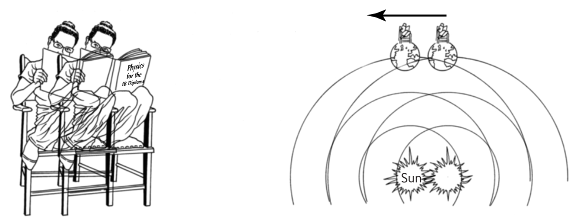
Reference frames

OBSERVERS AND FRAMES OF REFERENCE

The proper treatment of large velocities involves an understanding of Einstein's theory of relativity and this means thinking about space and time in a completely different way. The reasons for this change are developed in the following pages, but they are surprisingly simple. They logically follow from two straightforward assumptions. In order to see why this is the case we need to consider what we mean by an object in motion in the first place.

A person sitting in a chair will probably think that they are at rest. Indeed from their point of view this must be true, but this is not the only way of viewing the situation. The Earth is in orbit around the Sun, so from the Sun's point of view the person sitting in the chair must be in motion. This example shows that an object's motion (or lack of it) depends on the observer.

The calculation of relative velocity was considered on page 9. This treatment, like all the mechanics in this book so far, assumes that the velocities are small enough to be able to apply Newton's laws to different frames of reference.

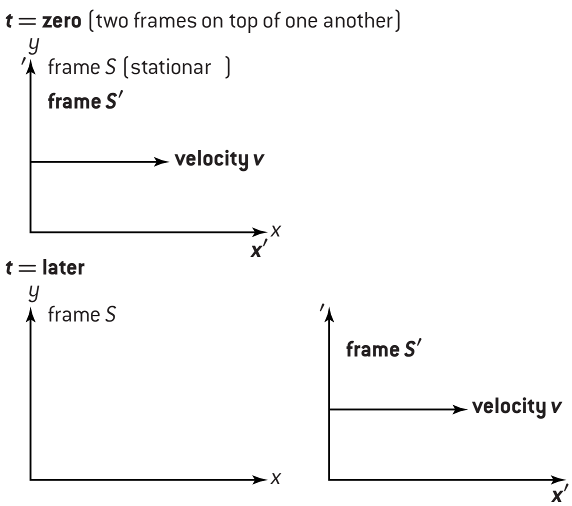


Is this person at rest or moving at great velocity?

GALILEAN TRANSFORMATIONS

It is possible to formalize the relationship between two different frames of reference. The idea is to use the measurement in one frame of reference to work out the measurements that would be recorded in another frame of reference. The equations that do this without taking the theory of relativity into consideration are called **Galilean transformations**.

The simplest situation to consider is two frames of reference (S and S') with one frame (S') moving past the other one (S) as shown below.



Each frame of reference can record the position and time of an event. Since the relative motion is along the x-axis, most measurements will be the same:

$y' = y;\ z' = z;\ t' = t$

If an event is stationary according to one frame, it will be moving according to the other frame – the frames will record different values for the x measurement. The transformation between the two is given by

$x' = x - vt$

We can use these equations to formalize the calculation of velocities. The frames will agree on any velocity measured in the y or z direction, but they will disagree on a velocity in the x-direction. Mathematically,

$u' = u - v$

For example, if the moving frame is going at 4 m s^{-1}, then an object moving in the same direction at a velocity of 15 m s^{-1} as recorded in the stationary frame will be measured as travelling at 11 m s^{-1} in the moving frame.

Newton's 3 laws of motion describe how an object's motion is effected. An assumption (Newton's Postulates) underlying these laws is that the time interval between two events is the same for all observers. Time is the same for all frames and the separation between events will also be the same in all frames. As a result, the same physical laws will apply in all frames.

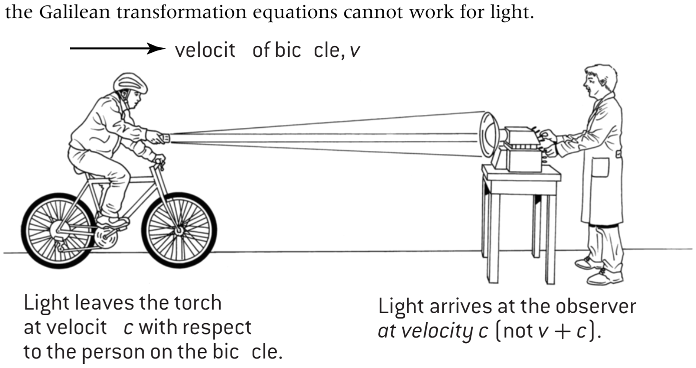
PION DECAY EXPERIMENTS

In 1964 an experiment at the European Centre for Nuclear Reseach (CERN) measured the speed of gamma-ray photons that had been produced by particles moving close to the speed of light and found these photons also to be moving at the speed of light. This is consistent with the speed of light being independent of the speed of its source, to a high degree of accuracy.

The experiment analysed the decay of a particle called the neutral pion into two gamma-ray photons. Energy considerations meant that the pions were known to be moving faster than 99.9% of the speed of light and the speed of the photons was measured to be $2.9977 \pm 0.0040 \times 10^{8}$ m s^{-1}.

FAILURE OF GALILEAN TRANSFORMATION EQUATIONS

If the speed of light has the same value for all observers (see box on left) then the Galilean transformation equations cannot work for light.



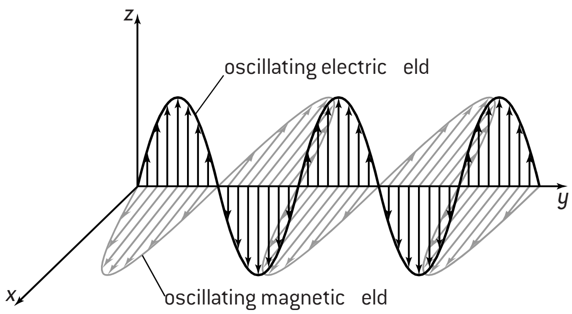
The theory of relativity attempts to work out what has gone wrong.

Max ell's equations

MAXWELL AND THE CONSTANCY OF THE SPEED OF LIGHT

In 1864 James Clerk Maxwell presented a new theory at the Royal Society in London. His ideas were encapsulated in a mathematical orm that elegantly expressed not only what was known at the time about the magnetic field B and the electric field E, but it also proposed a uni ying link between the two – electromagnetism. The 'rules' o electromagnetic interactions are summarized in our equations known as Maxwell's equations. These equations predict the nature o electromagnetic waves.

Most people know that light is an electromagnetic wave, but it is quite hard to understand what this actually means. A physical wave involves the oscillation o matter, whereas an electromagnetic wave involves the oscillation o electric and magnetic fields. The diagram below attempts to show this.



The changing electric and magnetic fields move through space – the technical way o saying this is that the fields **propagate** through space. The physics o how these fields propagate allows the speed o all electromagnetic waves (including light) to be predicted. It turns out that this can be done in terms o the electric and magnetic constants o the medium through which they travel.

$$c = \sqrt{\frac{1}{\varepsilon_0 \ _0}}$$

This equation does not need to be understood in detail. The only important idea is that the speed o light is **independent** o the velocity o the source o the light. In other words, a prediction rom Maxwell's equations is that the speed o light in a vacuum has the same value or all observers.

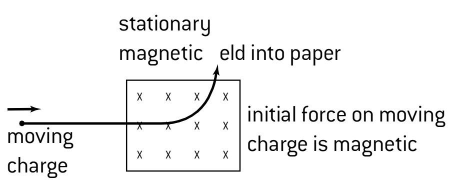
This prediction o the constancy o the speed o light highlights an inconsistency that cannot be reconciled with Newtonian mechanics (where the resultant speed o light would be equal to the addition o the relative speed o the source and the relative speed o light as measured by the source). Einstein's analysis orced long-held assumptions about the independence o space and time to be rejected.

COMPARING ELECTRIC AND MAGNETIC FIELDS

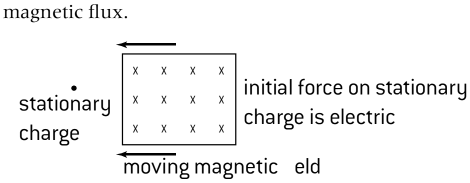
Electrostatic orces and magnetic orces appear very di erent to one another. Fundamentally, however, they are just di erent aspects o one orce – the electromagnetic interaction. The nature o which field is observed depends on the observer. For example:

a) A charge moving at right angles to a magnetic field.

An observer in a rame o re erence that is **at rest with respect to the magnetic field** will explain the orce acting on the charge (and its acceleration) in terms o a magnetic orce ($F_M = Bqv$) that acts on the moving charge.

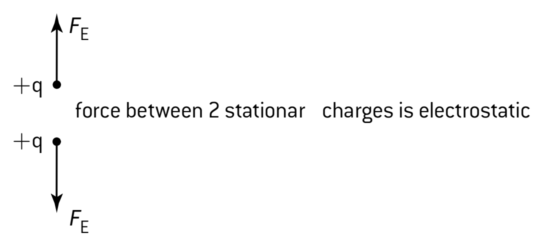


An observer in a rame o re erence that is **at rest with respect to the charge** will explain the initial orce acting on the charge (and its initial acceleration) in terms o an induced electric orce that results rom the cutting o magnetic flux.

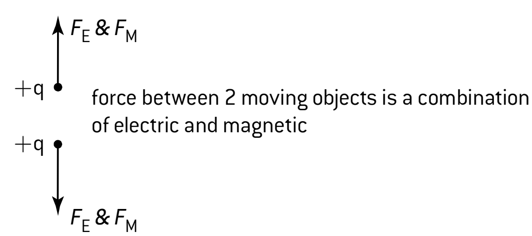


b) Two identically charged particles moving with parallel velocities according to a laboratory rame o re erence.

An observer in a rame o re erence that is **moving with the charged particles** will see the particles at rest. Thus this observer sees the orce o repulsion between the two charges as solely electrostatic in nature.



An observer in a rame o re erence where **the laboratory is at rest** will see the total orce between the two charges as a combination o electrostatic and magnetic. Moving charges are currents and thus each moving charge creates its own magnetic field which is stationary in the laboratory rame. Each charge is moving in the other's stationary magnetic field and will experience a magnetic orce.



Special relativit

POSTULATES OF SPECIAL RELATIVITY

The special theory of relativity is based on two fundamental assumptions or **postulates**. If either of these postulates could be shown to be wrong, then the theory of relativity would be wrong. When discussing relativity we need to be even more than usually precise with our use of technical terms.

One important technical phrase is an **inertial frame of reference**. This means a frame of reference in which the laws of inertia (Newton's laws) apply. Newton's laws do not apply in accelerating frames of reference so an inertial frame is a frame that is either stationary or moving with constant velocity.

An important idea to grasp is that there is no fundamental difference between being stationary and moving at constant velocity. Newton's laws link forces and accelerations. If there is no resultant force on an object then its acceleration will be zero. This could mean that the object is at **rest** or it could mean that the object is **moving at constant velocity**.

The two postulates of special relativity are:

- the speed of light in a vacuum is the same constant for all inertial observers
- the laws of physics are the same for all inertial observers.

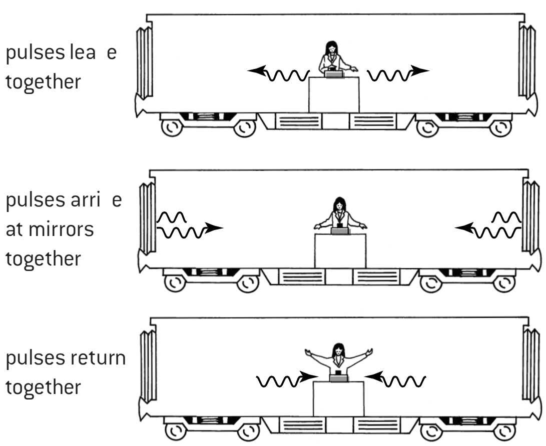
The first postulate leads on from Maxwell's equations and can be experimentally verified. The second postulate seems completely reasonable – particularly since Newton's laws do not differentiate between being at rest and moving at constant velocity. If both are accepted as being true then we need to start thinking about space and time in a completely different way. If in doubt, we need to return to these two postulates.

SIMULTANEITY

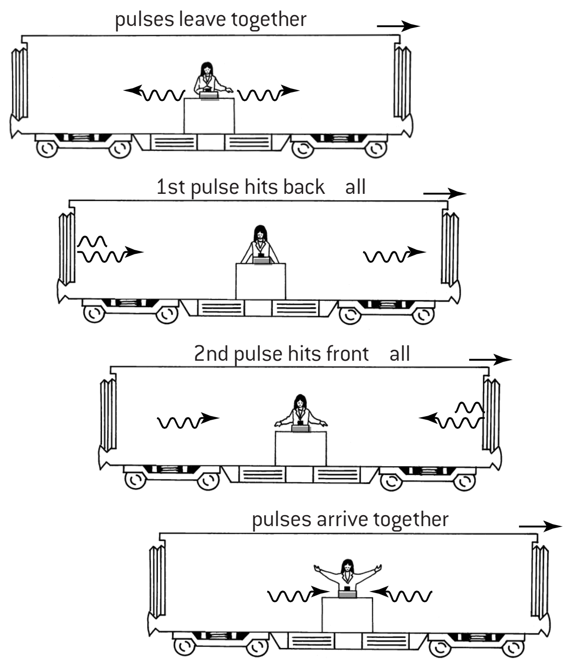
One example of how the postulates of relativity disrupt our everyday understanding of the world around us is the concept of simultaneity. If two events happen together we say that they are simultaneous. We would normally expect that if two events are **simultaneous** to one observer, they should be simultaneous to all observers – but this is not the case! A simple way to demonstrate this is to consider an experimenter in a train.

The experimenter is positioned **exactly** in the middle of a carriage that is moving at constant velocity. She sends out two pulses of light towards the ends of the train. Mounted at the ends are mirrors that reflect the pulses back towards the observer. As far as the experimenter is concerned, the whole carriage is at rest. Since she is in the middle, the experimenter will know that:

- the pulses were sent out simultaneously
- the pulses hit the mirrors simultaneously
- the pulses returned simultaneously.



The situation will seem very different if watched by a stationary observer (on the platform). This observer knows that light must travel at constant speed – both beams are travelling at the same speed as far as he is concerned, so they must hit the mirrors at different times. The left-hand end of the carriage is moving towards the beam and the right hand end is moving away. This means that the reflection will happen on the left-hand end first.



Interestingly, the observer on the platform does see the beams arriving back at the same time. The observer on the platform will know that:

- the pulses were sent out simultaneously
- the left-hand pulse hit the mirror before the right-hand pulse
- the pulses returned simultaneously.

In general, simultaneous events that take place at the same point in space will be simultaneous to all observers whereas events that take place at different points in space can be simultaneous to one observer but not simultaneous to another!

Do not dismiss these ideas because the experiment seems too fanciful to be tried out. The use of a pulse of light allowed us to rely on the first postulate. This conclusion is valid whatever event is considered.

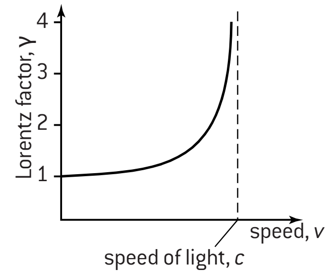
Lorentz transformations

LORENTZ FACTOR

The ormulae or special relativity all involve a actor that depends on the relative velocity between di erent observers, v.

We define the Lorentz actor, γ as ollows:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

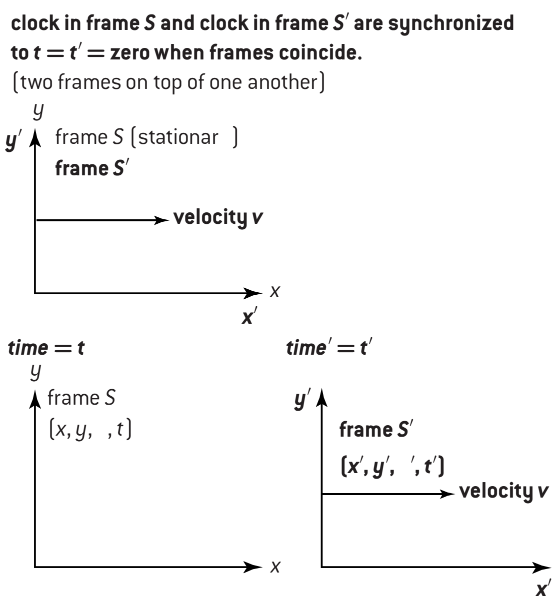


At low velocities, the Lorentz actor is approximately equal to one – relativistic e ects are negligible. It approaches infinity near the speed o light.

LORENTZ TRANSFORMATIONS

An observer defines a rame o re erence and di erent **events** can be characterized by di erent coordinates according to the observer's measurements o space and time. In a rame S, an event will be associated with a given position (x, y and z coordinates) and take place at a given time (t). Observers in relative uni orm motion disagree on the numerical values or these coordinates.

The Galilean trans ormations equations (page 131) allowed us to calculate what an observer in a second rame will record i we know the values in one rame but assume that the measurement o time is the same in both rames. Einstein has shown that this is not correct.



Because the rames were synchronized, the observers agree on the measurements o y and z. To switch between the other measurements made by di erent observers we need to use the Lorentz trans ormations. These all involve the Lorentz actor, γ, as defined above. The derivation o these equations is not required.

$$x' = \gamma(x - vt); \quad \Delta x' = \gamma\,(\Delta x - v\Delta t);$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right); \quad \Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

The reverse trans ormations also apply. These are just a consequence o the relative velocity o rame S (with respect to rame S') being in the opposite direction.

$$x = \gamma(x' + vt'); \quad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

LORENTZ TRANSFORMATION EXAMPLE

We can apply the Lorentz trans ormation equations to the situation shown on page 133. Suppose the experiment on the train measures the carriage to be 50.0 m long and the observer on the plat orm measures the speed o the train to be 2.7×10^8 m s^{-1} (0.90 c) to the right. In this situation, we know the times (t) and locations (x) are measured according to the experimenter on the train (rame S) and the experimenter on the plat orm is rame S'.

1. According to the experimenter on the train (rame S),

   Time taken or each pulse to reach mirror at end o carriage is given by:

   $$\Delta t = \frac{25.0}{3.0 \times 10^8} = 8.33 \times 10^{-8}\ \text{s}$$

   Total time taken or each pulse to complete the round journey to the experimenter is:

   $$\Delta t_{\text{total}} = \frac{50.0}{(3.0 \times 10^8)} = 1.67 \times 10^{-7}\ \text{s}$$

2. According to the experimenter on the plat orm (rame S'),

   $$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(0.9c)^2}{c^2}}}$$
   $$= \frac{1}{\sqrt{1 - 0.81}} = \frac{1}{\sqrt{0.19}} = 2.29$$

   Time taken or LH pulse to reach mirror at end o carriage is given by:

   $$\Delta t'_{(LH\ pulse)} = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

   where $\Delta t = 8.33 \times 10^{-8}$ s, $v = -2.7 \times 10^8$ m s^{-1}(relative velocity o plat orm is moving to the le t) and $\Delta x = -25.0$ m (pulse moving to le t)

   $$\Delta t'_{(LH\ pulse)} = 2.29\left(8.33 \times 10^{-8} - \frac{(-2.7 \times 10^8) \times (-25.0)}{(3.0 \times 10^8)^2}\right)$$
   $$= 1.91 \times 10^{-7} - 1.72 \times 10^{-7}$$
   $$= 1.9 \times 10^{-8}\ \text{s}$$

   Time taken or RH pulse to reach mirror at end o carriage is given by:

   $$\Delta t'_{(RH\ pulse)} = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$
   $$= 2.29\left(8.33 \times 10^{-8} - \frac{(-2.7 \times 10^8) \times 25.0}{(3.0 \times 10^8)^2}\right)$$
   $$= 1.91 \times 10^{-7} + 1.72 \times 10^{-7} = 3.63 \times 10^{-7}\ \text{s}$$

   Note that the time taken by each pulse is different they do not arrive simultaneously according to the experimenter on the platform.

   The return time or the LH pulse is the same as the time taken or the RH to initially reach the mirror (in each case, $\Delta x = 25.0$ m and $\Delta t = 8.33 \times 10^{-8}$ s)

   So total time taken or LH pulse to return to centre o carriage is

   $$\textit{total time}'_{(LH\ pulse)} = 1.9 \times 10^{-8} + 3.63 \times 10^{-7} = 3.82 \times 10^{-7}\ \text{s}$$

   This is the same as the total time taken or the RH pulse so both experimenters observe the return o the pulses to be simultaneous.

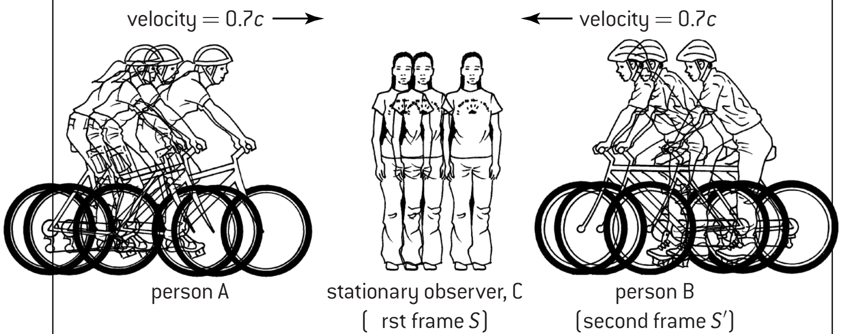
   Check: The above calculates that or rame S', the total time taken or the round trip is 3.82×10^{-7} s. The Lorentz trans ormation, can also be applied to the pulse's journey. In this situation, Δx (in rame S) = 0 as the pulse returns to its starting position.

   $$\text{total } \Delta t'_{(either\ pulse)} = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) = \gamma\,\Delta t$$
   $$= 2.29 \times 1.67 \times 10^{-7} = 3.82 \times 10^{-7}\ \text{s}$$

Velocit addition

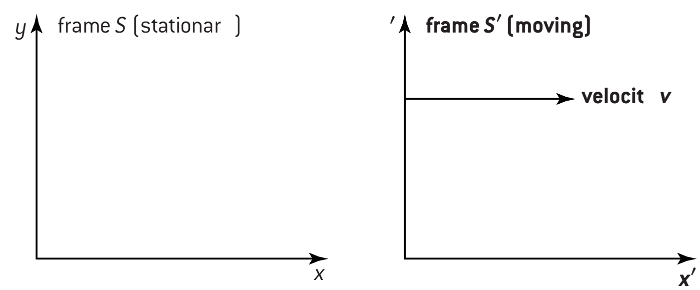
VELOCITY ADDITION

When two observers measure each other's velocity, they will always agree on the value. The calculation o relative velocity is not, however, normally straight orward. For example, an observer might see two objects approaching one another, as shown below.



I each object has a relative velocity o 0.7 c, the Galilean trans ormations would predict that the relative velocity between the two objects would be 1.4 c. This cannot be the case as the Lorentz actor can only be worked out or objects travelling at less than the speed o light.

The situation considered is one rame moving relative to another rame at velocity v.



Application o the Lorentz trans ormation gives the equation used to move between rames:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

u' – the velocity under consideration in the x-direction as measured in the second rame, S'

u – the velocity under consideration in the x-direction as measured in the first rame, S

v – the velocity o the second rame, S', as measured in the first rame, S

In each o these cases, a positive velocity means motion along the positive x-direction. I something is moving in the negative x-direction then a negative velocity should be substituted into the equation.

Example

In the example above, two objects approached each other with 70% o the speed o light. So u' is person A's velocity as measured in person B's rame o re erence.

u' = relative velocity o approach – to be calculated

$u = 0.7\ c$

$v = -0.7\ c$

$u' = \frac{1.4\ c}{(1 + 0.49)}$ *note the sign in the brackets*

$= \frac{1.4\ c}{1.49}$

$= 0.94$ c

COMPARISON WITH GALILEAN EQUATION

The top line o the relativistic addition o velocities equation can be compared with the Galilean equation or the calculation o relative velocities.

$u' = u - v$

At low values o v these two equations give the same value. The Galilean equation only starts to ail at high velocities.

At high velocities, the Galilean equation can give answers o greater than c, while the relativistic one always gives a relative velocity that is less than the speed o light.

Invariant quantities

SPACETIME INTERVAL

Relativity has shown that our Newtonian ideas of space and time are incorrect. Two inertial observers will generally disagree on their measurements of space and time but they will agree on a measurement of the speed of light. Is there anything else upon which they will agree?

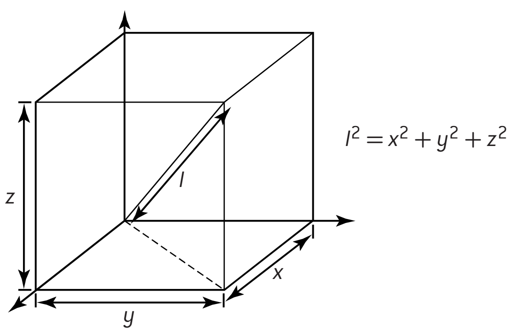
In relativity, a good way of imagining what is going on is to consider everything as different ‘events’ in something called **spacetime**. From one observer’s point of view, three co-ordinates (*x, y* and *z*) can define a position in space. One further ‘coordinate’ is required to define its position in time (*t*). An event is a given point specified by these four coordinates (*x, y, z, t*).

As a result of the Lorentz transformation, another observer would be expected to come up with totally different numbers for all of these four measurements – (x', y', z', t'). The amazing thing is that these two observers will agree on something. This is best stated mathematically:

$$(ct)^2 - x^2 - y^2 - z^2 = (ct')^2 - x'^2 - y'^2 - z^2$$

On normal axes, Pythagoras’s theorem shows us that the quantity $\sqrt{(x^2 + y^2 + z^2)}$ is equal to the length of the line from the origin, so $(x^2 + y^2 + z^2)$ is equal to (the length of the line)^2. In other words, it is the separation in space.

$$(\text{Separation in space})^2 = (x^2 + y^2 + z^2)$$



The two observers agree about something very similar to this, but it includes a coordinate of time. This can be thought of as the separation in imaginary four-dimensional spacetime.

$$(\text{Separation in spacetime})^2 = (ct)^2 - x^2 - y^2 - z^2$$

or

$$(\text{Separation in spacetime})^2 = (\text{time separation})^2 - (\text{space separation})^2$$

In 1 dimension, this is simplified to

$$(ct')^2 - (x')^2 = (ct)^2 - (x)^2$$

OTHER INVARIANT QUANTITIES

In addition to the spacetime interval between two events (see box above), all observers agree on the values of three other quantities associated with the separation between two events or with reference to a given object. These are:

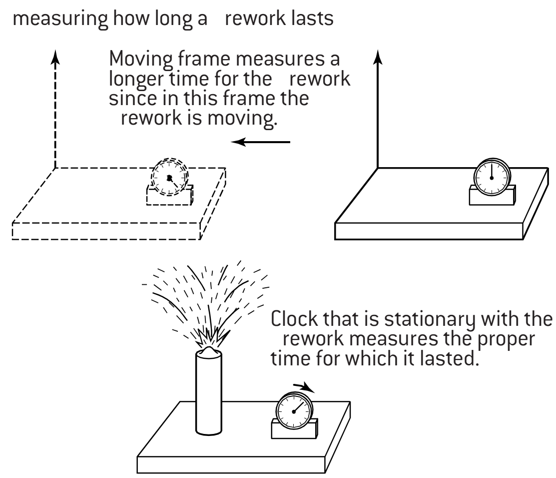
- Proper time interval Δt_0
- Proper length L_0
- Rest mass m_0

These four quantities are said to be **invariant** as they are always constant and do not vary with a change of observer. There are additional quantities, not associated with mechanics, that are also invariant e.g. electric charge.

PROPER TIME, PROPER LENGTH & REST MASS

a) Proper time interval Δt_0

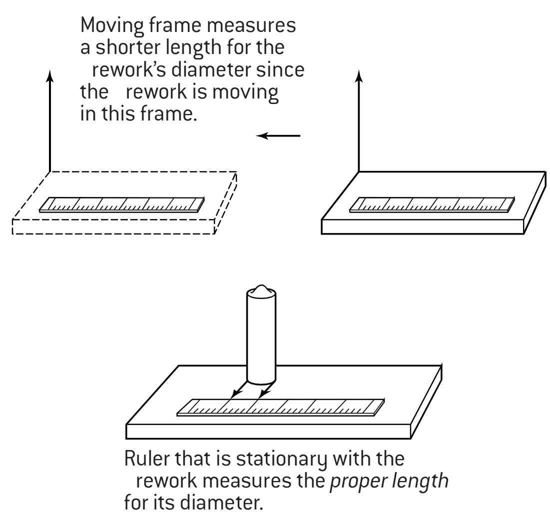
When expressing the time taken between events (for example the length of time that a firework is giving out light), the **proper time** is the time as measured in a frame where the events take place at the same point in space. It turns out to be the shortest possible time that any observer could correctly record for the event.



If A is moving past B then B will think that time is running slowly for A. From A’s point of view, B is moving past A. This means that A will think that time is running slowly for B. Both views are correct!

b) Proper length L_0

As before, different observers will come up with different measurements for the length of the same object depending on their relative motions. The **proper length** of an object is the length recorded in a frame where the object is at rest.



c) Rest mass m_0

The measurement of mass depends on relative velocity. Once again it is important to distinguish the measurement taken in the frame of the object from all other possible frames. The **rest mass** of an object is its mass as measured in a frame where the object is at rest. A particle’s rest mass does not change.

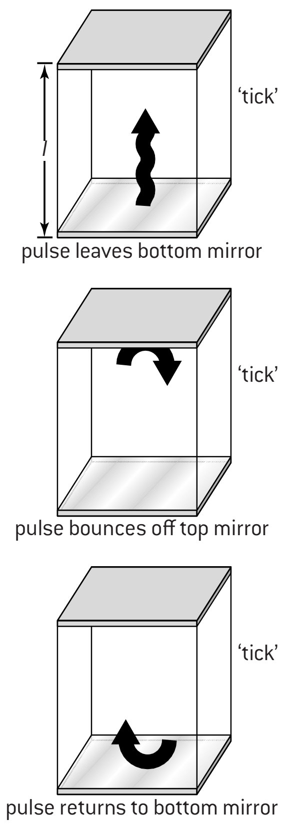
Time dilatio

LIGHT CLOCK

A **light clock** is an imaginary device. A beam o light bounces between two mirrors – the time taken by the light between bounces is one 'tick' o the light clock.

As shown in the derivation the path taken by light in a light clock that is moving at constant velocity is longer. We know that the speed o light is fixed so the time between the 'ticks' on a moving clock must also be longer. This e ect – that moving clocks run slow – is called **time dilation**.

The time between bounces Δt_0 is the proper time or this clock in the rame where the clock is at rest.



DERIVATION OF EFFECT FROM LORENTZ TRANSFORMATION

I rame S is a rame where two events take place at the same point in space, then the time interval between these two events must be the proper time interval, Δt_0.

Time dilation is then a direct consequence o the Lorentz trans ormation:

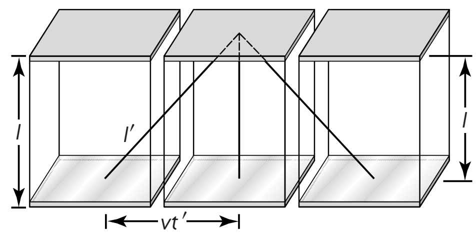
$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta}{c^2}\right)$$

Where $\Delta t = \Delta t_0$, (the proper time interval) and Δ = zero (same point in space)

time interval in rame S', $\Delta t' = \gamma \Delta t_0$

DERIVATION OF THE EFFECT FROM FIRST PRINCIPLES

I we imagine a stationary observer with one light clock then t is the time between 'ticks' on their stationary clock. In **this stationary frame**, a moving clock runs **slowly** and t' is the time between 'ticks' on the moving clock: t' is greater than t.



In the time t',

the clock has moved on a distance $= vt'$

Distance travelled by the light, $l' = \sqrt{((vt')^2 + l^2)}$

$$t' = \frac{l'}{c}$$

$$= \frac{\sqrt{(vt')^2 + l^2}}{c}$$

$$t'^2 = \frac{v^2 t'^2 + l^2}{c^2}$$

$$t'^2\left(1 - \frac{v^2}{c^2}\right) = \frac{l^2}{c^2}$$

but $$\frac{l^2}{c^2} = t^2$$

$$t'^2\left(1 - \frac{v^2}{c^2}\right) = t^2$$

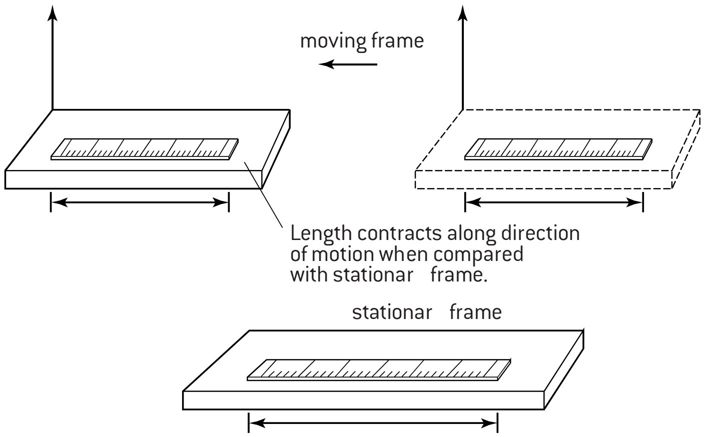
or $$t' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \times t \quad \text{or} \quad t' = \gamma t$$

This equation is true or all measurements o time, whether they have been made using a light clock or not.

Length contraction and evidence to support special relativity

EFFECT OF LENGTH CONTRACTION

Time is not the only measurement that is a ected by relative motion. There is another relativistic e ect called **length contraction**. According to a (stationary) observer, the separation between two points in space contracts i there is relative motion in that direction. The contraction is in the same direction as the relative motion.



Length contracts by the same proportion as time dilates – the Lorentz actor is once again used in the equation, but this time there is a division rather than a multiplication.

$$L = \frac{L_0}{\gamma}$$

EXAMPLE

An unstable particle has a li etime o 4.0×10^{-8} s in its own rest rame. I it is moving at 98% o the speed o light calculate:

a) Its li etime in the laboratory rame.

b) The length travelled in both rames.

a) $\gamma = \sqrt{\frac{1}{1 - (0.98)^2}}$

$= 5.025$

$\Delta t = \gamma \Delta t_0$

$= 5.025 \times 4.0 \times 10^{-8}$

$= 2.01 \times 10^{-7}$ s

b) In the laboratory rame, the particle moves

Length = speed × time

$= 0.98 \times 3 \times 10^8 \times 2.01 \times 10^{-7}$

$= 59.1$ m

In the particle's rame, the laboratory moves

$\Delta l = \frac{59.1}{\gamma}$

$= 11.8$ m

(alternatively: length = speed × time

$= 0.98 \times 3 \times 10^8 \times 4.0 \times 10^{-8}$

$= 11.8$ m)

DERIVATION OF LENGTH CONTRACTION FROM LORENTZ TRANSFORMATION

When we measure the length o a moving object, then we are recording the position o each end o the object at one given instant o time according to that rame o re erence. In other words the time interval measured in rame S between these two events will be zero, $\Delta t = 0$. In this case, the length measured Δx is the length o the moving object L_0.

Length contraction is then a direct consequence o the Lorentz trans ormation, as, i we move into the rame, S', where the object is at rest, we will be measuring the proper length L_0:

$$\Delta x' = \gamma(\Delta x - v\Delta t)$$

Where $\Delta x' = L_0$ (the proper length) and

$\Delta t =$ zero (simultaneous measurements o position o end o object)

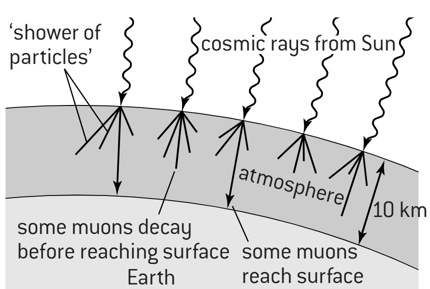
Length in rame S', $L_0 = \gamma(L)$

$$L = \frac{L_0}{\gamma}$$

THE MUON EXPERIMENT

Muons are leptons (see page 78) – they can be thought o as a more massive version o an electron. They can be created in the laboratory but they quickly decay. Their average li etime is 2.2×10^{-6} s as measured in the rame in which the muons are at rest.

Muons are also created high up (10 km above the sur ace) in the atmosphere. Cosmic rays rom the Sun can cause them to be created with huge velocities – perhaps 0.99 c. As they travel towards the Earth some o them decay but there is still a detectable number o muons arriving at the sur ace o the Earth.



Without relativity, no muons would be expected to reach the sur ace at all. A particle with a li etime o 2.2×10^{-6} s which is travelling near the speed o light (3×10^8 m s^{-1}) would be expected to travel less than a kilometre be ore decaying ($2.2 \times 10^{-6} \times 3 \times 10^8 = 660$ m).

The moving muons are e ectively moving 'clocks'. Their high speed means that the Lorentz actor is high.

$$\gamma = \sqrt{\frac{1}{1 - 0.99^2}} = 7.1$$

There ore an average li etime o 2.2×10^{-6} s in the muons' rame o re erence will be time dilated to a longer time as ar as a stationary observer on the Earth is concerned. From this rame o re erence they will last, on average, 7.1 times longer. Many muons will still decay but some will make it through to the sur ace – this is exactly what is observed.

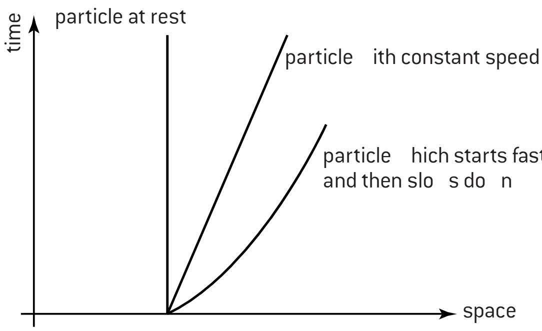
In the muons' rame they exist or 2.2×10^{-6} s on average. They make it down to the sur ace because the atmosphere (and the Earth) is moving with respect to the muons. This means that the atmosphere will be length-contracted. The 10 km distance as measured by an observer on the Earth will only be $\frac{10}{7.1} = 1.4$ km. A significant number o muons will exist long enough or the Earth to travel this distance.

Spacetime diagrams (Minkowski diagrams) 1

SPACETIME DIAGRAMS

Spacetime separation was introduced on page 136. A spacetime diagram is a visual way of representing the geometry. Measurements can be taken from the diagram to calculate actual values.

We cannot represent all four dimensions on the one diagram, so we usually limit the number of dimensions of space that we represent. The simplest representation has only one dimension of space and one of time as shown below.



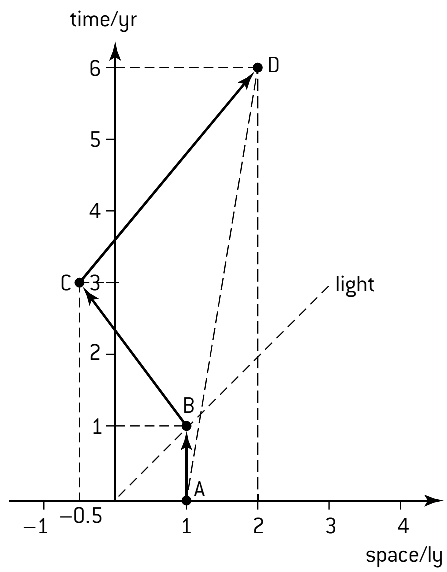
An object (moving or stationary) is always represented as a line in spacetime.

Note that:

- The values on the spacetime diagram are as would be measured by an observer whose worldline is represented by the vertical axis.
- The vertical axis in the above spacetime diagram is time *t*. An alternative is to plot (speed of light × time), *ct*. This means that both axes can have the same units (m, light-years or equivalent).
- Whatever axes are being used, by convention, the path of a beam of light is represented by a line at 45° to the axes.
- The advance of proper time for any traveller can be calculated from the overall separation in spacetime. In the traveller s frame of reference, they remained stationary so the separation between two events can be calculated as shown below.

EXAMPLE 1 OF SPACETIME DIAGRAMS

The advance of proper time for the journey between the events A→B→C→D can be calculated from the values on the spacetime diagram.



A journey through spacetime

Journey	Space separation (x)/ly	Time separation (t)/yr	(Spacetime separation)^2 $(ct)^2 - (x)^2$/ly^2	Advance of proper time according to traveller / yr $t' = \frac{\sqrt{(ct)^2 - (x)^2}}{c}$
A→B	0.0	1.0	$1^2 - 0^2 = 1$	$\sqrt{1.00} = 1.00$
B→C	1.5	2.0	4 – 2.25 = 1.75	$\sqrt{1.75} = 1.32$
C→D	2.5	3.0	9 − 6.25 = 2.75	$\sqrt{2.75} = 1.66$

The total advance of proper time for the traveller is 1.00 + 1.32 + 1.66 = 3.98 yr. This compares with the advance of 6.0 years according to an observer whose worldline is a vertical line on this spacetime diagram. This difference is an example of **time dilation** (see page 137).

The alternative journey direct from A → D shows a greater elapsed proper time.

Journey	Space separation (x)/ly	Time separation (t)/yr	(Spacetime separation)^2 $(ct)^2 - (x)^2$/ly^2	Advance of proper time according to traveller / yr $t' = \frac{\sqrt{(ct)^2 - (x)^2}}{c}$
A→D	1.0	6.0	36 − 1 = 35	$\sqrt{35} = 5.92$

This is always true. A direct worldline always has a greater amount of elapsed proper time than an indirect worldline.

Spacetime diagrams 2

CALCULATION OF TIME DILATION AND LENGTH CONTRACTION

Time dilation and **length contraction** are quantitatively represented on spacetime diagrams. Refer to diagram on page 139.

a) **Time dilation**: In the journey direct from B → C, the relative velocity between the traveller and the stationary observer is $\frac{1.5 \text{ ly}}{2.0 \text{ yrs}} = 0.75$ c. The Lorentz gamma factor is:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.75^2}} = 1.51$$

The journey takes 2 yrs according to the observer at rest. This means the proper time as measured by the traveller will be:

$$\Delta t = \gamma \Delta t_0 \Rightarrow \Delta t_0 = \frac{\Delta t}{\gamma} = \frac{2.0}{1.51} = 1.32 \text{ yr}$$

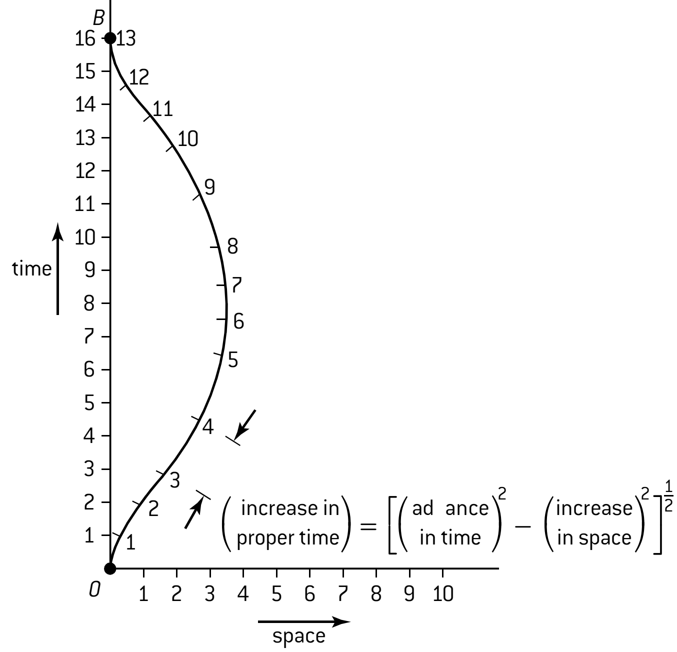
as shown in the table on page 139.

b) **Length contraction**: The observer at rest measures the journey length from B→C to be 1.5 ly. The journey will be length contracted to be

$$L = \frac{L_0}{\gamma} = \frac{1.5}{1.51} = 0.99 \text{ ly}$$

The relative velocity of travel is 0.75 c, and the time taken to go from from B → C, in the traveller s frame of reference, is 1.32 yr. This makes the distance according to the traveller to be 0.75 $c \times$ 1.32 yr = 0.99 ly as shown above.

EXAMPLE 2 – CURVED WORLDLINE

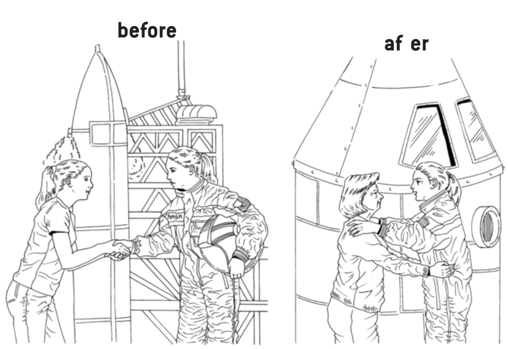


Proper time along a curved worldline from event O to event B is smaller than the proper time along the straight line from O to B.

The t in paradox 1

As mentioned on page 136, the theory of relativity gives no preference to different inertial observers – the time dilation effect (moving clocks run slowly) is always the same. This leads to the '**twin paradox** . In this imaginary situation, two identical twins compare their views of time. One twin remains on Earth while the other twin undergoes a very fast trip out to a distant star and back again.

As far as the twin on the Earth is concerned the other twin is a moving observer. This means that the twin that remains on the Earth will think that time has been running slowly for the other twin. When they meet up again, the returning twin should have aged less.



This seems a very strange prediction, but it is correct according to the time dilation formula. Remember that:

- This is a relativistic effect – time is running at different rates because of the **relative velocity** between the two twins and **not** because of the **distance** between them.
- The difference in ageing is relative. Neither twin is getting younger; as far as both of them are concerned, time has been passing at the normal rate. It s just that the moving twin thinks that she has been away for a shorter time than the time as recorded by the twin on the Earth.

The paradox is that, according to the twin who made the journey, the twin on the Earth was moving all the time and so the twin left on the Earth should have aged less. Whose version of time is correct?

The solution to the paradox comes from the realization that the equations of special relativity are only symmetrical when the two observers are in constant relative motion. For the twins to meet back up again, one of them would have to turn around. This would involve external forces and acceleration. If this is the case then the situation is no longer symmetrical for the twins. The twin on the Earth has not accelerated so her view of the situation must be correct.

The resolution of the twin paradox using a spacetime diagram is on page 141.

T in paradox 2

RESOLVING THE TWIN PARADOX USING SPACETIME DIAGRAMS

The diagram below is a spacetime diagram or a journey to a distant planet ollowed by an immediate return.

According to the twin remaining on Earth:

- the distance to the planet = 3.0 ly
- relative velocity o traveller is 0.6 c
- each leg o the journey takes $\frac{3.0}{0.6} = 5.0$ yr
- Total journey time = 10.0 yr

The gamma actor is

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.6^2}} = 1.25$$

So according to the twin undertaking the journey:

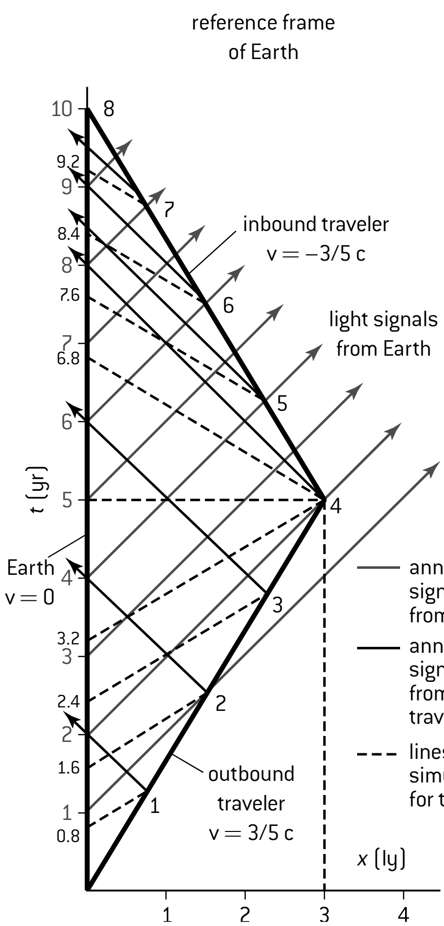
- each leg o the journey takes $\frac{5.0}{1.25} = 4.0$ yr
- Total journey time = 8.0 yr
- the distance to the planet = $\frac{3.0}{1.25} = 2.4$ ly
- relative velocity o Earth = $\frac{4.8}{8.0} = 0.6\ c$

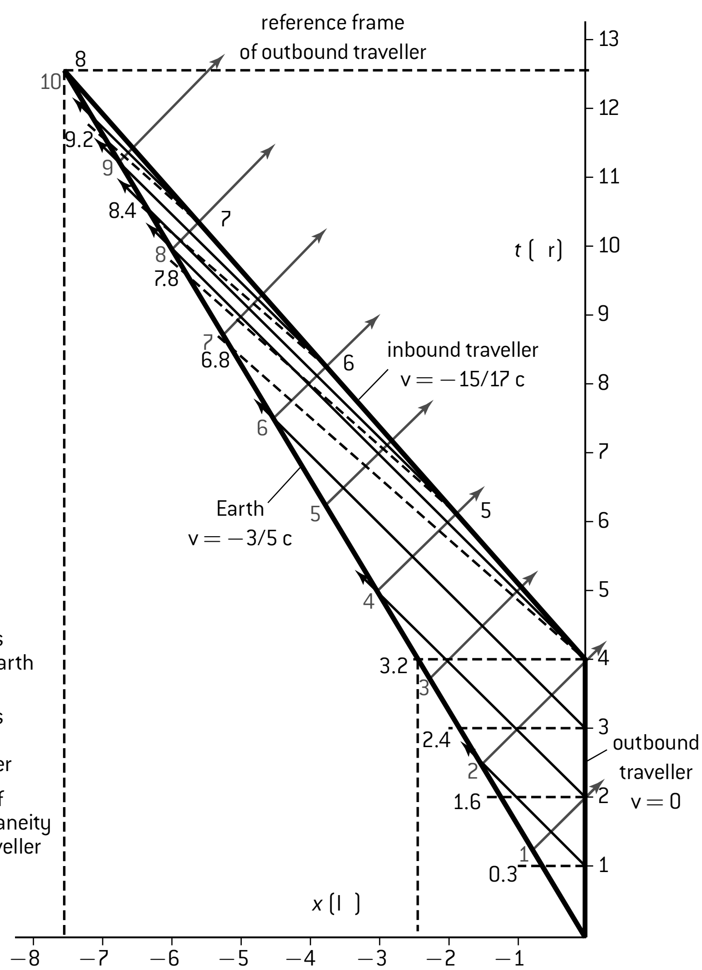
In order to check whose version o time 'is correct', **they agree to send light signals every year**. The spacetime diagram or this situation in the Earth's rame o re erence is shown below (le t).

Note that there is no paradox; they agree on the number o signals sent and received; the travelling twin has aged less than the twin that stayed on Earth.

A more complicated spacetime diagram can be drawn or the re erence rame o the outbound traveller (below right). Note that:

- The first our years has the travelling twin's worldline vertical i.e. stationary.
- When the travelling twin turns round, she leaves her original rame o re erence and changes to a rame where the Earth is moving towards her at $\frac{3}{5}\ c$ (= 0.6 c).
- Her relative velocity towards the Earth with respect to her original rame o re erence can be calculated rom the velocity trans ormation equations as $\frac{15}{17}\ c$ (= 0.88 c) back.
- In this rame o re erence, the total time or the round trip would be measured as 12.5 yr





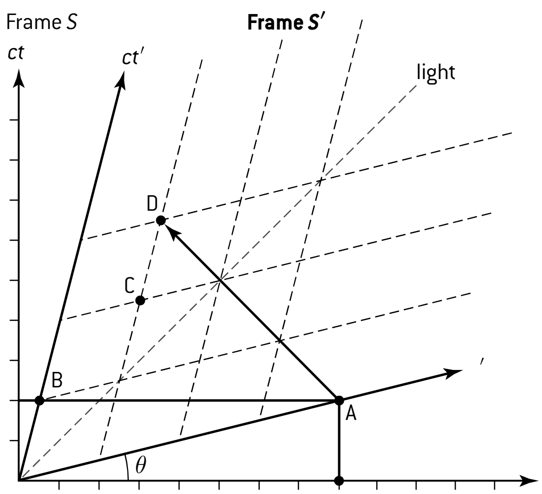
Spacetime diagrams 3

REPRESENTING MORE THAN ONE INERTIAL FRAME ON THE SAME SPACETIME DIAGRAM

The Lorentz transformations describe how measurements of space and time in one frame can be converted into the measurements observed in another frame of reference. The situation in each frame of reference can be visualized by using separate spacetime diagrams for each frame of reference (see page 141 for examples).

It is also possible to represent two inertial frames on the same spacetime diagram. A frame S' (coordinates x' and ct') is moving at relative constant velocity $+v$ according to a frame S (coordinates x and ct). The principles are as follows:

- The same worldline applies to both sets of coordinate axes (that is, to x and ct, as well as to x' and ct').
- The Lorentz transformation is made by *changing the coordinate s stem for frame S'* rather than the position of the worldline.
- The spacetime axes for frame S has x and ct at right angles to one another as normal.
- The spacetime axes for frame S' has its x' and ct' axes both angled in towards the $x = ct$ line (which represents a path of a beam of light.
- The coordinates of a spacetime event in S are read from the x and ct axes directly.
- The coordinates of a spacetime event in S' are measured by drawing lines parallel to the ct' and x' axes until they hit the x' and ct' axes.



1. Events A & B are simultaneous in frame S but are *not* simultaneous in frame S' (A occurs before B)

   $\tan\theta = \frac{2}{8} = 0.25$

   relative velocity of frames S' and $S = 0.25\ c$
2. Events C & D occur at same location in frame S'.

   Events C & D occur at different locations in frame S.
3. A pulse of light emitted by event A arrives at event D according to both frames of reference. It cannot arrive at events B or C.

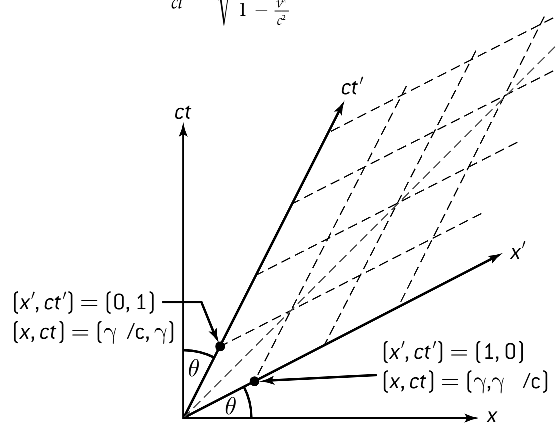
Mathematically for the above process to agree with the Lorentz transformation calculations, the following must apply:

- The angle between the ct' axis (the worldline for the origin of S') and the ct axis is the same as the angle between the x' axis and the x axis. It is:

  $$\theta = \tan^{-1}\left(\frac{v}{c}\right)$$

- The scales used by the axes in S' are different to the scales used by the axes in S.
- A given value is represented by a greater length on the ct' axis when compared with the ct axis.
- A given value is represented by a greater length on the x' axis when compared with the x axis.
- The ratio of the measurements on the axes depends on the relative velocity between the frames. The equation (which does not need to be recalled) is:

  $$\text{ratio of units } \frac{ct'}{ct} = \sqrt{\frac{1 + \frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}}}$$



Summar

- At greater speed:

  the S' axes swing towards the $x = ct$ line as the angle θ increases.

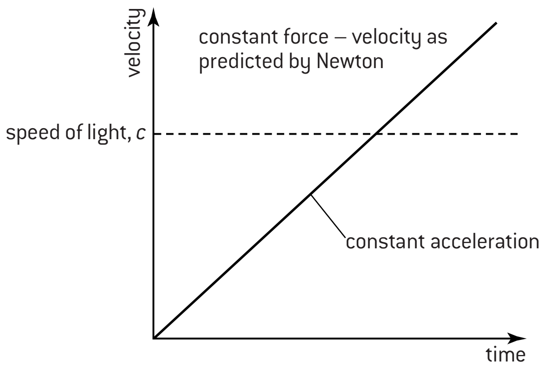
  the ct' and x' axes are more stretched when compared with the ct and x axes.
- Events that are simultaneous in S are on the same horizontal line.
- Events that are simultaneous in S' are on a line parallel to the x' axis.

HL Mass and energ

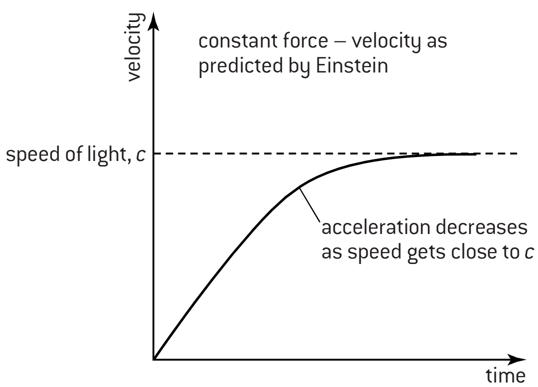
$E = MC^2$

The most famous equation in all of physics is surely Einstein's mass–energy relationship $E = mc^2$, but where does it come from? By now it should not be a surprise that if time and length need to be viewed in a different way, then so does energy.

According to Newton's laws, a constant force produces a constant acceleration. If this was always true then any velocity at all should be achievable – even faster than light. All we have to do is apply a constant force and wait.



In practice, this does not happen. As soon as the speed of an object starts to approach the speed of light, the acceleration gets less and less even if the force is constant.



The force is still doing work (= force × distance), therefore the object must still be gaining kinetic energy and a new relativistic equation is needed for energy:

$$E = \gamma m_0 c^2$$

Note that some textbooks compare this equation with the definition of rest energy ($E_0 = m_0 c^2$) in order to define a concept of relativistic mass that varies with speed ($m = \gamma m_0$). The current IB syllabus does not encourage this approach.

The preferred approach is to see rest mass as invariant and to adopt a new relativistic formula for kinetic energy:

Total energy = rest energy + kinetic energy = $\gamma m_0 c^2$

rest energy = $m_0 c^2$

so, kinetic energy $E_K = (\gamma - 1)m_0 c^2$

MASS AND ENERGY

Mass and energy are equivalent. This means that energy can be converted into mass and vice versa. Einstein's mass–energy equation can always be used, but one needs to be careful about how the numbers are substituted. Newtonian equations (such as $KE = \frac{1}{2}m\ ^2$ or momentum = m) will take different forms when relativity theory is applied.

The energy needed to create a particle at rest is called the rest energy E_0 and can be calculated from the rest mass:

$$E_0 = m_0 c^2$$

If this particle is given a velocity, it will have a greater total energy.

$$E = \gamma m_0 c^2$$

HL Relativistic momentum and energy

EQUATIONS

The laws o conservation o momentum and conservation o energy still apply in relativistic situations. However the concepts o ten have to be refined to take into account the new ways o viewing space and time.

For example, in Newtonian mechanics, momentum p is defined as the product o mass and velocity.

$$p = mv$$

In relativity it has a similar orm, but the Lorentz actor needs to be taken into consideration.

$$p = \gamma\, m_0\, v$$

The momentum o an object is related to its total energy. In relativistic mechanics, the relationship can be stated as

$$E^2 = p^2c^2 + m_0^{\ 2} c^4$$

In Newtonian mechanics, the relationship between energy and momentum is

$$E = \frac{p^2}{2m}$$

Do not be tempted to use the standard Newtonian equations – i the situation is relativistic, then you need to use the relativistic equations.

UNITS

SI units can be applied in these equations. Sometimes, however, it is use ul to use other units instead.

At the atomic scale, the joule is a huge unit. O ten the electronvolt (eV) is used. One electronvolt is the energy gained by one electron i it moves through a potential di erence o 1 volt. Since

$$\text{Potential di erence} = \frac{\text{energy di erence}}{\text{charge}}$$

$$1 \text{ eV} = 1 \text{ V} \times 1.6 \times 10^{-19} \text{ C}$$

$$= 1.6 \times 10^{-19} \text{ J}$$

In act the electronvolt is too small a unit, so the standard SI multiples are used

1 keV = 1000 eV

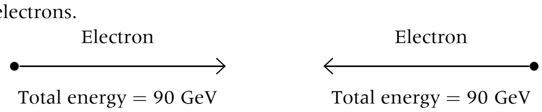
1 MeV = 10^6 eV   etc.

Since mass and energy are equivalent, it makes sense to have comparable units or mass. The equation that links the two ($E = mc^2$) defines a new unit or mass – the MeV c^{-2}. The speed o light is included in the unit so that no change o number is needed when switching between mass and energy – I a particle o mass o 5 MeV c^{-2} is converted completely into energy, the energy released would be 5 MeV. It would also be possible to use keV c^{-2} or GeV c^{-2} as a unit or mass.

In a similar way, the easiest unit or momentum is the MeV c^{-1}. This is the best unit to use i using the equation which links relativistic energy and momentum.

EXAMPLE

The Large Electron / Positron (LEP) collider at the European Centre or Nuclear Research (CERN) accelerates electrons to total energies o about 90 GeV. These electrons then collide with *positrons* moving in the opposite direction as shown below. Positrons are identical in rest mass to electrons but carry a positive charge. The positrons have the same energy as the electrons.



a) Use the equations o special relativity to calculate,

(i) the velocity o an electron (with respect to the laboratory);

Total energy = 90 GeV = 90000 MeV

Rest mass = 0.5 MeVc^{-2} $\therefore \gamma$ = 18000 (huge)

$\therefore \quad \simeq c$

(ii) the momentum o an electron (with respect to the laboratory).

$$p^2 c^2 = E^2 - m_0^{\ 2} c^4$$

$$\simeq E^2$$

$$p \simeq 90 \text{ GeV}c^{-1}$$

b) For these two particles, estimate their relative velocity o approach.

since γ so large

relative velocity $\simeq c$

c) What is the total momentum o the system (the two particles) be ore the collision?

zero

d) The collision causes new particles to be created.

(i) Estimate the maximum total rest mass possible or the new particles.

Total energy available = 180 GeV

$\therefore$ max total rest mass possible = 180 GeVc^{-2}

(ii) Give one reason why your answer is a *maximum*.

Above assumes that particles were created at rest

HL Relativistic mechanics examples

PARTICLE ACCELERATION AND ELECTRIC CHARGE

In a particle accelerator (e.g. a linear accelerator or cyclotron), charged particles are accelerated up to very high energies. The basic principle is to pass the charged particles through a series of potential differences and each time, the particle s total energy increases as a result. The increase in kinetic energy (ΔE_K) as a result of a charge q passing through a potential difference V is given by:

$$qV = \Delta E_K$$

EXAMPLE

An electron is accelerated through a pd of 1.0×10^6 V. Calculate its velocity.

$$\text{Energy gained} = 1.0 \times 10^6 \times 1.6 \times 10^{-19}\text{ J}$$
$$= 1.6 \times 10^{-13}\text{ J}$$
$$E_0 = m_0c^2 = 9.11 \times 10^{-31} \times (3 \times 10^8)^2$$
$$= 8.2 \times 10^{-14}\text{ J}$$
$$\text{Total energy} = 1.6 \times 10^{-13} + 8.2 \times 10^{-14}$$
$$= 2.42 \times 10^{-13}\text{ J}$$
$$\gamma = \frac{2.42 \times 10^{-13}}{8.2 \times 10^{-14}} = 2.95$$
$$\text{velocity} = \sqrt{1 - \frac{1}{\gamma^2}} \cdot c$$
$$= 0.94\ c$$

PHOTONS

Photons are particles that have a zero rest mass and travel at the speed of light, c. Their total energy and their frequency f is linked by Planck s constant h:

$$E = hf$$

The relativistic equation that links total energy, E and momentum, p, must also apply to photons:

$$E^2 = p^2c^2 + m_0^{\,2}c^4$$

The rest mass of a photon is zero so the momentum of a photon is:

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

EXAMPLE: DECAY OF A PION

A neutral pion (π^0) is a meson of rest mass $m_0 = 135.0$ MeV c^{-2}. A typical mode of decay is to convert into two photons:

$$\pi^0 \rightarrow 2\gamma$$

The wavelength of these photons can be calculated:

a) Decay at rest

If the pion was at rest when it decayed, each photon would have half the total energy of the pion:

$$E = 67.5\text{ MeV} = 67.5 \times 10^6 \times 1.6 \times 10^{-19}\text{ J}$$
$$= 1.08 \times 10^{-11}\text{ J}$$

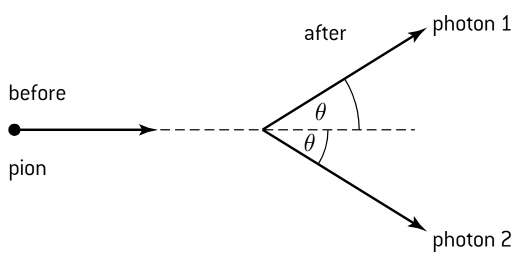
Planck s constant can be used to calculate the wavelength of one of the photons:

$$E = h\frac{c}{\lambda}$$
$$\lambda = h\frac{c}{E} = 6.63 \times 10^{-34} \times \frac{3.0 \times 10^8}{1.08 \times 10^{-11}}$$
$$= 1.84 \times 10^{-14}\text{ m}$$

The momentum of the pion was initially zero as it was at rest. Conservation of momentum means that the photons will be emitted in opposite directions. The total momentum of each photon add together to give a total, once again, of zero.

b) Decay while moving

Suppose the pion was moving forward when it decayed with a total energy 270.0 MeV c^{-2}; the photons will be emitted as shown below:



Note that in this example, total energy $2m_0c^2$, so $\gamma = 2$ so $= 0.866\ c$

Each photon will have a total energy of 135 MeV $= 2.16 \times 10^{-11}$ J

and a momentum of 135 MeV c^{-1}. The wavelengths of the photons will be:

$$\lambda = h\frac{c}{E} = 6.63 \times 10^{-34} \times \frac{3.0 \times 10^8}{2.16 \times 10^{-11}}$$
$$= 9.21 \times 10^{-15}\text{ m}$$

Initial total momentum for the pion in the forward direction can be calculated from

$$E^2 = p^2c^2 + m_0^{\,2}\,c^4$$
$$p^2\,c^2 = E^2 - m_0^{\,2}\,c^4 = (4 - 1)m_0^{\,2}\,c^4$$
$$p = \sqrt{3}\ m_0\ c = 1.73 \times 135.0 = 233.8\text{ MeV } c^{-1}$$

So conservation of momentum in forward direction is:

$$233.8 = 2 \times 135 \times \cos\theta$$
$$\cos\theta = \frac{233.8}{270} = 0.866$$
$$\theta = 30°$$

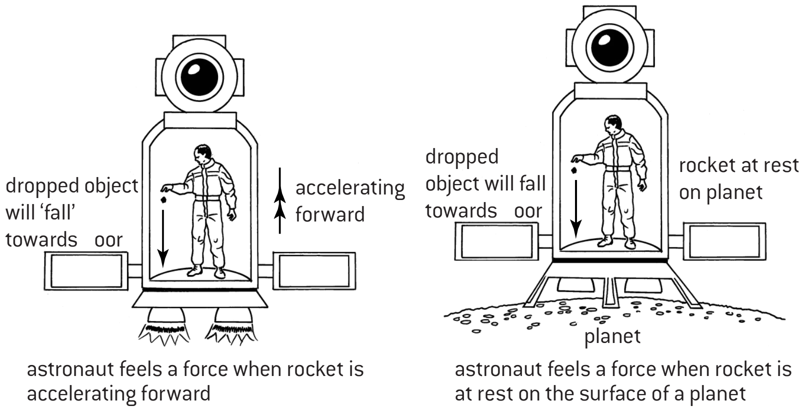
General relativity – the equivalence principle

PRINCIPLE OF EQUIVALENCE

One of Einstein's 'thought experiments' considers how an observer's view of the world would change if they were accelerating. The example below considers an observer inside a closed spaceship.

There are two possible situations to compare.

- The rocket could be far away from any planet but accelerating forwards.
- The rocket could be at rest on the surface of a planet.



Although these situations seem completely different, the observer **inside** the rocket would interpret these situations as being identical.

This is Einstein's 'principle of equivalence' – a postulate that states that there is no difference between an accelerating frame of reference and a gravitational field.

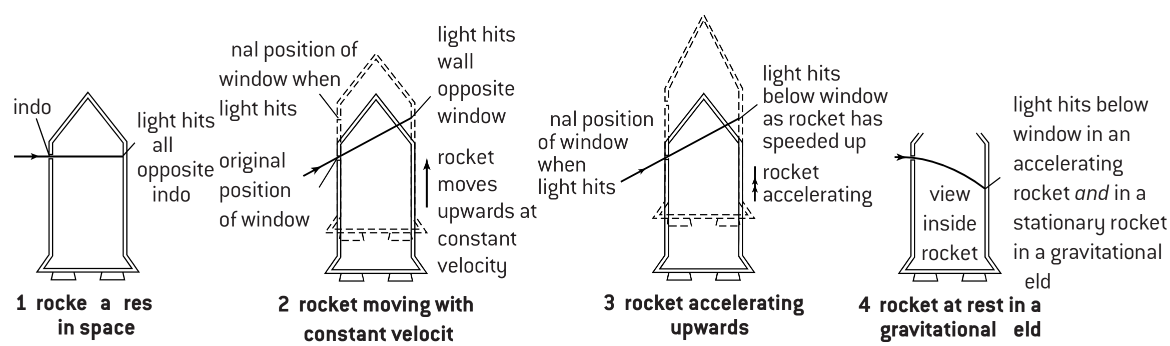
From the principle of equivalence, it can be deduced that light rays are bent in a gravitational field (see below) and that time slows down near a massive body (see page 147).

BENDING OF LIGHT

Einstein's principle of equivalence suggests that a gravitational field should bend light rays! There is a small window high up in the rocket that allows a beam of light to enter.

In both of the cases in diagrams 1 and 2, the observer is an **inertial** observer and would see the light shining on the wall at the point that is exactly opposite the small window. If, however, the rocket was accelerating upwards (see diagram 3) then the beam of light would hit a point on the wall **belo** the point that is opposite the small window.

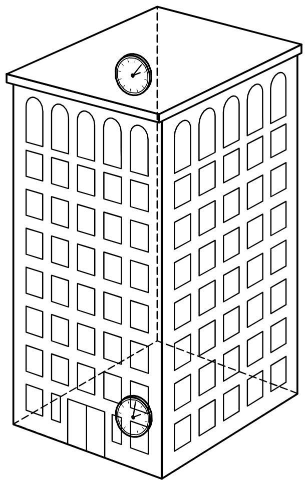
But Einstein's principle of equivalence states that there is no difference between an accelerating observer and inertial observer in a gravitational field. If this is true then light should follow a curved path in a gravitational field as shown in diagram 4. This effect does happen!



HL Gravitational red shift

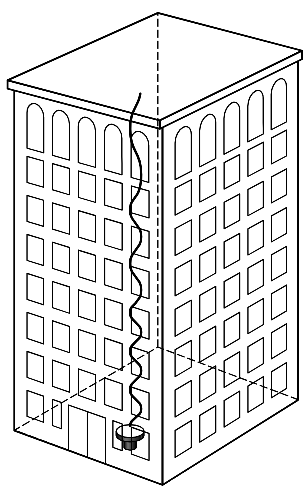
CONCEPT

The general theory o relativity makes other predictions that can be experimentally tested. One such e ect is **gravitational red shift** – clocks slow down in a gravitational field. In other words a clock on the ground floor o a building will run slowly when compared with a clock in the attic – the attic is urther away rom the centre o the Earth.



A clock on the ground oor runs slow when compared with a clock in the attic

The same e ect can be imagined in a di erent way. We have seen that a gravitational field a ects light. I light is shone away rom a mass (or example the Sun), the photons o light must be increasing their gravitational potential energy as they move away. This means that they must be decreasing their total energy. Since requency is a measure o the energy o a photon, the observed requency away rom the source must be less than the emitted requency.



At the top of the building, the photon has less energ , and so a lower frequenc , than when it was at the bottom.

The oscillations o the light can be imagined as the pulses o a clock. An observer at the top o the building would perceive the clock on the ground floor to be running slowly.

MATHEMATICS

This gravitational time dilation e ect can be mathematically worked out or a uni orm gravitational field g. The change in requency Δf is given by

$$\frac{\Delta f}{f} = \frac{g\Delta}{c^2}$$

where

- f is the requency emitted at the source
- g is the gravitational field strength (assumed to be constant)
- Δ is the height di erence and
- c is the speed o light.

EXAMPLE

A UFO travels at such a speed to remain above one point on the Earth at a height o 200 km above the Earth's sur ace. A radio signal o requency o 110 MHz is sent to the UFO.

(i) What is the requency received by the UFO?

(ii) I the signal was reflected back to Earth, what would be the observer requency o the return signal? Explain your answer.

(i) $f = 1.1 \times 10^8$ Hz

$g = 10$ m s^{-2}

$\Delta = 2.0 \times 10^5$m

$$\therefore \Delta f = \frac{10 \times 2.0 \times 10^5}{(3 \times 10^8)^2} \times 1.1 \times 10^8 \text{ Hz}$$

$= 2.4 \times 10^{-3}$ Hz

$\therefore f$ received $= 1.1 \times 10^8 - 2.4 \times 10^{-3}$

$= 109999999.998$ Hz

$\approx 1.1 \times 10^8$ Hz

(ii) The return signal will be gravitationally blue shi ted. There ore it will arrive back at **exactly** the same requency as emitted.

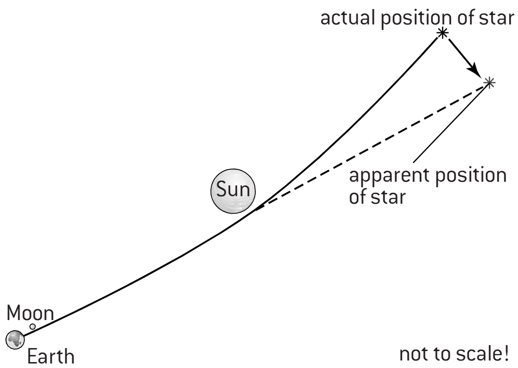
Supporting evidence

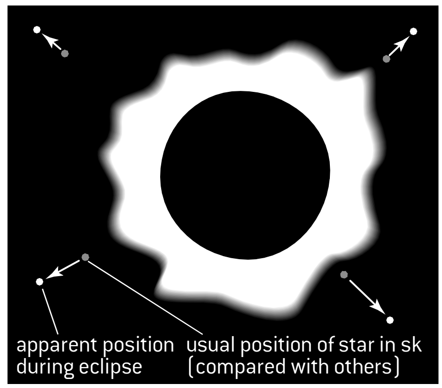
EVIDENCE TO SUPPORT GENERAL RELATIVITY

Bending of star light

The predictions of general relativity, just like those of special relativity, seem so strange that we need strong experimental evidence. One main prediction was the bending of light by a gravitational field. One of the first experiments to check this effect was done by a physicist called Arthur Eddington in 1919.

The idea behind the experiment was to measure the deflection of light (from a star) as a result of the Sun's mass. During the day, the stars are not visible because the Sun is so bright. During a solar eclipse, however, stars are visible during the few minutes when the Moon blocks all of the light from the Sun. If the positions of the stars during the total eclipse were compared with the positions of the same stars recorded at a different time, the stars that appeared near the edge of the Sun would appear to have moved.

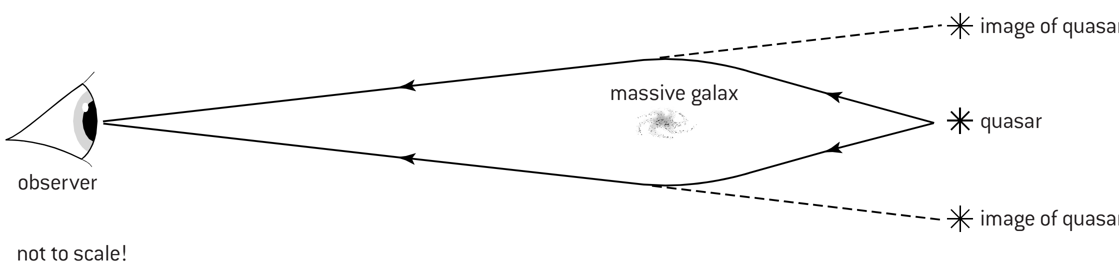




The angle of the shift of these stars turned out to be exactly the angle as predicted by Einstein's general theory of relativity.

Gravitational lensing

The bending of the path of light or the warping of spacetime (depending on which description you prefer) can also produce some very extreme effects. Massive galaxies can deflect the light from quasars (or other very distance sources of light) so that the rays bend around the galaxy as shown below.



In this strange situation, the galaxy is acting like a lens and we can observe multiple images of the distant quasar.

EVIDENCE TO SUPPORT GRAVITATIONAL RED SHIFT

Pound–Rebka–Snider experiment

The decrease in the frequency of a photon as it climbs out of a gravitational field can be measured in the laboratory. The measurements need to be very sensitive, but they have been successfully achieved on many occasions. One of the experiments to do this was done in 1960 and is called the **Pound Rebka** experiment. The frequencies of gamma-ray photons were measured after they ascended or descended Jefferson Physical Laboratory Tower at Harvard University.

The original **Pound Rebka** experiment was repeated with greater accuracy by Pound and Snider.

Atomic clock frequency shift

Because they are so sensitive, comparing the difference in time recorded by two identical atomic clocks can provide a direct measurement of gravitational red shift. One of the clocks is taken to high altitude by a rocket, whereas a second one remains on the ground. The clock that is at the higher altitude will run faster.

Global positioning system

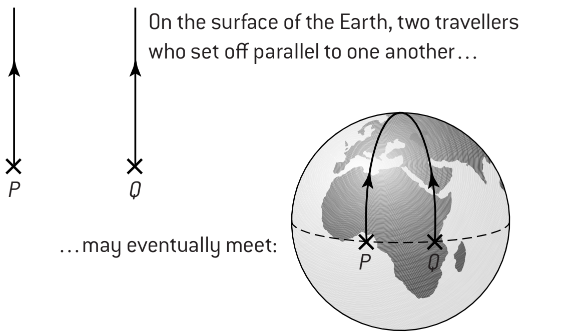
For the global positioning system to be so accurate, general relativity must be taken into account in calculating the details of the satellite's orbit.

Curvature of spacetime

HL

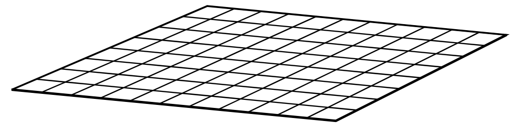
EFFECT OF GRAVITY ON SPACETIME

The Newtonian way of describing gravity is in terms of the forces between two masses. In general relativity the way of thinking about gravity is not to think of it as a force, but as changes in the shape (warping) of spacetime. The warping of spacetime is caused by mass. Think about two travellers who both set off from different points on the Earth's equator and travel north.



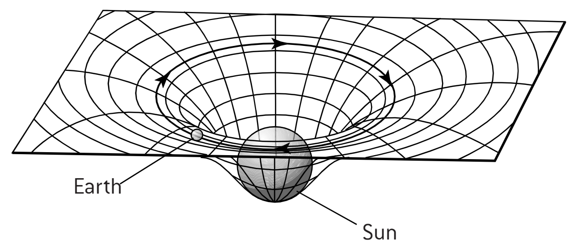
As they travel north they will get closer and closer together. They could explain this coming together in terms of a force of attraction between them or they could explain it as a consequence of the surface of the Earth being curved. The travellers have to move in straight lines across the surface of the Earth so their paths come together.

Einstein showed how spacetime could be thought of as being curved by mass. The more matter you have, the more curved spacetime becomes. Moving objects follow the curvature of spacetime or in other words, they take the shortest path in spacetime. As has been explained, it is very hard to imagine the four dimensions of spacetime. It is easier to picture what is going on by representing spacetime as a flat two-dimensional sheet.



spacetime represented by at sheet

Any mass present warps (or bends) spacetime. The more mass you have the greater the warping that takes place. This warping of spacetime can be used to describe the orbit of the Earth around the Sun. The diagram below represents how Einstein would explain the situation. The Sun warps spacetime around itself. The Earth orbits the Sun because it is travelling along the shortest possible path in spacetime. This turns out to be a curved path.



- Mass 'tells' spacetime how to curve.
- Spacetime 'tells' matter how to move.

APPLICATIONS OF GENERAL RELATIVITY TO THE UNIVERSE AS A WHOLE

General relativity is now fundamental to understanding how the objects in the Universe interact with spacetime and thus how they affect each other. This allows far-reaching predictions to be created about the future development and fate of the Universe – see cosmology sections of the astrophysics option (option D).

The development of the Universe can be modelled in detail. Many current aspects (e.g. its large-scale structure, the creation of the elements and the presence of cosmic background radiation) are predicted.

Very large mass black holes may exist at the centres of many galaxies. General relativity predicts how these may interact with matter and astronomers are searching for appropriate evidence.

General relativity predicts the existence of gravitational waves associated with high energy events such as the collision of two black holes. Experimental evidence for the existence of these waves is being sought.

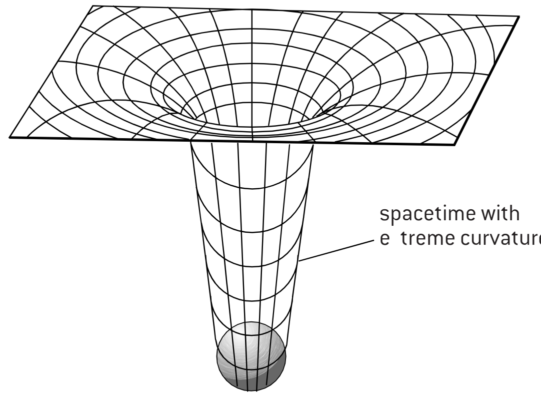
HL Black holes

DESCRIPTION

When a star has used up all o its nuclear uel, the orce o gravity makes it collapse down on itsel (see the astrophysics option or more details). The more it contracts the greater the density o matter and thus the greater the gravitational field near the collapsing star. In terms o general relativity, this would be described in terms o the spacetime near a collapsing star becoming more and more curved. The curvature o spacetime becomes more and more severe depending on the mass o the collapsing star.

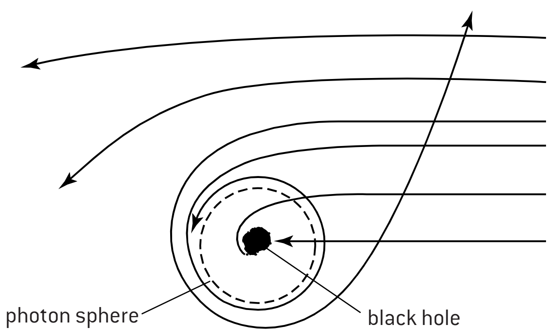
I the collapsing star is less than about 1.4 times the mass o the Sun, then the electrons play an important part in eventually stopping this contraction. The star that is le t is called a white dwar . I the collapsing star is greater than this, the electrons cannot halt the contraction. A contracting mass o up to three times the mass o the Sun can also be stopped – this time the neutrons play an important role and the star that is le t is called a neutron star. The curvature o spacetime near a neutron star is more extreme than the curvature near a white dwar .

At masses greater than this we do not know o any process that can stop the contraction. Spacetime around the mass becomes more and more warped until eventually it becomes so great that it olds in over itsel . What is le t is called a black hole. All the mass is concentrated into a point – the **singularity**.

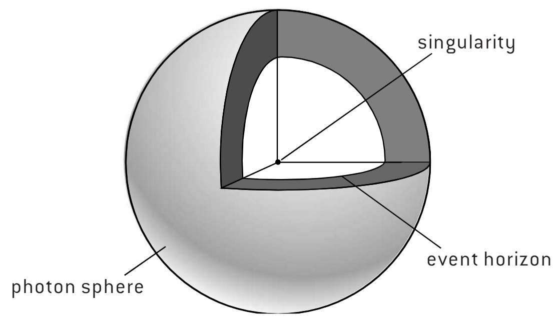


SCHWARZCHILD RADIUS

The curvature o spacetime near a black hole is so extreme that nothing, not even light, can escape. Matter can be attracted into the hole, but nothing can get out since nothing can travel aster than light. The gravitational orces are so extreme that light would be severely deflected near a black hole.



I you were to approach a black hole, the gravitational orces on you would increase. The first thing o interest would be the **photon sphere**. This consists o a very thin shell o light photons captured in orbit around the black hole. As we all urther in, the gravitational orces increase and so the escape velocity at that distance also increases.



At a particular distance rom the centre, called the **Schwarzchild radius**, we get to a point where the escape velocity is equal to the speed o light. Newtonian mechanics predicts that the escape velocity v rom a mass M o radius r is given by the ormula

$$v = \sqrt{\frac{2GM}{r}}$$

I the escape velocity is the speed o light, c, then the Schwarzchild radius would be given by

$$R_s = \frac{2GM}{c^2}$$

It turns out that this equation is also correct i we use the proper equations o general relativity. I we cross the Schwarzchild radius and get closer to the singularity, we would no longer be able to communicate with the Universe outside. For this reason crossing the Schwarzchild radius is sometimes called crossing the **event horizon**. An observer watching an object approaching a black hole would see time slowing down or the object.

The observed time dilation is worked out rom

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{R_s}{r}}}$$

where r is the distance rom the black hole.

EXAMPLE

Calculate the size o a black hole that has the same mass as our Sun (1.99×10^{30} kg).

$$R_{Sch} = \frac{2 \times 6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{(3 \times 10^8)^2}$$

$$= 2949.6 \text{ m}$$

$$= 2.9 \text{ km}$$

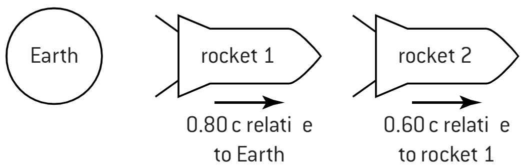
IB Quest ons – opt on A – relat v ty

1. In the laboratory frame of reference, a slow moving alpha particle travels parallel to stationary metal wire that carries an electric current. In this frame of reference, the velocity of the alpha particle and the drift velocity of the electrons in the wire are identical. Explain the origin of the force on the alpha particle in the frame of reference of

   a) The alpha particle [2]

   b) The laboratory [2]

2. Two identical rockets are moving along the same straight line as viewed from Earth. Rocket 1 is moving away from the Earth at speed 0.80 c **relative to the Earth** and rocket 2 is moving away from rocket 1 at speed 0.60 c **relative to rocket 1**.



   a) Calculate the velocity of rocket 2 relative to the Earth, using the

      (i) Galilean transformation equation. [1]

      (ii) relativistic transformation equation. [2]

   b) Comment on your answers in (a). [2]

   c) The rest mass of rocket 1 is 1.0×10^3 kg. Determine the **relativistic** kinetic energy of rocket 1, as measured by an observer on Earth. [3]

3. The spacetime diagram below shows two events, A and B, as observed in a reference frame *S*. Each event emits a light signal.

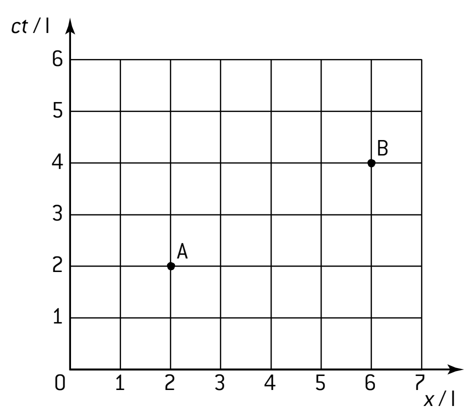
   Use the diagram to calculate, according to frame *S*,

   a) The time between event A and event B [2]

   b) The time taken for the light signal leaving event A to arrive at the position of event B. [2]

   c) The location of a stationary observer who receives the light signal from events A simultaneously with receiving the light signal from event B. [2]

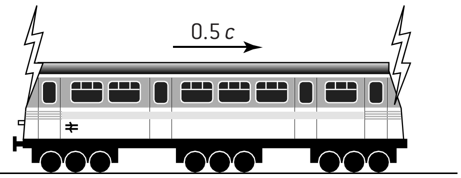
   d) The velocity of a moving frame of reference in which event A and event B occurred simultaneously. [4]



4. *Relativit and simultaneit*

   a) State two postulates of the special theory of relativity. [2]

   Einstein proposed a 'thought experiment' along the following lines. Imagine a train of proper length 100 m passing through a station at half the speed of light. There are two lightning strikes, one at the front and one at the rear of the train, leaving scorch marks on both the train and the station platform. Observer S is standing on the station platform midway between the two strikes, while observer T is sitting in the middle of the train. Light from each strike travels to both observers.



   b) If observer S on the station concludes from his observations that the two lightning strikes occurred simultaneously, explain why observer T on the train will conclude that they did **not** occur simultaneously. [4]

   c) Which strike will T conclude occurred first? [1]

   d) What will be the distance between the scorch marks on the *train*, according to T and according to S? [3]

   e) What will be the distance between the scorch marks on the *platform*, according to T and according to S? [2]

HL

5. In a laboratory experiment two identical particles (P and Q), each of rest mass m_0, collide. In the **laboratory frame of reference**, they are both moving at a velocity of 2/3 *c*. The situation before the collision is shown in the diagram below.

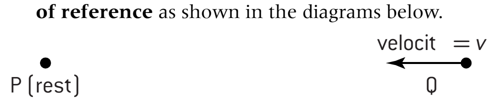
   Before:

   a) In the laboratory frame of reference,

      (i) what is the total momentum of P and Q? [1]

      (ii) what is the total energy of P and Q? [3]

   The same collision can be viewed according to **P's frame of reference** as shown in the diagrams below.



   b) In P's frame of reference,

      (i) what is Q's velocity, v? [3]

      (ii) what is the total momentum of P and Q? [3]

      (iii) what is the **total energy** of P and Q? [3]

   c) As a result of the collision, many particles and photons are formed, but the total energy of the particles depends on the frame of reference. Do the observers in each frame of reference agree or disagree on the number of particles and photons formed in the collision? Explain your answer. [2]

6. The concept of gravitational red-shift indicates that clocks run slower as they approach a black hole.

   a) Describe what is meant by

      (i) gravitational red-shift. [2]

      (ii) spacetime. [1]

      (iii) a black hole with reference to the concept of spacetime. [2]

   b) A particular black hole has a Schwarzschild radius *R*. A person at a distance of 2*R* from the event horizon of the black hole measures the time between two events to be 10 s. Deduce that for a person a very long way from the black hole the time between the events will be measured as 12 s. [1]