separation o the tip and the sur ace and can be used to visualize atomic structure.

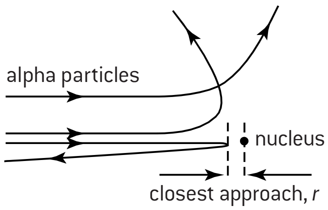


The nucleus

THE NUCLEUS – SIZE

In the example below, alpha particles are allowed to bombard gold atoms.

As they approach the gold nucleus, they eel a orce o repulsion. I an alpha particle is heading directly or the nucleus, it will be reflected straight back along the same path. It will have got as close as it can. Note that none o the alpha particles actually collides with the nucleus – they do not have enough energy.

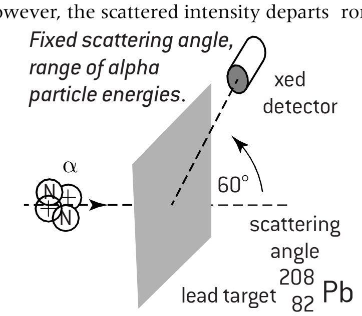


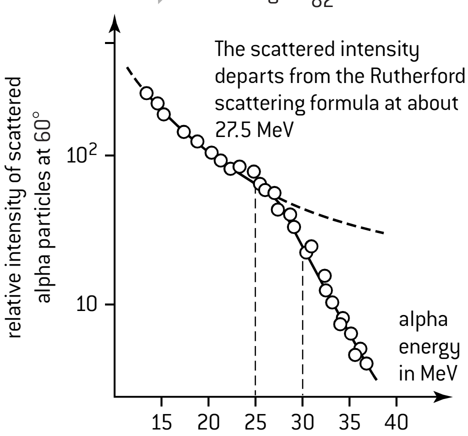
Alpha particles are emitted rom their source with a known energy. As they come in they gain electrostatic potential energy and lose kinetic energy (they slow down). At the closest approach, the alpha particle is temporarily stationary and all its energy is potential.

Since electrostatic energy $= \frac{q_1 q_2}{4\pi\varepsilon_0 r}$, and we know q_1, the charge on an alpha particle and q_2, the charge on the gold nucleus we can calculate r.

DEVIATIONS FROM RUTHERFORD SCATTERING IN HIGH ENERGY EXPERIMENTS

Ruther ord scattering is modelled in terms o the coulomb repulsion between the alpha particle and the target nucleus. At relatively low energies, detailed analysis o this model accurately predicts the relative intensity o scattered alpha particles at given angles o scattering. At high energies, however, the scattered intensity departs rom predictions.





Eisberg, R. M. and Porter, C. E., Rev. Mod. Phys. 33, 190 (1961)

At these high energies the alpha particles are beginning to get close enough to the target nucleus or the *strong nuclear force to* begin to have an e ect. In order to investigate the size o the nucleus in more detail, high energy electrons can be used (see box on the right).

EXAMPLE

I the α particles have an energy o 4.2 MeV, the closest approach to the gold nucleus ($Z = 79$) is given by

$$\frac{(2 \times 1.6 \times 10^{-19})\ (79 \times 1.6 \times 10^{-19})}{4 \times \pi \times 8.85 \times 10^{-12} \times r}$$

$$= 4.2 \times 10^{6} \times 1.6 \times 10^{-19}$$

$$\therefore\ r = \frac{2 \times 1.6 \times 10^{-19} \times 79}{4 \times \pi \times 8.85 \times 10^{-12} \times 4.2 \times 10^{6}}$$

$$= 5.4 \times 10^{-14}\ \text{m}$$

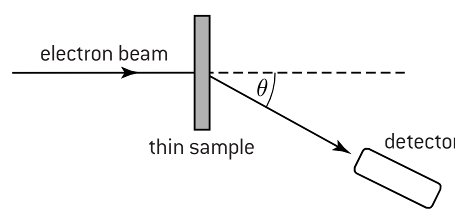
NUCLEAR SCATTERING EXPERIMENT INVOLVING ELECTRONS

Electrons, as leptons, do not eel the strong orce. High-energy electrons have a very small de Broglie wavelength which can be o the right order to di ract around small objects such as nuclei. The di raction pattern around a circular object o diameter D has its first minimum at an angle θ given by:

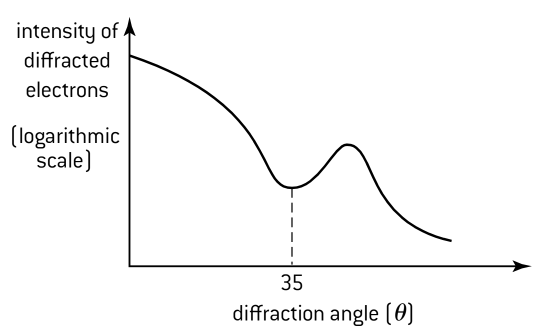
$$\sin\theta \approx \frac{\lambda}{D}$$

[Note that this small angle approximation is usually not appropriate to use to determine the location o the minimum intensity but this is being used to give an approximate answer around a spherical object. A more exact expression that is sometimes used or circular objects is $\sin\theta = 1.22\,\frac{\lambda}{D}$]

High energy (400 MeV) electrons are directed at a target containing carbon-12 nuclei:



The results are shown below:



The first minimum is $\theta = 35°$

The de Broglie wavelength or the electrons is e ectively:

$$\lambda = \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^{8}}{400 \times 10^{6} \times 1.6 \times 10^{-19}} = 3.1 \times 10^{-15}\ \text{m}$$

$$D \approx \frac{\lambda}{\sin\theta} = \frac{3.1 \times 10^{-15}}{\sin 35} = 5.4 \times 10^{-15}\ \text{m}$$

So radius o nucleus $\approx 2.7 \times 10^{-15}$ m

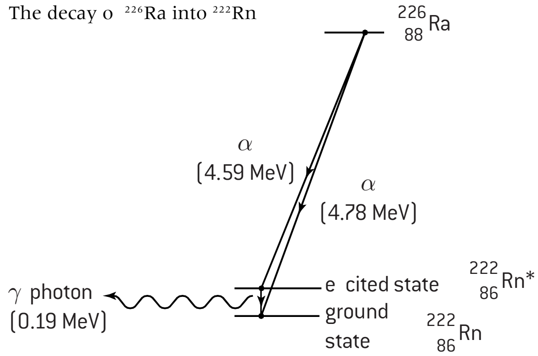
Nuclear energ   levels and radioactive deca

ENERGY LEVELS

The energy levels in a nucleus are higher than the energy levels o the electrons but the principle is the same. When an alpha particle or a gamma photon is emitted rom the nucleus only discrete energies are observed. These energies correspond to the di erence between two **nuclear energy levels** in the same way that the photon energies correspond to the di erence between two **atomic energy levels**.

Beta particles are observed to have a continuous spectrum o energies. In this case there is another particle (the antineutrino in the case o beta minus decay) that shares the energy. Once again the amount o energy released in the decay is fixed by the di erence between the nuclear energy levels involved. The beta particle and the antineutrino can take varying proportions o the energy available. The antineutrino, however, is very di ficult to observe (see box below).

The decay o ^{226}Ra into ^{222}Rn

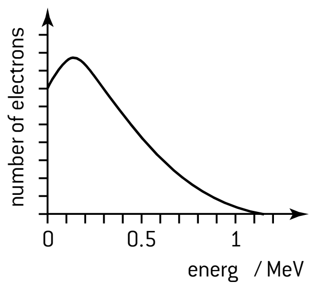


NEUTRINOS AND ANTINEUTRINOS

Understanding beta decay properly requires accepting the existence o a virtually undetectable particle, the neutrino. It is needed to account or the 'missing' energy and (angular) momentum when analysing the decay mathematically. Calculations involving mass di erence mean that we know how much energy is available in beta decay. For example, an isotope o hydrogen, tritium, deçays as ollows:

$$^{3}_{1}\text{H} \rightarrow\ ^{3}_{2}\text{He} +\ ^{0}_{-1}\beta$$

The mass di erence or the decay is 19.5 keV c^{-2}. This means that the beta particles should have 19.5 keV o kinetic energy. In act, a ew beta particles are emitted with this energy, but all the others have less than this. The average energy is about hal this value and there is no accompanying gamma photon. All beta decays seem to ollow a similar pattern.



The energy distribution o the electrons emitted in the beta decay o bismuth-210. The kinetic energy o these electrons is between zero and 1.17 MeV.

The neutrino (and antineutrino) must be electrically neutral. Its mass would have to be very small, or even zero. It carries away the excess energy but it is very hard to detect. One o the triumphs o the particle physics o the last century was to be able to design experiments that confirmed its existence. The ull equation or the decay o tritium is:

$$^{3}_{1}\text{H} \rightarrow\ ^{3}_{2}\text{He} +\ ^{0}_{-1}\beta + \bar{\nu}$$

where $\bar{\nu}$ is an antineutrino

As has been mentioned be ore, another orm o radioactive decay can also take place, namely positron decay. In this decay, a proton within the nucleus decays into a neutron and the antimatter version o an electron, a positron, which is emitted.

$$^{1}_{1}\text{p} \rightarrow\ ^{1}_{0}\text{n} +\ ^{0}_{+1}\beta^{+} + \nu$$

In this case, the positron, β^{+}, is accompanied by a neutrino.

The antineutrino is the antimatter orm o the neutrino.

e.g. $^{19}_{10}\text{Ne} \rightarrow\ ^{19}_{9}\text{F} +\ ^{0}_{+1}\beta^{+} + \nu$

$^{14}_{6}\text{C} \rightarrow\ ^{14}_{7}\text{N} +\ ^{0}_{-1}\beta + \bar{\nu}$

MATHEMATICS OF EXPONENTIAL DECAY

The basic relationship that defines exponential decay as a random process is expressed as ollows:

$$\frac{dN}{dt} \propto -N$$

The constant o proportionality between the rate o decay and the number o nuclei available to decay is called the decay constant and given the symbol λ. Its units are time^{-1} i.e. s^{-1} or yr^{-1} etc.

$$\frac{dN}{dt} = -\lambda N$$

The solution o this equation is:

$$N = N_0 e^{-\lambda t}$$

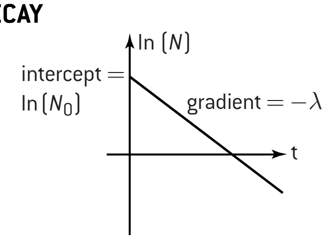
The activity o a source, A, $A = -\frac{dN}{dt}$

$$A = A_0 e^{-\lambda t} = \lambda N_0 e^{-\lambda t}$$

It is use ul to take natural logarithms:

$$\ln (N) = \ln (N_0 e^{-\lambda t})$$
$$= \ln (N_0) + \ln (e^{-\lambda t})$$
$$= \ln (N_0) - \lambda t \ln (e)$$
$$\ln (N) = \ln (N_0) - \lambda t \quad (\text{since } \ln (e) = 1)$$

This is o the orm $= c + mx$ so a graph o ln N vs t will give a straight-line graph.



$$N = N_0 e^{-\lambda t}$$

I $t = T_{\frac{1}{2}}$

$$N = \frac{N_0}{2}$$

So $$\frac{N_0}{2} = N_0 e^{-\lambda T_{\frac{1}{2}}}$$

$$\frac{1}{2} = e^{-\lambda T_{\frac{1}{2}}}$$

$$\ln ½ = -\lambda T_{\frac{1}{2}}$$

$$-\lambda T_{\frac{1}{2}} = -\ln ½$$
$$= \ln 2$$

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

EXAMPLE

The hal -li e o a radioactive isotope is 10 days. Calculate the raction o a sample that remains a ter 25 days.

$$T_{\frac{1}{2}} = 10 \text{ days}$$

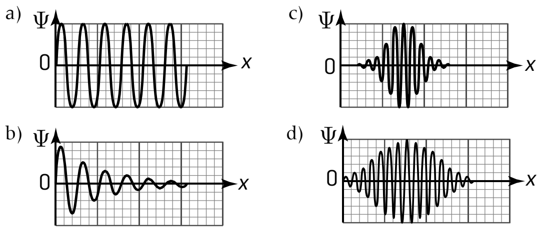
$$\lambda = \frac{\ln 2}{T_{\frac{1}{2}}}$$
$$= 6.93 \times 10^{-2} \text{ day}^{-1}$$

$$N = N_0 e^{-\lambda t}$$

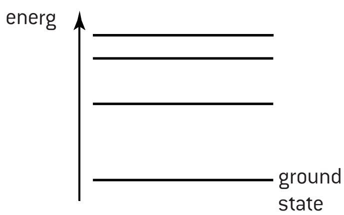
$$\text{Fraction remaining} = \frac{N}{N_0}$$
$$= e^{-(6.93 \times 10^{-2} \times 25)}$$
$$= 0.187$$
$$= 18.7\%$$

HL IB Questions – quantum and nuc ear physics

1. The diagrams show the variation with distance x of the wavefunction Ψ of four different electrons. The scale on the horizontal axis in all four diagrams is the same. For which electron is the uncertainty in the momentum the largest?

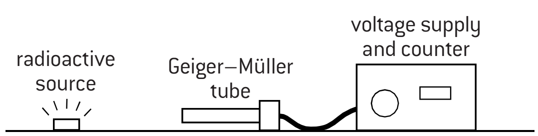


2. The diagram represents the available energy levels of an atom. How many emission lines could result from electron transitions between these energy levels?



A. 3 B. 6 C. 8 D. 12

3. A medical physicist wishes to investigate the decay of a radioactive isotope and determine its decay constant and half-life. A Geiger–Müller counter is used to detect radiation from a sample of the isotope, as shown.



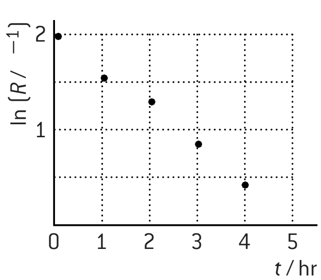
a) Define the activity of a radioactive sample. [1]

Theory predicts that the activity A of the isotope in the sample should decrease exponentially with time t according to the equation $A = A_0 e^{-\lambda t}$, where A_0 is the activity at $t = 0$ and λ is the decay constant for the isotope.

b) Manipulate this equation into a form which will give a straight line if a semi-log graph is plotted with appropriate variables on the axes. State what variables should be plotted. [2]

The Geiger counter detects a proportion of the particles emitted by the source. The physicist records the count-rate R of particles detected as a function of time t and plots the data as a graph of ln R versus t, as shown below.

c) Does the plot show that the experimental data are consistent with an *exponential* law? Explain. [1]



d) The Geiger counter does not measure the total activity A of the sample, but rather the count-rate R of those particles that enter the Geiger tube. Explain why this will not matter in determining the decay constant of the sample. [1]

e) From the graph, determine a value for the decay constant λ. [2]

The physicist now wishes to calculate the half-life.

f) Define the half-life of a radioactive substance. [1]

g) Derive a relationship between the decay constant λ and the half-life . [2]

h) Hence calculate the half-life of this radioactive isotope. [1]

4. This question is about the quantum concept.

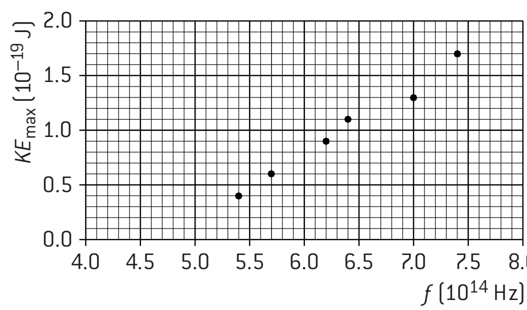
A biography of Schrödinger contains the following sentence: 'Shortly after de Broglie introduced the concept of *matter waves* in 1924, Schrödinger began to develop a new atomic theory.'

a) Explain the term '*matter waves*'. State what quantity determines the wavelength of such waves. [2]

b) Electron diffraction provides evidence to support the existence of matter waves. What is electron diffraction? [2]

5. Light is incident on a clean metal surface in a vacuum. The maximum kinetic energy KE_{max} of the electrons ejected from the surface is measured for different values of the frequency f of the incident light.

The measurements are shown plotted below.



a) Draw a line of best fit for the plotted data points. [1]

b) Use the graph to determine

(i) the Planck constant [2]

(ii) the minimum energy required to eject an electron from the surface of the metal (the *work function*). [3]

c) Explain briefly how Einstein's photoelectric theory accounts for the fact that no electrons are emitted from the surface of this metal if the frequency of the incident light is less than a certain value. [3]

6. Thorium-227 (Th-227) undergoes *a*-decay with a half-life of 18 days to form radium-223 (Ra-223). A sample of Th-227 has an initial activity of 3.2×10^5 Bq.

Determine the activity of the remaining thorium-227 after 50 days. [4]

7. Explain:

a) The role of angular momentum in the Bohr model for hydrogen [3]

b) Pair production and ahnihilation [3]

c) Quantum tunnelling [3]