

Moment of Inertia



Hoop

$$I = mr^2$$

Thin rod

$$I = \frac{1}{12} mr^2$$

Sphere

$$I = \frac{2}{5} mr^2$$

Board



Disk

$$I = \frac{1}{2} mr^2$$

$$I = m \left(\frac{l^2 + h^2}{12} \right)$$

Donut

$$I = \frac{1}{2} m(a^2 + b^2)$$



1. a. $\Gamma = ?$ $I = 0.85 \text{ kg m}^2$ $\omega_i = 0 \frac{\text{rad}}{\text{s}}$ $\omega_f = 9.4 \frac{\text{rad}}{\text{s}}$
 $t = 6.0 \text{ s}$

$$\Gamma = I \alpha$$

$$\Gamma = (0.85 \text{ kg m}^2) \left(1.56 \frac{\text{rad}}{\text{s}^2} \right)$$

$$\Gamma = 1.3 \text{ Nm}$$

$$\omega_f = \omega_i + \alpha t$$

$$\frac{\omega_f - \omega_i}{t} = \alpha$$

$$\alpha = \frac{9.4 - 0}{6}$$

$$\alpha = 1.56 \frac{\text{rad}}{\text{s}^2}$$

$$1. b. \quad r = .25 \text{ m} \quad \omega_i = 9.4 \frac{\text{rad}}{\text{s}} \quad \omega_f = 0 \frac{\text{rad}}{\text{s}} \quad \theta = 4 \text{ rev} \Rightarrow 8\pi$$

$$F = ma \quad I = \frac{1}{2} m r^2$$

$$m = \frac{2I}{r^2} = \frac{2(.85 \text{ kg m}^2)}{(.25 \text{ m})^2} = 27.2 \text{ kg}$$

$$F = (27.2 \text{ kg})(0.44 \text{ m/s}^2)$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

$$F = 11.95 \text{ N} \Rightarrow$$

$$\boxed{10 \text{ N}}$$

$$\frac{\omega_f^2 - \omega_i^2}{2\theta} = \alpha = \frac{9.4^2 - 0^2}{2(8\pi)} = 1.76 \frac{\text{rad}}{\text{s}^2}$$

$$a = \alpha r = 1.76(.25) = 0.44 \frac{\text{m}}{\text{s}^2}$$

$$2. \quad f_1 = \frac{100 \text{ rev}}{3 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = \frac{5}{9} \frac{\text{rev}}{\text{s}} \quad \omega_1 = \frac{10\pi}{9} \frac{\text{rad}}{\text{s}}$$

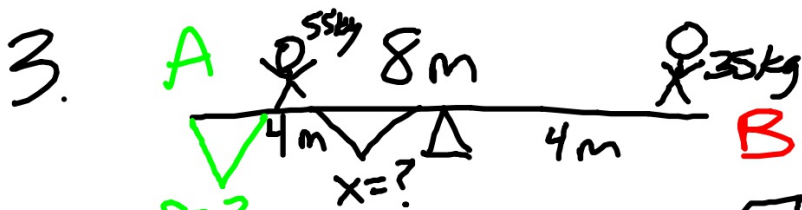
$$f_2 = \frac{45 \text{ rev}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = \frac{3}{4} \frac{\text{rev}}{\text{s}} \quad \omega_2 = \frac{3\pi}{2} \frac{\text{rad}}{\text{s}}$$

$$t = 2.5 \text{ s} \Rightarrow \omega = 2\pi f$$

$$\Theta = \frac{(\omega_1 + \omega_2) t}{2}$$

$$\Theta = \frac{\left(\frac{10\pi}{9} + \frac{3\pi}{2}\right) 2.5 \text{ s}}{2} = 10.25 \text{ rad}$$

$$\boxed{= 10 \text{ rad.}}$$



$S = ?$
 $S = 4 - x$

$$\tau_A = \tau_B$$

$$\tau_A = (55\text{kg})(g)(x)$$

$$\tau_B = (35\text{kg})(g)(4\text{m})$$

$$55gx = 35g(4)$$

$$55x = 35(4)$$

$$x = 2.54\text{m}$$

$$\begin{array}{r} 4\text{m} \\ - 2.54\text{m} \\ \hline 1.46\text{m} \Rightarrow \boxed{1\text{m}} \end{array}$$

$$4. a. \quad m = 5.0 \text{ kg} \quad \tau = 18 \text{ Nm} \quad r = .15 \text{ m}$$

$$\tau = I\alpha \quad I = \frac{2}{5}mr^2$$

$$\alpha = ? \quad \frac{\tau}{\frac{2}{5}Mr^2} = \alpha$$

$$\frac{18}{\frac{2}{5}(\cancel{5})(.15)^2} = \alpha = 4.0(10^2) \frac{\text{rad}}{\text{s}^2}$$

4b.

$$\alpha_{15} = 400$$

$$a = \alpha r$$

$$a_{15} = (400)(.15)$$

$$a_{15} = 60 \text{ m/s}^2$$

$$= 36 \text{ m/s}^2$$

$$\boxed{24 \text{ m/s}^2}$$

$$\frac{\text{m}}{\text{s}^2} = \alpha$$

$$\frac{18}{\frac{2}{5}(\cancel{5.0})(.25)^2} = 144$$

$$a_{.25} = \alpha r$$

$$a = (144)(.25)$$

$$a_{.25} = 36 \text{ m/s}^2$$

$$5a. \quad s = 25m \quad \alpha = .75 \frac{\text{rad}}{\text{s}^2} \quad \omega_0 = 0 \frac{\text{rad}}{\text{s}}$$

$$r = .80m$$

$$m = 50kg$$

$$v = \omega r = 0 \frac{m}{s}$$

$$a = \alpha r = (.75)(.8) = .6 \frac{m}{s^2}$$

$$v^2 = v_0^2 + 2as$$

$$v = \sqrt{v_0^2 + 2as}$$

$$v = \sqrt{0^2 + 2(.6)(25)}$$

$$v = 5.477 \Rightarrow 5 \frac{m}{s}$$

5b. $\Gamma = ?$ $r = .80 \text{ m}$ $\alpha = .75 \frac{\text{rad}}{\text{s}^2}$
 $m = 50 \text{ kg}$

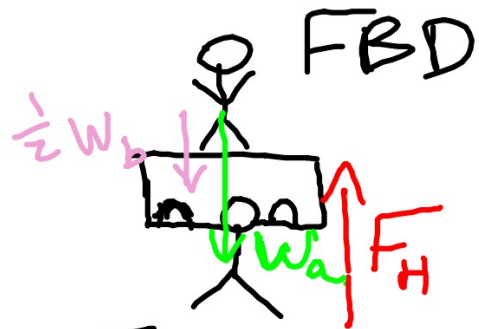
$$I = \frac{1}{2} m r^2$$

$$\Gamma = I \alpha$$

$$\Gamma = \frac{1}{2} m r^2 \alpha$$

$$\Gamma = \frac{1}{2} (50) (.8)^2 (.75)$$

$$\Gamma = 12 \text{ Nm} \Rightarrow \boxed{10 \text{ Nm}}$$



$$F_H = W_a + \frac{1}{2} W_b$$

$$= 65(9.81) + \frac{1}{2}(15)(9.81)$$

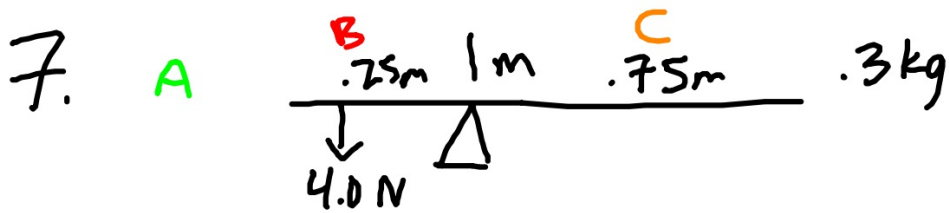
$$F_{bH} = F_{bB} \quad \frac{11m}{2} = r_{bH} = r_{bB}$$

$$F_H = 46915.09875$$

$$F_H = 47000 \text{ N}$$

$$F_{bH} = F_{bB}$$

$$F_{bH} = \frac{1}{2} W_b$$



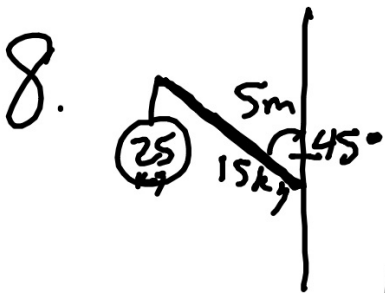
$$\sum A + \sum B = \sum C$$

$$(4.0 \text{ N})(x \text{ m}) + (.3 \text{ kg}) \left(\frac{25}{100} \right) \left(\frac{.25}{2} \right) \left(9.8 \frac{\text{m}}{\text{s}^2} \right) =$$

$$(4x + .09197 = .8277)$$

$$4x = .73575$$

$$x = .184 \Rightarrow \boxed{18 \text{ cm}}$$



$$\tau = \tau_{\text{mass}} + \tau_{\text{pole}}$$

$$\tau = (25 \text{ kg})(9.81 \text{ m/s}^2)(5 \text{ m})(\sin 45) +$$

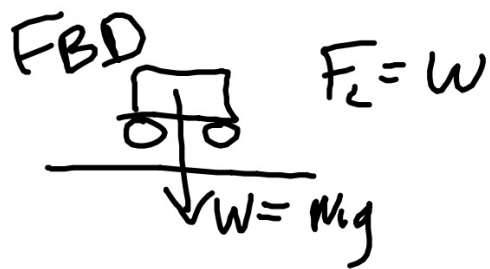
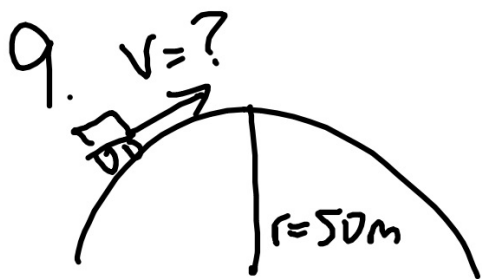
$$\tau = F r \sin \theta$$

$$= m a r \sin \theta$$

$$(15 \text{ kg})(9.81 \text{ m/s}^2)\left(\frac{5}{2} \text{ m}\right)(\sin 45)$$

$$\tau = 1127 \text{ Nm}$$

$$\tau = 1100 \text{ Nm}$$



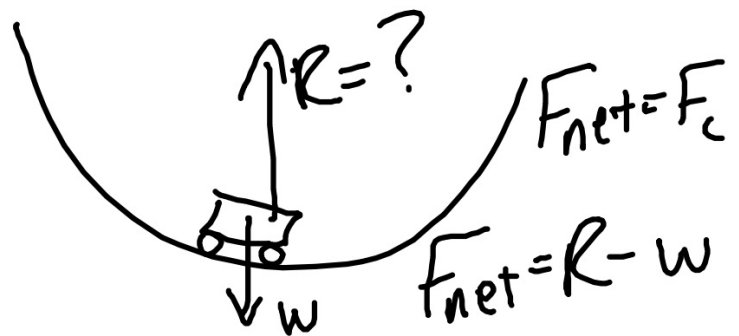
$$mg = \frac{mv^2}{r}$$

$$v = \sqrt{rg}$$

$$v = 22.147$$

$$v = 22 \text{ m/s}$$

10. $v = 22.147 \text{ m/s}$



$$R = 68670$$

$$R = 69000 \text{ N}$$

$$\frac{mv^2}{r} + mg = R$$

$$R = \frac{3500(22)^2}{50} + 3500(9.81)$$

