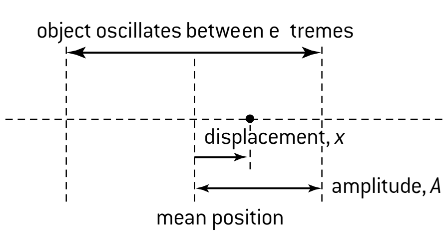
Oscilla ions

DEFINITIONS

Many systems involve vibrations or oscillations; an object continually moves to-and-fro about a fixed average point (the **mean position**) retracing the same path through space taking a fixed time between repeats. Oscillations involve the interchange of energy between kinetic and potential.

	Kinetic energy	**Potential energy store**
Mass moving between two horizontal springs	Moving mass	Elastic potential energy in the springs
Mass moving on a vertical spring	Moving mass	Elastic potential energy in the springs and gravitational potential energy
Simple pendulum	Moving pendulum bob	Gravitational potential energy of bob
Buoy bouncing up and down in water	Moving buoy	Gravitational PE of buoy and water
An oscillating ruler as a result of one end being displaced while the other is fixed	Moving sections of the ruler	Elastic PE of the bent ruler

	Definition
Displacement, x	The instantaneous distance (SI measurement: m) of the moving object from its mean position (in a specified direction)
Amplitude, A	The maximum displacement (SI measurement: m) from the mean position
Frequency, f	The number of oscillations completed per unit time. The SI measurement is the number of cycles per second or Hertz (Hz).
Period, T	The time taken (SI measurement: s) for one complete oscillation. $T = \frac{1}{f}$
Phase difference, ø	This is a measure of how 'in step' different particles are. If moving together they are **in phase**. ø is measured in either degrees (°) or radians (rad). 360° or 2π rad is one complete cycle so 180° or π rad is completely **out o phase** by half a cycle. A phase difference of 90° or $\pi/2$ rad is a quarter of a cycle.



SIMPLE HARMONIC MOTION (SHM)

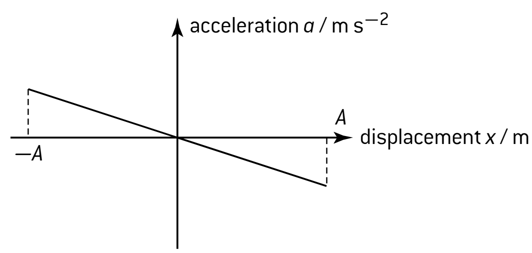
Simple harmonic motion is defined as the motion that takes place when the acceleration, a, of an object is always directed towards, and is proportional to, its displacement from a fixed point. This acceleration is caused by a **restoring orce** that must always be pointed towards the mean position and also proportional to the displacement from the mean position.

$F \quad -x$ or $F = -$ (constant) $\times x$

Since $F = ma$

$a \quad -x$ or $a = -$ (constant) $\times x$

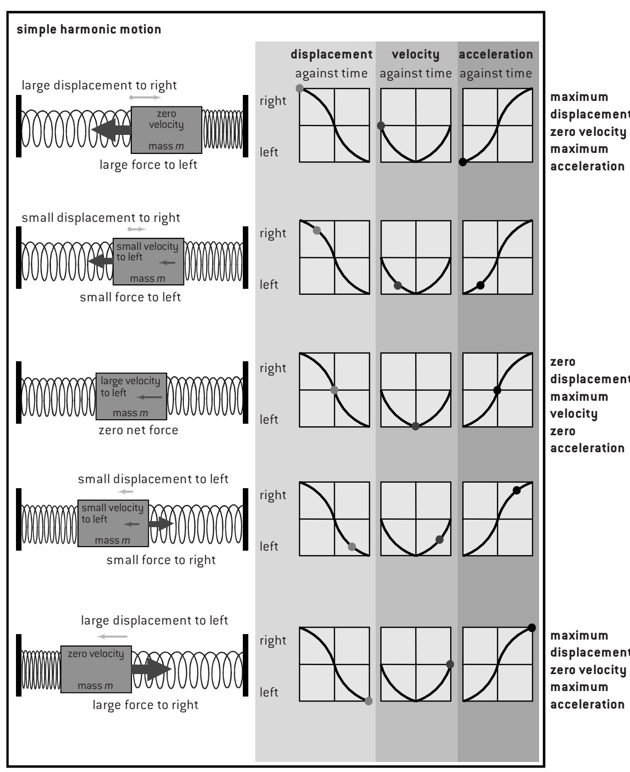
The negative sign signifies that the acceleration is always pointing back towards the mean position.



Points to note about SHM:

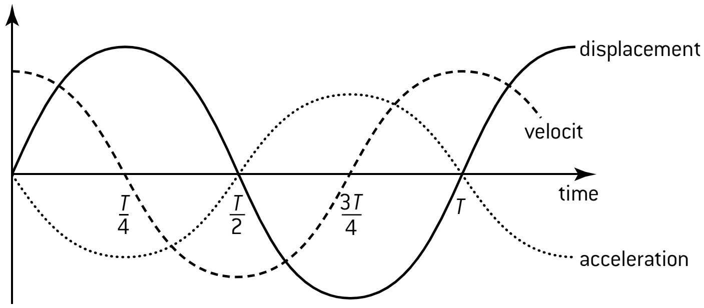
- The time period T does not depend on the amplitude A. It is **isochronous**.
- Not all oscillations are SHM, but there are many everyday examples of natural SHM oscillations.

EXAMPLE OF SHM: MASS BETWEEN TWO SPRINGS



Graphs of simple harmonic mo ion

ACCELERATION, VELOCITY AND DISPLACEMENT DURING SHM



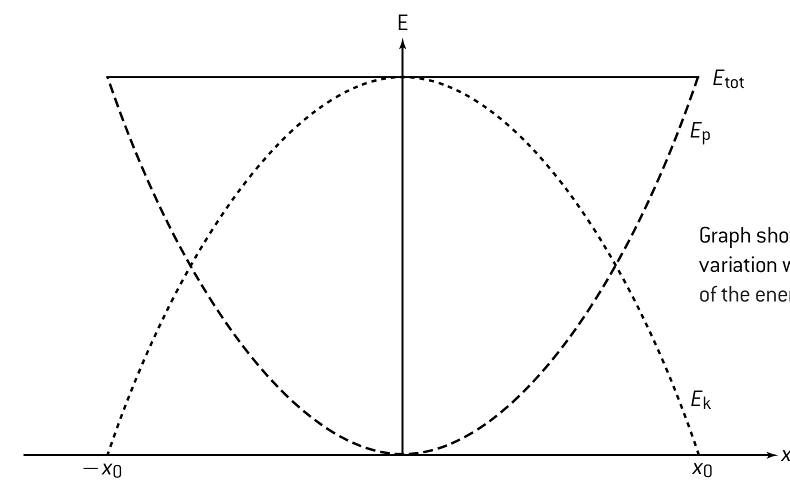
- acceleration leads velocity by 90°
- velocity leads displacement by 90°
- acceleration and displacement are 180° out of phase
- displacement lags velocity by 90°
- velocity lags acceleration by 90°

ENERGY CHANGES DURING SIMPLE HARMONIC MOTION

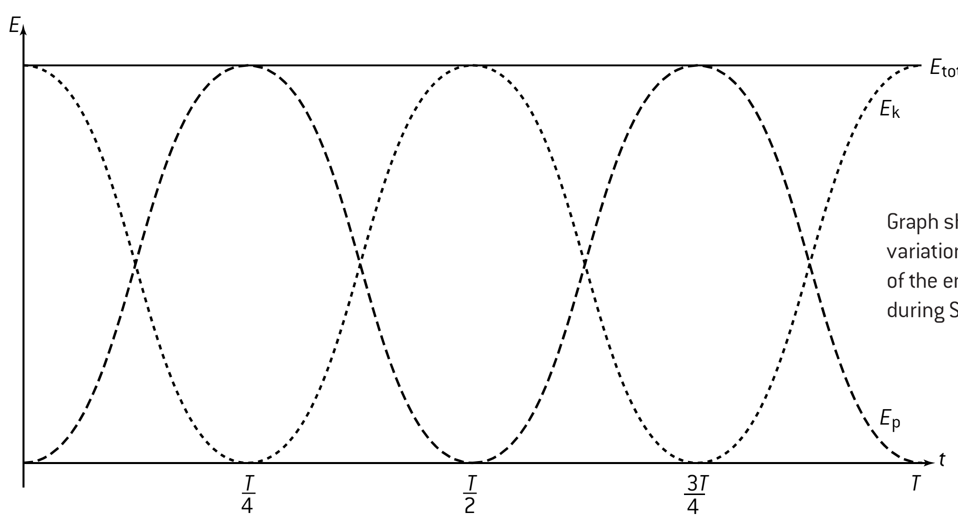
During SHM, energy is interchanged between KE and PE. Providing there are no resistive forces which dissipate this energy, the total energy must remain constant. The oscillation is said to be **undamped**.

Energy in SHM is proportional to:

- the mass m
- the (amplitude)^2
- the (frequency)^2



Graph showing the variation with distance, x of the energ during SHM



Graph showing the variation with time, t of the energ during SHM

Travelling aves

INTRODUCTION – RAYS AND WAVE FRONTS

Light, sound and ripples on the surface of a pond are all examples of wave motion.

- They all transfer energy from one place to another.
- They do so without a net motion of the medium through which they travel.
- They all involve oscillations (vibrations) of one sort or another. The oscillations are SHM.

A **continuous wave** involves a succession of individual oscillations. A **wave pulse** involves just one oscillation. Two important categories of wave are **transverse** and **longitudinal** (see below). The table gives some examples.

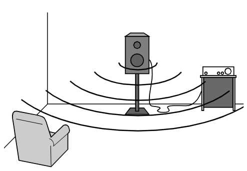
The following pages analyse some of the properties that are common to all waves.

	Example of energy transfer
Water ripples (Transverse)	A floating object gains an 'up and down' motion.
Sound waves (Longitudinal)	The sound received at an ear makes the eardrum vibrate.
Light wave (Transverse)	The back of the eye (the retina) is stimulated when light is received.
Earthquake waves (Both T and L)	Buildings collapse during an earthquake.
Waves along a stretched rope (Transverse)	A 'sideways pulse' will travel down a rope that is held taut between two people.
Compression waves down a spring (Longitudinal)	A compression pulse will travel down a spring that is is held taut between two people.

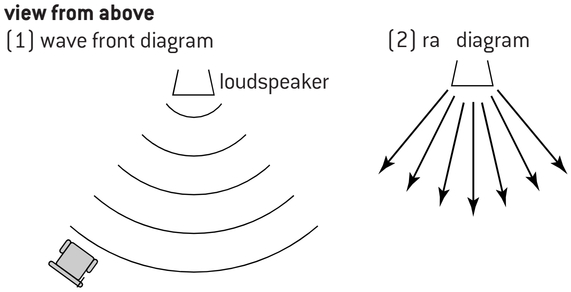
LONGITUDINAL WAVES

Sound is a longitudinal wave. This is because the oscillations are **parallel** to the direction of energy transfer.

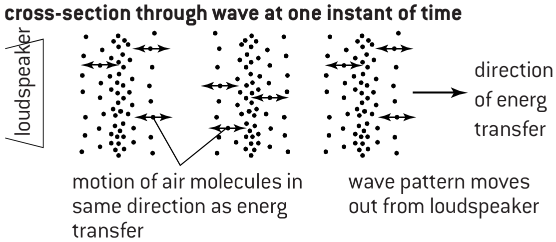
situation



view from above



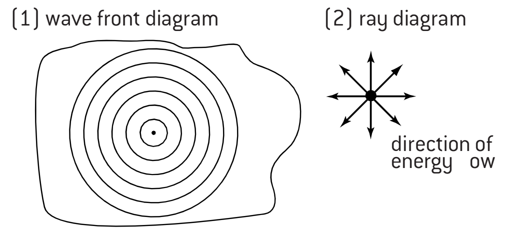
cross-section through wave at one instant of time



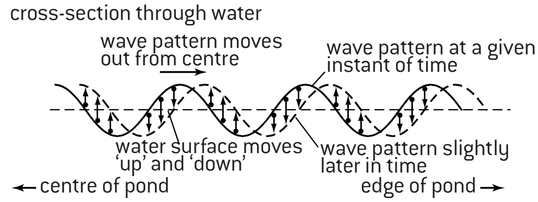
TRANSVERSE WAVES

Suppose a stone is thrown into a pond. Waves spread out as shown below.

situation



cross-section through water



The top of the wave is known as the **crest**, whereas the bottom of the wave is known as the **trough**.

Note that there are several aspects to this wave that can be studied. These aspects are important to all waves.

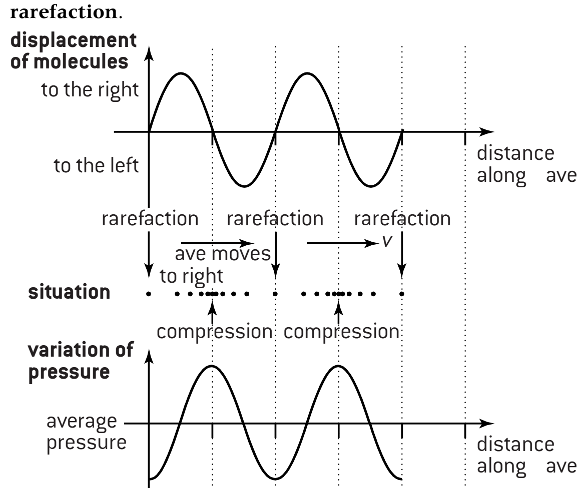
- The movement of the wave pattern. The **wave fronts** highlight the parts of the wave that are moving together.
- The direction of energy transfer. The **rays** highlight the direction of energy transfer.
- The oscillations of the medium.

It should be noted that the rays are at right angles to the wave fronts in the above diagrams. This is always the case.

This wave is an example of a transverse wave because the oscillations are **at right angles** to the direction of energy transfer.

Transverse mechanical waves cannot be propagated through fluids (liquids or gases).

A point on the wave where everything is 'bunched together' (high pressure) is known as a **compression**. A point where everything is 'far apart' (low pressure) is known as a **rarefaction**.



Relationship between displacement and pressure graphs

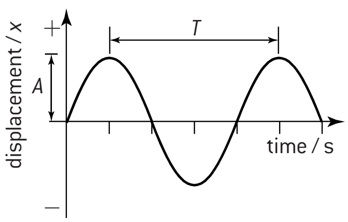
Wave charac eris ics

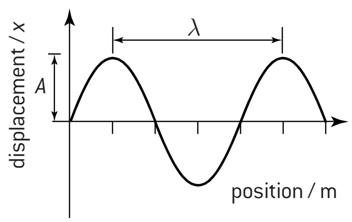
DEFINITIONS

There are some use ul terms that need to be defined in order to analyse wave motion in more detail. The table below attempts to explain these terms and they are also shown on the graphs.

Because the graphs seem to be identical, you need to look at the axes o the graphs care ully.

- The displacement–time graph on the le t represents the oscillations or one point on the wave. All the other points on the wave will oscillate in a similar manner, but they will not start their oscillations at exactly the same time.
- The displacement–position graph on the right represents a 'snapshot' o all the points along the wave at one instant o time. At a later time, the wave will have moved on but it will retain the same shape.
- The graphs can be used to represent longitudinal AND transverse waves because the y-axis records only the value o the displacement. It does NOT speci y the direction o this displacement. So, i this displacement were parallel to the direction o the wave energy, the wave would be a longitudinal wave. I this displacement were at right angles to the direction o the wave energy, the wave would be a transverse wave.





Term	Symbol	Definition
Displacement	x	This measures the change that has taken place as a result o a wave passing a particular point. Zero displacement re ers to the mean (or average) position. For mechanical waves the displacement is the distance (in metres) that the particle moves rom its undisturbed position.
Amplitude	A	This is the maximum displacement rom the mean position. I the wave does not lose any o its energy its amplitude is constant.
Period	T	This is the time taken (in seconds) or one complete oscillation. It is the time taken or one complete wave to pass any given point.
Frequency	f	This is the number o oscillations that take place in one second. The unit used is the hertz (Hz). A requency o 50 Hz means that 50 cycles are completed every second.
Wavelength	λ	This is the shortest distance (in metres) along the wave between two points that are **in phase** with one another. 'In phase' means that the two points are moving exactly in step with one another. For example, the distance rom one crest to the next crest on a water ripple or the distance rom one compression to the next one on a sound wave.
Wave speed	c	This is the speed (in m s^{-1}) at which the wave ronts pass a stationary observer.
Intensity	I	The intensity o a wave is the power per unit area that is received by the observer. The unit is W m^{-2}. The intensity o a wave is proportional to the square o its amplitude: $I \propto A^2$.

The period and the requency o any wave are inversely related. For example, i the requency o a wave is 100 Hz, then its period must be exactly $\frac{1}{100}$ o a second.

In symbols,

$$T = \frac{1}{f}$$

WAVE EQUATIONS

There is a very simple relationship that links wave speed, wavelength and requency. It applies to all waves.

The time taken or one complete oscillation is the period o the wave, T.

In this time, the wave pattern will have moved on by one wavelength, λ.

This means that the speed o the wave must be given by

$$c = \frac{\text{distance}}{\text{time}} = \frac{\lambda}{T}$$

Since $\frac{1}{T} = f$

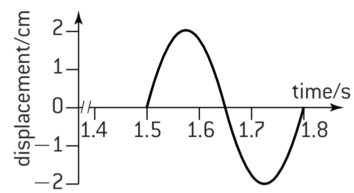
$$c = f\lambda$$

In words,

velocity = frequency × wavelength

EXAMPLE

A stone is thrown onto a still water sur ace and creates a wave. A small floating cork 1.0 m away rom the impact point has the ollowing displacement–time graph (time is measured rom the instant the stone hits the water):



a) the amplitude o the wave:

2 cm

b) the speed o the wave:

$$c = \frac{d}{t} = \frac{1.0}{1.5} = 0.67 \text{ m s}^{-1}$$

c) the requency o the wave:

$$f = \frac{1}{T} = \frac{1}{0.3} = 3.33 \text{ Hz}$$

d) the wavelength o the wave:

$$\lambda = \frac{c}{f} = \frac{0.666}{3.33} = 0.2 \text{ m}$$

Elec romagne ic spec rum

ELECTROMAGNETIC WAVES

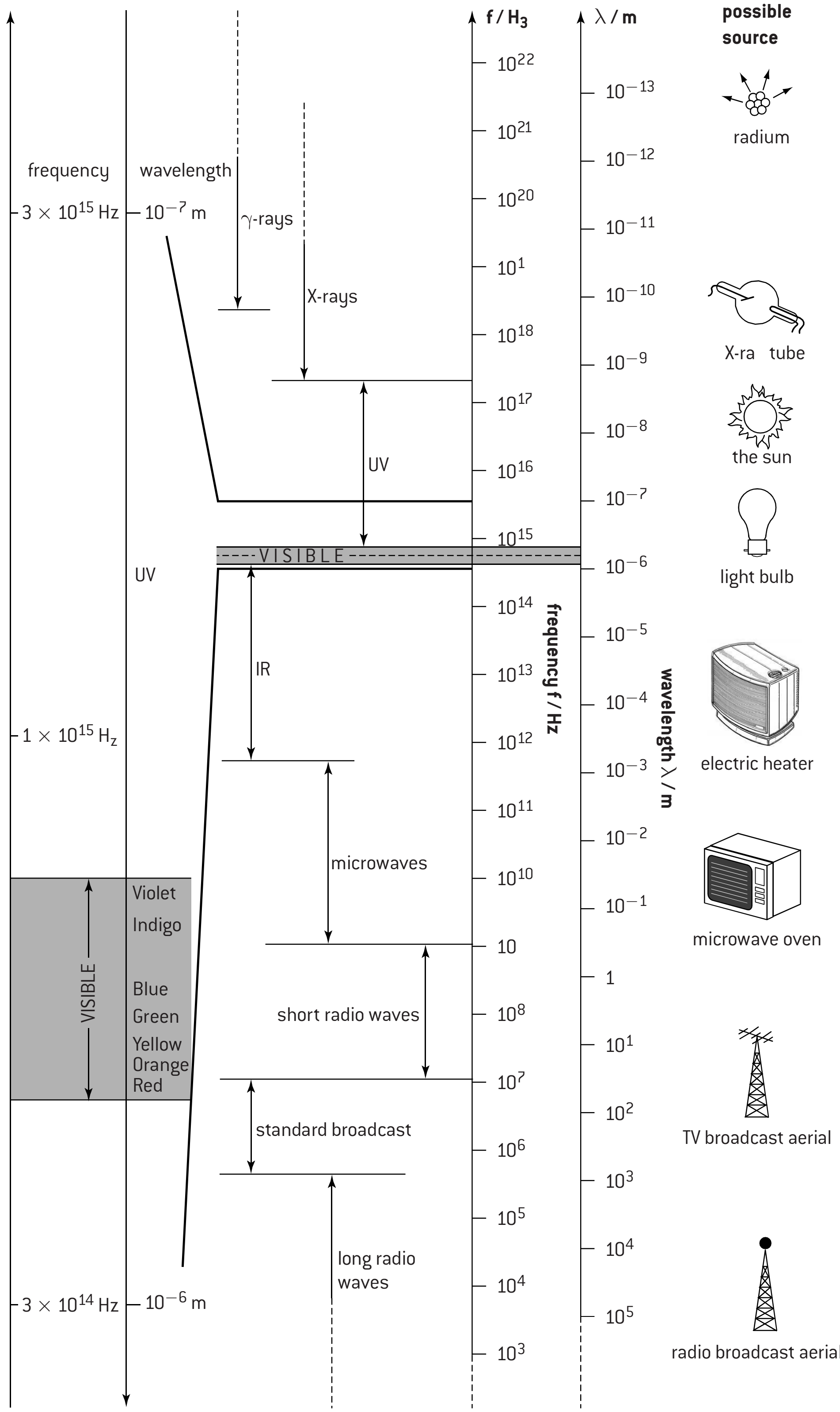
Visible light is one part of a much larger spectrum of similar waves that are all electromagnetic.

Charges that are accelerating generate electromagnetic fields. If an electric charge oscillates, it will produce a varying electric and magnetic field at right angles to one another.

These oscillating fields propagate (move) as a transverse wave through space. Since no physical matter is involved in this propagation, they can travel through a vacuum. The speed of this wave can be calculated from basic electric and magnetic constants and it is the same for all electromagnetic waves, 3.0×10^8 m s^{-1}.

Although all electromagnetic waves are identical in their nature, they have very different properties. This is because of the huge range of frequencies (and thus energies) involved in the electromagnetic spectrum.

See page 132 (option A) for more details.



Investigating speed of sound experimentally

1. DIRECT METHODS

The most direct method to measure the speed of sound is to record the time taken t for sound to cover a known distance d: speed $c = \frac{d}{t}$. In air at normal pressures and temperatures, sound travels at approximately 330 m s^{-1}. Given the much larger speed of light (3×10^8 m s^{-1}), a possible experiment would be to use a stop watch to time the difference between seeing an event (e.g. the firing of a starting pistol for a race or seeing two wooden planks being hit together) and hearing the same event some distance away (100 m or more).

Echoes can be used to put the source and observer of the sound in the same place. Standing a distance d in front of a tall wall (e.g. the side of a building that is not surrounded by other buildings) can allow the echo from a pulse of sound (e.g. a single clap of the hands) to be heard. With practice, it is possible for an experimenter to adjust the frequency of clapping to synchronize the sound of the claps with their echoes. When this is achieved, the frequency of clapping f can be recorded (counting the number of claps in a given time) and the time period T between claps is just $T = \frac{1}{f}$. In this time, the sound travels to the wall **and back.** The speed of sound is thus $c = 2df$.

In either of the above situations a more reliable result will be achieved if a range of distances, rather than one single value is used. A graph of distance against time will allow the speed of sound to be calculated from the gradient of the best-fit straight line (which should go through the origin).

Timing pulses of sound over smaller distances requires small time intervals to be recorded with precision. It is possible to automate the process using electronic timers and / or data loggers. This equipment would allow, for example, the speed of a sound wave along a metal rod or through water to be investigated.

2. INDIRECT METHODS

Since $c = f\lambda$, the speed of sound can be calculated if we measure a sound's frequency and wavelength.

Frequency measurement

a) A microphone and a cathode ray oscilloscope (CRO) [page 116] can display a graph of the oscillations of a sound wave. Appropriate measurements from the graph allow the time period and hence the frequency to be calculated.

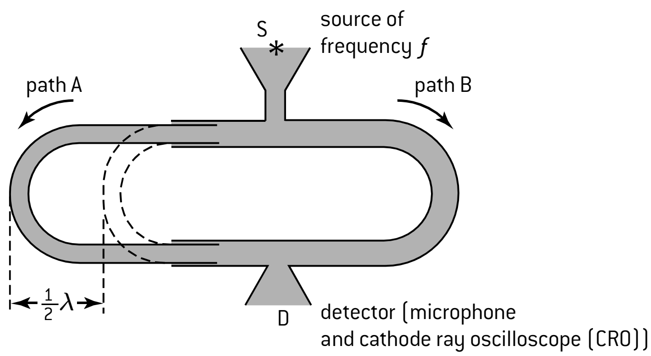
b) **Stroboscopic** techniques (e.g. flashing light of known frequency) can be used to measure the frequency of the vibrating object (e.g. a tuning fork) that is the source of the sound.

c) Frequency of sound can be controlled at source using a known frequency source (e.g. a standard tuning fork) or a calibrated electronic frequency generator.

d) Comparisons can also be made between the unknown frequency and a known frequency.

Wavelength measurement

a) The interference of waves (see page 40) can be employed to find the path difference between consecutive positions of destructive interference. The path difference between these two situations will be λ.



b) Standing waves (see page 48) in a gas can be employed to find the location of adjacent nodes. The positions in an enclosed tube can be revealed either:

- in the period pattern made by dust in the tube
- electronically using a small movable microphone.

c) A resonance tube (see page 49) allows the column length for different maxima to be recorded. The length distance between adjacent maxima will be $\frac{\lambda}{2}$.

3. FACTORS THAT AFFECT THE SPEED OF SOUND

Factors include:

- Nature of material
- Density
- Temperature (for an ideal gas, $c \propto \sqrt{T}$)
- Humidity (for air).

Intensit

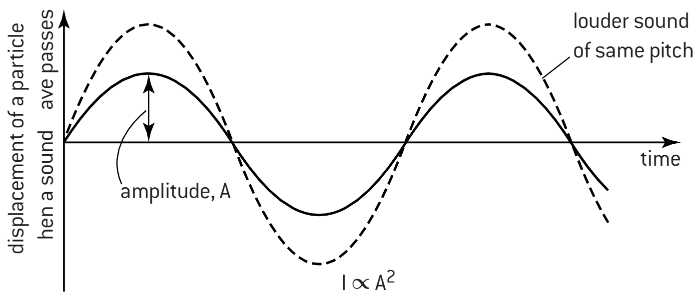
INTENSITY

The **sound intensity**, I, is the amount of energy that a sound wave brings to a unit area every second. The units of sound intensity are W m^{-2}.

It depends on the amplitude of the sound. A more intense sound (one that is louder) must have a larger amplitude.

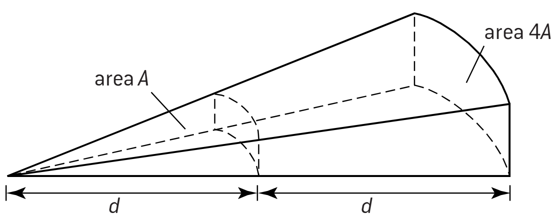
Intensity $\propto$ (amplitude)^2

This relationship between intensity and amplitude is true for all waves.



INVERSE SQUARE LAW OF RADIATION

As the distance of an observer from a point source of light increases, the power received by the observer will decrease as the energy spreads out over a larger area. A doubling of distance will result in the reduction of the power received to a quarter of the original value.



The surface area A of a sphere of radius r is calculated using:

$$A = 4\pi r^2$$

If the point source radiates a total power P in all directions, then the power received per unit area (the **intensity *I***) at a distance r away from the point source is:

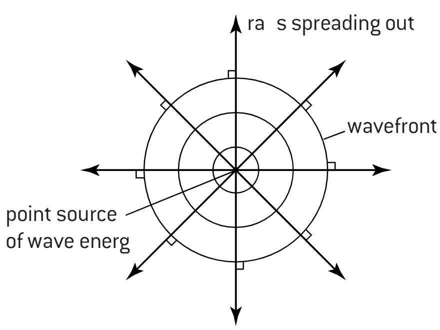
$$I = \frac{P}{4\pi r^2}$$

For a given area of receiver, the intensity of the received radiation is inversely proportional to the square of the distance from the point source to the receiver. This is known as the **inverse square law** and applies to all waves.

$$I \propto x^{-2}$$

WAVEFRONTS AND RAYS

As introduced on page 35, waves can be described in terms of the motion of a wavefront and/or in terms of rays.



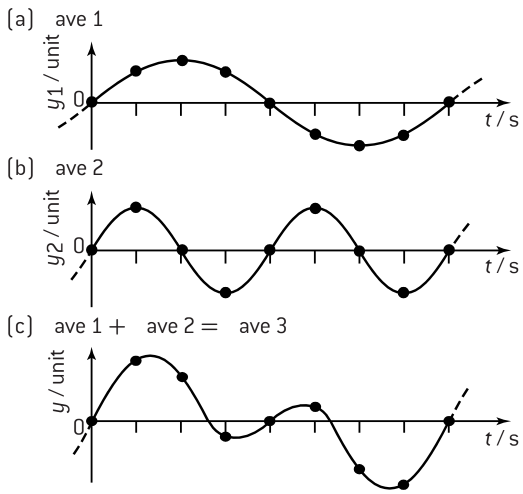
A **ray** is the path taken by the wave energy as it travels out from the source.

A **wavefront** is a surface joining neighbouring points where the oscillations are in phase with one another. In two dimensions, the wavefront is a line and in one dimension, the wavefront is a point.

Superposition

INTERFERENCE OF WAVES

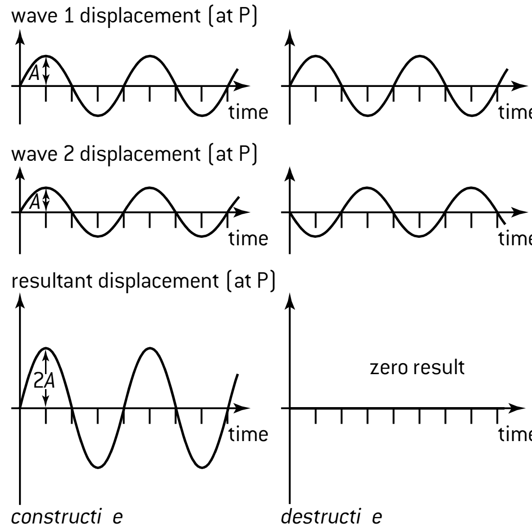
When two waves of the same type meet, they **interfere** and we can work out the resulting wave using the principle of superposition. The overall disturbance at any point and at any time where the waves meet is the vector sum of the disturbances that would have been produced by each of the individual waves. This is shown below.



Wave superposition

If the waves have the same amplitude and the same frequency then the interference **at a particular point** can be **constructive** or **destructive.**

graphs



TECHNICAL LANGUAGE

Constructive interference takes place when the two waves are 'in step' with one another – they are said to be **in phase**. There is a zero **phase difference** between them. Destructive interference takes place when the waves are exactly 'out of step' – they are said to be **out of phase**. There are several different ways of saying this. One could say that the phase difference is equal to 'half a cycle' or '180 degrees' or 'π radians'.

Interference can take place if there are two possible routes for a ray to travel from source to observer. If the path difference between the two rays is a whole number of wavelengths, then constructive interference will take place.

path difference $= n\,\lambda \rightarrow$ constructive

path difference $= (n + \frac{1}{2})\,\lambda \rightarrow$ destructive

$n = 0, 1, 2, 3 \ldots$

For constructive or destructive interference to take place, the sources of the waves must be phase linked or **coherent.**

EXAMPLES OF INTERFERENCE

Water aves

A ripple tank can be used to view the interference of water waves. Regions of large-amplitude waves are constructive interference. Regions of still water are destructive interference.

Sound

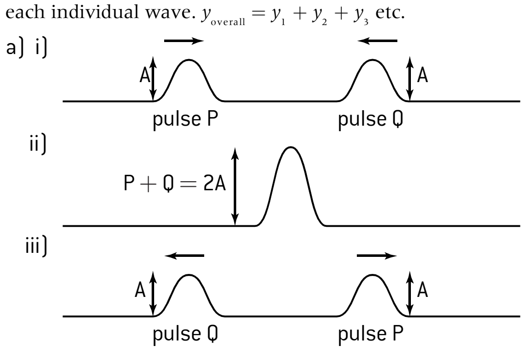
It is possible to analyse any noise in terms of the component frequencies that make it up. A computer can then generate exactly the same frequencies but of different phase. This 'antisound' will interfere with the original sound. An observer in a particular position in space could have the overall noise level reduced if the waves superimposed destructively at that position.

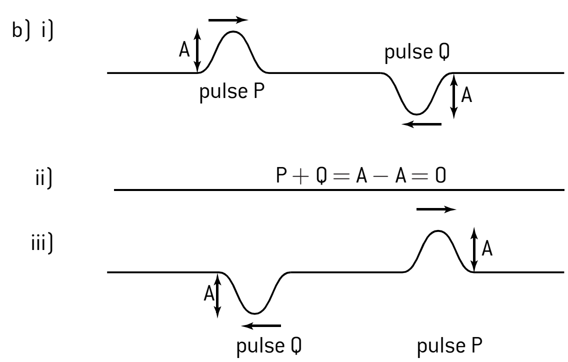
Light

The colours seen on the surface of a soap bubble are a result of constructive and destructive interference of two light rays. One ray is reflected off the outer surface of the bubble whereas the other is reflected off the inner surface.

SUPERPOSITION OF WAVE PULSES

Whenever wave pulses meet, the principle of superposition applies: At any instant in time, the net displacement that results from different waves meeting at the same point in space is just the vector sum of the displacements that would have been produced by each individual wave. $y_{overall} = y_1 + y_2 + y_3$ etc.

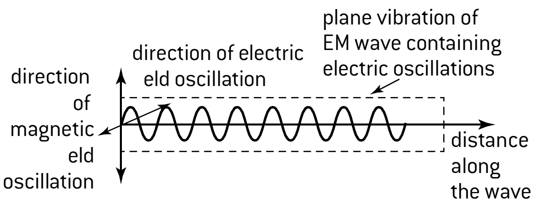




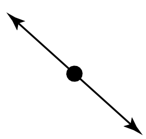
Polarization

POLARIZED LIGHT

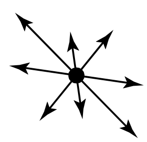
Light is part of the electromagnetic spectrum. It is made up of oscillating electric and magnetic fields that are at right angles to one another (for more details see page 132). They are transverse waves; both fields are at right angles to the direction of propagation. The **plane of vibration** of electromagnetic waves is defined to be the plane that contains the electric field and the direction of propagation.



There are an infinite number of ways for the fields to be oriented. Light (or any EM wave) is said to be **unpolarized** if the plane of vibration varies randomly whereas **plane-polarized** light has a fixed plane of vibration. The diagrams below represent the electric fields of light when being viewed 'head on'.



polarized light: over a period of time, the electric eld only oscillates in one direction



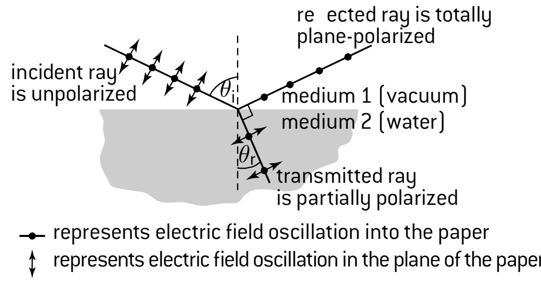
unpolarized light: over a period of time, the electric eld oscillates in random directions

A mixture of polarized light and unpolarized light is **partially plane-polarized**. If the plane of polarization rotates uniformly the light is said to be **circularly polarized**.

Most light sources emit unpolarized light whereas radio waves, radar and laboratory microwaves are often plane-polarized as a result of the processes that produce the waves. Light can be polarized as a result of reflection or selective absorption. In addition, some crystals exhibit **double refraction** or **birefringence** where an unpolarized ray that enters a crystal is split into two plane-polarized beams that have mutually perpendicular planes of polarization.

BREWSTER'S LAW

A ray of light incident on the boundary between two media will, in general, be reflected and refracted. The reflected ray is always partially plane-polarized. If the reflected ray and the refracted ray are at right angles to one another, then the reflected ray is totally plane-polarized. The angle of incidence for this condition is known as the **polarizing angle**.



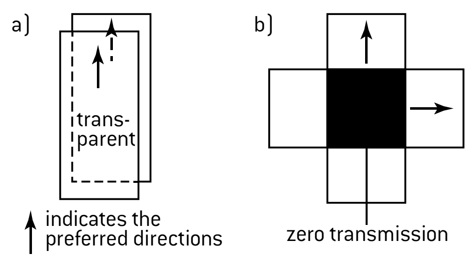
$\theta_i + \theta_r = 90°$

Brewster's law relates the refractive index of medium 2, n, to the incident angle θ_i:

$$n = \frac{\sin\theta_i}{\sin\theta_r} = \frac{\sin\theta_i}{\cos\theta_i} = tan\,\theta_i$$

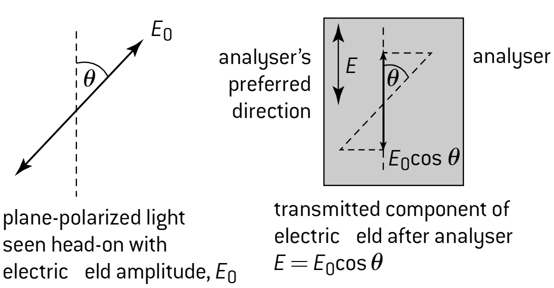
A **polarizer** is any device that produces plane-polarized light from an unpolarized beam. An **analyser** is a polarizer used to detect polarized light.

Polaroid is a material which preferentially absorbs any light in one particular plane of polarization allowing transmission only in the plane at 90° to this.



MALUS'S LAW

When plane-polarized light is incident on an analyser, its preferred direction will allow a component of the light to be transmitted:



The intensity of light is proportional to the (amplitude)^2.

Transmitted intensity $I \propto E^2$

$I \propto E_0{}^2 \cos^2\theta$ as expressed by **Malus's law:**

$I = I_0 \cos^2\theta$

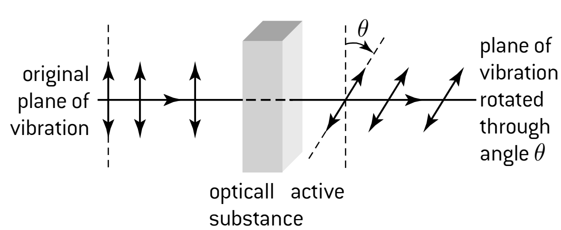
I is transmitted intensity of light in W m^{-2}

I_0 is incident intensity of light in W m^{-2}

θ is the angle between the plane of vibration and the analyser's preferred direction

OPTICALLY ACTIVE SUBSTANCES

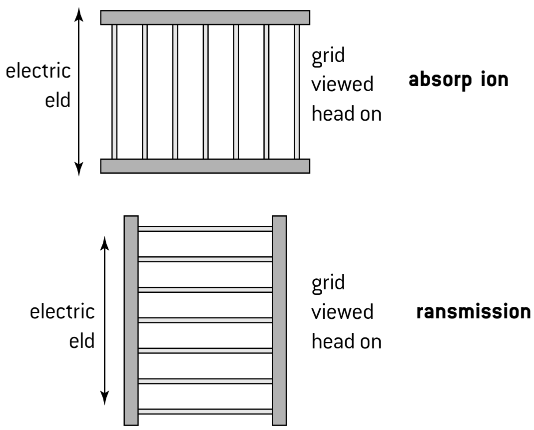
An **optically active** substance is one that rotates the plane of polarization of light that passes through it. Many solutions (e.g. sugar solutions of different concentrations) are optically active.



Uses of polarization

POLAROID SUNGLASSES

Polaroid is a material containing long chain molecules. The molecules selectively absorb light that have electric fields aligned with the molecules in the same way that a grid of wires will selectively absorb microwaves.



When worn normally by a person standing up, Polaroid dark glasses allow light with vertically oscillating electric fields to be transmitted and absorb light with horizontally oscillating electric fields.

- The absorption will mean that the overall light intensity is reduced.
- Light that has re ected from horizontal surfaces will be horizontally plane-polarized to some extent.
- Polaroid sunglasses will preferentially absorb re ected light, reducing 'glare' from horizontal surfaces.

CONCENTRATION OF SOLUTIONS

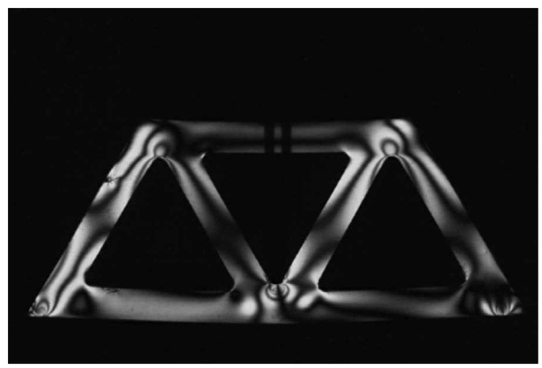
For a given optically active solution, the angle θ through which the plane of polarization is rotated is proportional to:

- The length of the solution through which the plane-polarized light passes.
- The concentration of the solution.

A polarimeter is a device that measures θ for a given solution. It consists of two polarizers (a polarizer and an analyser) that are initially aligned. The optically active solution is introduced between the two and the analyser is rotated to find the maximum transmitted light.

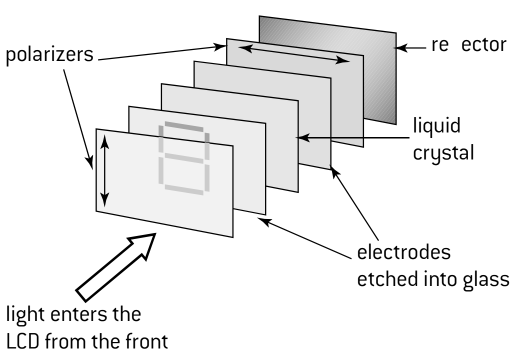
STRESS ANALYSIS

Glass and some plastics become birefringent (see page 41) when placed under stress. When polarized white light is passed through stressed plastics and then analysed, bright coloured lines are observed in the regions of maximum stress.



LIQUID-CRYSTAL DISPLAYS (LCDS)

LCDs are used in a wide variety of different applications that include calculator displays and computer monitors. The liquid crystal is sandwiched between two glass electrodes and is birefringent. One possible arrangement with crossed polarizers surrounding the liquid crystal is shown below:



- With no liquid crystal between the electrodes, the second polarizer would absorb all the light that passed through the first polarizer. The screen would appear black.
- The liquid crystal has a twisted structure and, in the absence of a potential difference, causes the plane of polarization to rotate through 90°.
- This means that light can pass through the second polarizer, reach the reflecting surface and be transmitted back along its original direction.
- With no pd between the electrodes, the LCD appears light.
- A pd across the liquid crystal causes the molecules to align with the electric field. This means less light will be transmitted and this section of the LCD will appear darker.
- The extent to which the screen appears grey or black can be controlled by the pd
- Coloured filters can be used to create a colour image.
- A picture can be built up from individual picture elements.

FURTHER POLARIZATION EXAMPLES

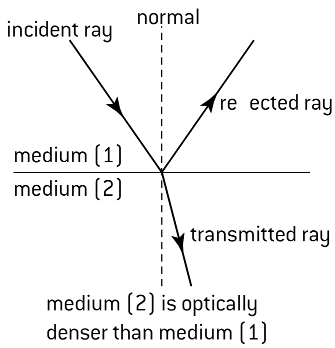
Only transverse waves can be polarized. Page 41 has concentrated on the polarization of light but all EM waves that are transverse are able, in principle, to be polarized.

- Sound waves, being longitudinal waves, cannot be polarized.
- The nature of radio and TV broadcasts means that the signal is often polarized and aerials need to be properly aligned if they are to receive the maximum possible signal strength.
- Microwave radiation (with a typical wavelength of a few cm) can be used to demonstrate wave characteristics in the laboratory. Polarization can be demonstrated using a grid of conducting wires. If the grid wires are aligned parallel to the plane of vibration of the electric field the microwaves will be absorbed. Rotation of the grid through 90° will allow the microwaves to be transmitted.

Wave behaviour – reflec ion

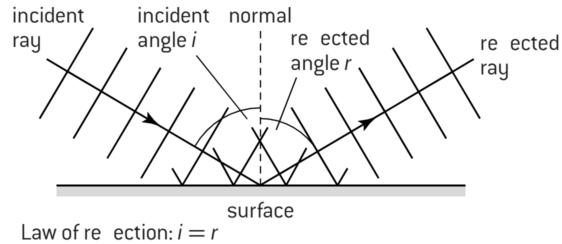
REFLECTION AND TRANSMISSION

In general, when any wave meets the boundary between two different media it is partially reflected and partially transmitted.



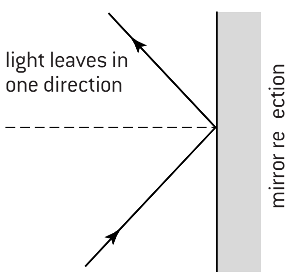
REFLECTION OF TWO-DIMENSIONAL PLANE WAVES

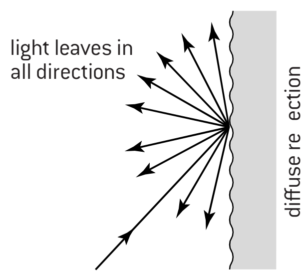
The diagram below shows what happens when plane waves are reflected at a boundary. When working with rays, by convention we always measure the angles between the rays and the **normal**. The normal is a construction line that is drawn at right angles to the surface.



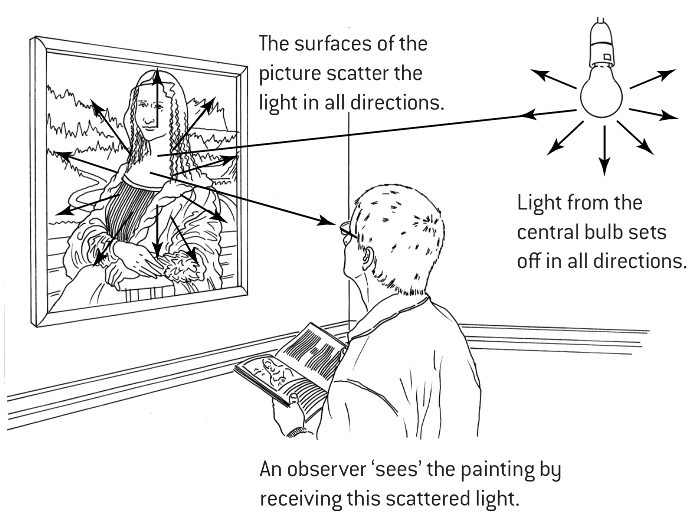
TYPES OF REFLECTION

When a single ray of light strikes a smooth mirror it produces a single reflected ray. This type of 'perfect' reflection is very different to the reflection that takes place from an uneven surface such as the walls of a room. In this situation, a single incident ray is generally scattered in all directions. This is an example of a **diffuse** reflection.





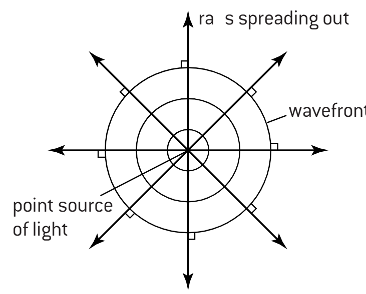
We see objects by receiving light that has come from them. Most objects do not give out light by themselves so we cannot see them in the dark. Objects become visible with a source of light (e.g. the Sun or a light bulb) because diffuse reflections have taken place that scatter light from the source towards our eyes.



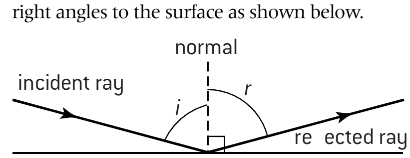
Our brains are able to work out the location of the object by assuming that rays travel in straight lines.

LAW OF REFLECTION

The location and nature of optical images can be worked out using ray **diagrams** and the principles of **geometric optics**. A ray is a line showing the direction in which light energy is propagated. The ray must always be at right angles to the wavefront. The study of geometric optics ignores the wave and particle nature of light.



When a mirror reflection takes place, the direction of the reflected ray can be predicted using the laws of reflection. In order to specify the ray directions involved, it is usual to measure all angles with respect to an imaginary construction line called the **normal**. For example, the incident angle is always taken as the angle between the incident ray and the normal. The normal to a surface is the line at right angles to the surface as shown below.



The laws of reflection are that:

- the incident angle is equal to the reflected angle
- the incident ray, the reflected ray and the normal all lie in the same plane (as shown in the diagram).

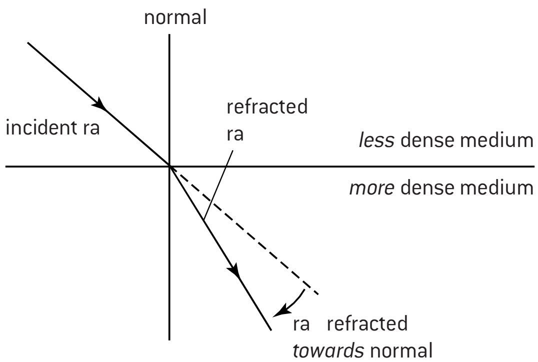
The second statement is only included in order to be precise and is often omitted. It should be obvious that a ray arriving at a mirror (such as the one represented above) is not suddenly reflected in an odd direction (e.g. out of the plane of the page).

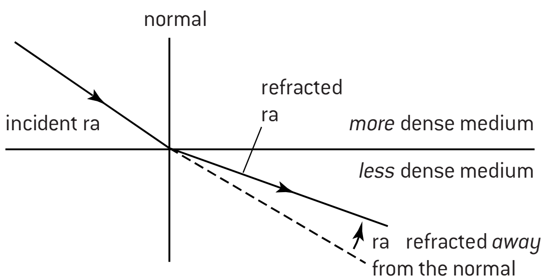
Snell's law and refractive inde

REFRACTIVE INDEX AND SNELL'S LAW

Re raction takes place at the boundary between two media. In general, a wave that crosses the boundary will undergo a change o direction. The reason or this change in direction is the change in wave speed that has taken place.

As with reflection, the ray directions are always specified by considering the angles between the ray and **the normal**. I a ray travels into an optically denser medium (e.g. rom air into water), then the ray o light is re racted **towards** the normal. I the ray travels into an optically less dense medium then the ray o light is re racted **away from** the normal.





Snell's law allows us to work out the angles involved. When a ray is re racted between two di erent media, the ratio $\frac{\sin(\text{angle o incidence})}{\sin(\text{angle o re raction})}$ is a constant.

The constant is called the re ractive index n between the two media. This ratio is equal to the ratio o the speeds o the waves in the two media.

$$\frac{\sin i}{\sin r} = n$$

I the re ractive index or a particular substance is given as a particular number and the other medium is not mentioned then you can assume that the other medium is air (or to be absolutely correct, a vacuum). Another way o expressing this is to say that the re ractive index o air can be taken to be 1.0.

For example the re ractive index or a type o glass might be given as

$$n_{\text{glass}} = 1.34$$

This means that a ray entering the glass rom air with an incident angle o 40° would have a re racted angle given by

$$\sin r = \frac{\sin 40°}{1.34} = 0.4797$$

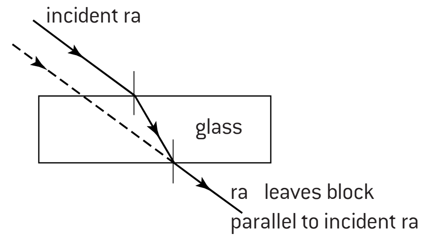
$$\therefore r = 28.7°$$

$$n_{\text{glass}} = \frac{n_{\text{glass}}}{n_{\text{air}}} = \frac{\sin \theta_{\text{air}}}{\sin \theta_{\text{glass}}} = \frac{V_{\text{air}}}{V_{\text{glass}}}$$

EXAMPLES

1. Parallel-sided block

A ray will always leave a parallel-sided block travelling in a parallel direction to the one with which it entered the block. The overall e ect o the block has been to move the ray sideways. An example o this is shown below.



2. Ray travelling between two media

I a ray goes between two di erent media, the two individual re ractive indices can be used to calculate the overall re raction using the ollowing equation

$n_1 \sin \theta_1 = n_2 \sin \theta_2$ or $\frac{n_1}{n_2} = \frac{\sin \theta_2}{\sin \theta_1}$

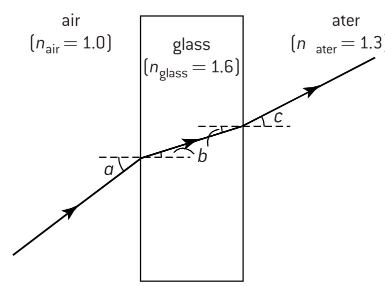
n_1 re ractive index o medium 1

θ_1 angle in medium 1

n_2 re ractive index o medium 2

θ_2 angle in medium 2

Suppose a ray o light is shone into a fish tank that contains water. The re raction that takes place would be calculated as shown below:



1st re raction:

$$n_{\text{glass}} = \frac{\sin a}{\sin b}$$

2nd re raction:

$$n_{\text{glass}} \times \sin b = n_{\text{water}} \times \sin c$$

$$\frac{n_{\text{glass}}}{n_{\text{water}}} = \frac{\sin c}{\sin b}$$

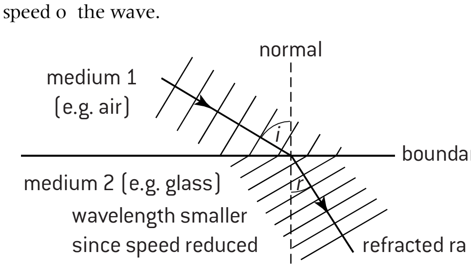
Overall the re raction is rom incident angle a to re racted angle c.

i.e. $$n_{\text{overall}} = \frac{\sin a}{\sin c} = \frac{\sin a}{\sin b} \times \frac{\sin b}{\sin c}$$

$$= n_{\text{water}}$$

REFRACTION OF PLANE WAVES

The reason or the change in direction in re raction is the change in speed o the wave.



Snell's law (an experimental law o re raction) states that

the ratio $\frac{\sin i}{\sin r}$ = constant, or a given requency.

The ratio is equal to the ratio o the speeds in the di erent media

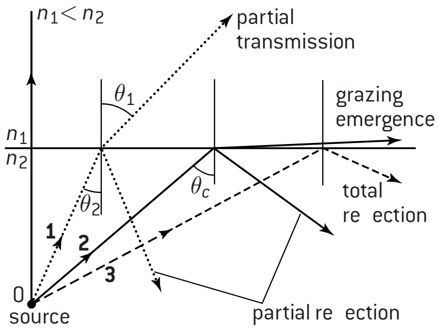
$$\frac{n_1}{n_2} = \frac{\sin \theta_2}{\sin \theta_1} = \frac{V_2}{V_1} \begin{array}{l} \leftarrow \text{speed o wave in medium 2} \\ \leftarrow \text{speed o wave in medium 1} \end{array}$$

Refrac ion and cri ical angle

TOTAL INTERNAL REFLECTION AND CRITICAL ANGLE

In general, both reflection and re raction can happen at the boundary between two media.

It is, under certain circumstances, possible to guarantee complete (total) reflection with no transmission at all. This can happen when a ray meets the boundary and it is travelling in the denser medium.



Ray1 This ray is partially reflected and partially re racted.

Ray2 This ray has a re racted angle o nearly 90°. The **critical ray** is the name given to the ray that has a re racted angle o 90°. The **critical angle** is the angle o incidence θ_c or the critical ray.

Ray3 This ray has an angle o incidence **greater** than the critical angle. Re raction cannot occur so the ray must be totally reflected at the boundary and stay inside medium 2. The ray is said to be **totally internally reflected**.

The critical angle can be worked out as ollows. For the critical ray,

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

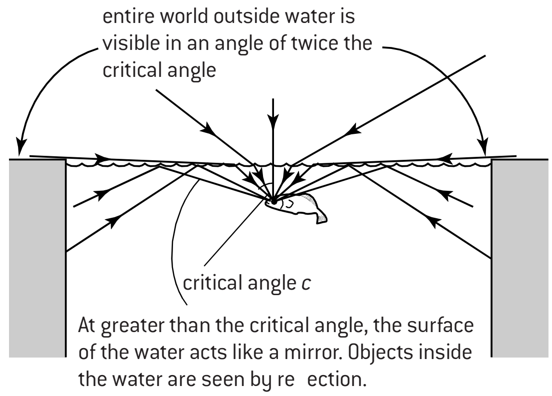
$$\theta_1 = 90°$$

$$\theta_2 = \theta_c$$

$$\therefore \sin \theta_c = \frac{1}{n_2}$$

EXAMPLES

1. What a sh sees under water

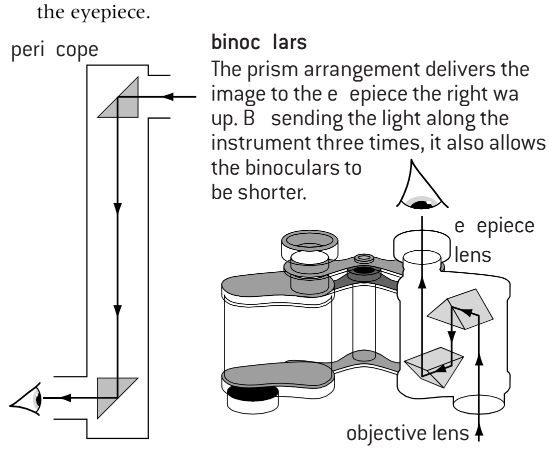


2. Prismatic reflectors

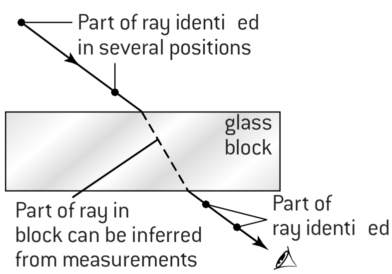
A prism can be used in place o a mirror. I the light strikes the sur ace o the prism at greater than the critical angle, it must be totally internally reflected.

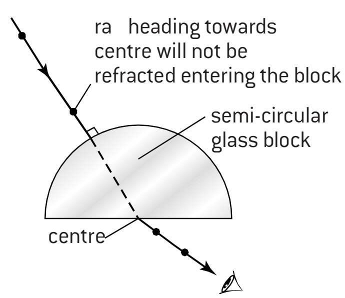
Prisms are used in many optical devices. Examples include:

- periscopes – the double reflection allows the user to see over a crowd.
- binoculars – the double reflection means that the binoculars do not have to be too long
- SLR cameras – the view through the lens is reflected up to the eyepiece.



METHODS FOR DETERMINING REFRACTIVE INDEX EXPERIMENTALLY





1. Locate paths taken by di erent rays either by sending a ray through a solid and measuring its position or aligning objects by eye. Uncertainties in angle measurement are dependent on protractor measurements. (See diagrams on le t)
2. Use a travelling microscope to measure real and apparent depth and apply ollowing ormula:

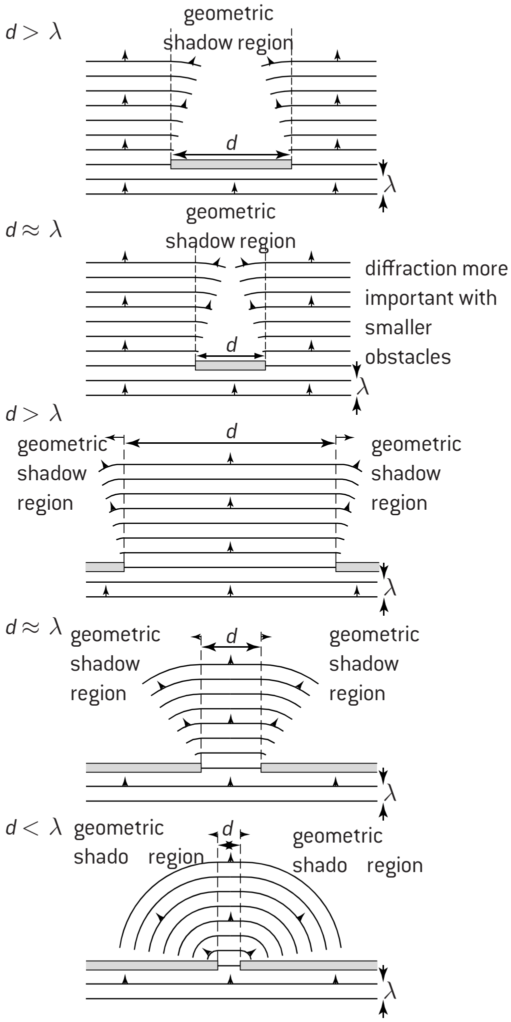
$$n = \frac{\text{real depth o object}}{\text{apparent depth o object}}$$

3. Very accurate measurements o angles o re raction can be achieved using a prism o the substance and a custom piece o equipment call a *spectrometer*.

Diffrac ion

DIFFRACTION

When waves pass through apertures they tend to spread out. Waves also spread around obstacles. This wave property is called **diffraction**.



d = width of obstacle/gap

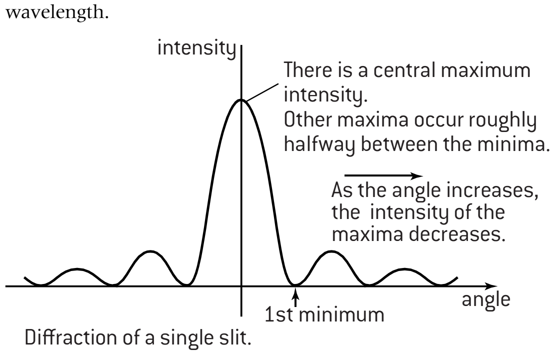
Diffraction – wave energy is received in geometric shadow region.

There are some important points to note from these diagrams.

- Diffraction becomes relatively more important when the wavelength is large in comparison to the size of the aperture (or the object).
- The wavelength needs to be of the same order of magnitude as the aperture for diffraction to be noticeable.

BASIC OBSERVATIONS

Diffraction is a wave effect. The objects involved (slits, apertures, etc.) have a size that is of the same order of magnitude as the wavelength.



Diffraction of a single slit.

PRACTICAL SIGNIFICANCE OF DIFFRACTION

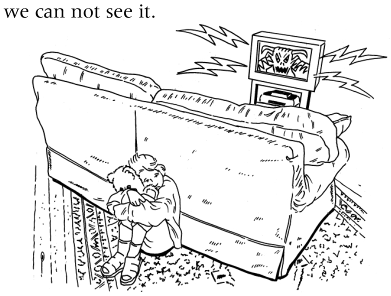
Whenever an observer receives information from a source of electromagnetic waves, diffraction causes the energy to spread out. This spreading takes place as a result of any obstacle in the way and the width of the device receiving the electromagnetic radiation. Two sources of electromagnetic waves that are angularly close to one another will both spread out and interfere with one another. This can affect whether or not they can be resolved (see page 101).

Diffraction effects mean that it is impossible ever to see atoms because they are smaller than the wavelength of visible light, meaning that light will diffract around the atoms. It is, however, possible to image atoms using smaller wavelengths. Practical devices where diffraction needs to be considered include:

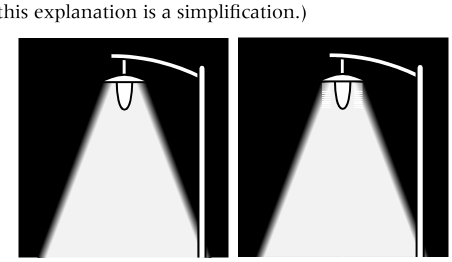
- CDs and DVDs – the maximum amount of information that can be stored depends on the size and the method used for recording information.
- The electron microscope – resolves items that cannot be resolved using a light microscope. The electrons have an effective wavelength that is much smaller than the wavelength of visible light (see page 127).
- Radio telescopes – the size of the dish limits the maximum resolution possible. Several radio telescopes can be linked together in an array to create a virtual radio telescope with a greater diameter and with a greater ability to resolve astronomical objects. (See page 181)

EXAMPLES OF DIFFRACTION

Diffraction provides the reason why we can hear something even if we can not see it.



If you look at a distant street light at night and then squint your eyes the light spreads sideways – this is as a result of diffraction taking place around your eyelashes! (Needless to say, this explanation is a simplification.)



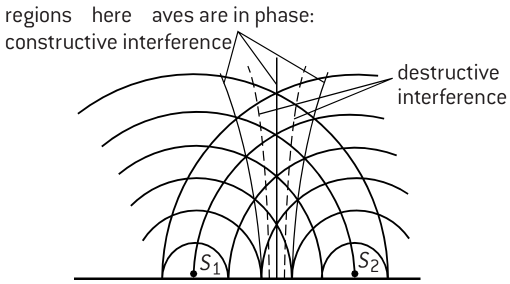
T o-source interference of aves

PRINCIPLES OF THE TWO-SOURCE INTERFERENCE PATTERN

Two-source interference is simply another application of the principle of superposition, for two coherent sources having roughly the same amplitude.

Two sources are coherent if:

- they have the same frequency
- there is a constant phase relationship between the two sources.

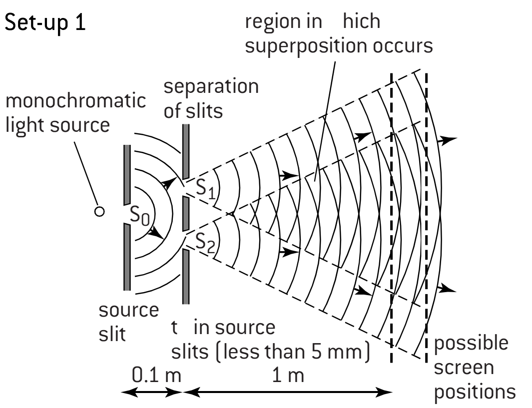


Two dippers in water moving together are coherent sources. This forms regions of water ripples and other regions with no waves.

Two loudspeakers both connected to the same signal generator are coherent sources. This forms regions of loud and soft sound.

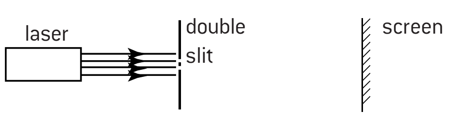
A set-up for viewing two-source interference with light is shown below. It is known as **Young's double slit** experiment. A **monochromatic** source of light is one that gives out only one frequency. Light from the twin slits (the sources) interferes and patterns of light and dark regions, called **fringes**, can be seen on the screen.

Set-up 1

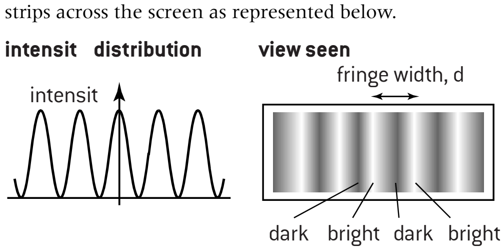


Set-up 2

The use of a laser makes the set-up easier.



The experiment results in a regular pattern of light and dark strips across the screen as represented below.

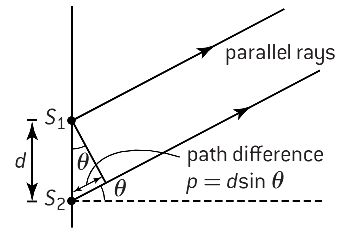


MATHEMATICS

The location of the light and dark fringes can be mathematically derived in one of two ways. The derivations do not need to be recalled.

Method 1

The simplest way is to consider two parallel rays setting off from the slits as shown below.



If these two rays result in a bright patch, then the two rays must arrive in phase. The two rays of light started out in phase but the light from source 2 travels an extra distance. This extra distance is called the **path difference**.

Constructive interference can only happen if the path difference is a whole number of wavelengths. Mathematically,

Path difference $= n\,\lambda$

[where n is an integer e.g. 1, 2, 3 etc.]

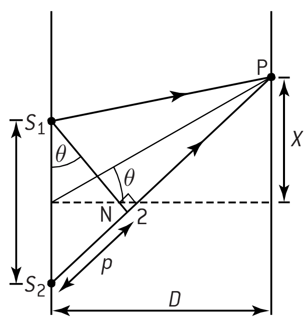
From the geometry of the situation

Path difference $= d \sin\theta$

In other words $n\,\lambda = d \sin\theta$

Method 2

If a screen is used to make the fringes visible, then the rays from the two slits cannot be absolutely parallel, but the physical set-up means that this is effectively true.



$\sin\theta = \frac{p}{s}$

$\tan\theta = \frac{X}{D}$

If θ is small $\sin\theta$ $\tan\theta$

so $\frac{p}{s} = \frac{X}{D}$

$\therefore \quad p = \frac{Xs}{D}$

For constructive interference:

$p = n\lambda$

$\therefore \quad n\lambda = \frac{X_n s}{D}$

$\therefore \quad X_n = \frac{n\lambda D}{s}$

fringe separation $d = X_{n+1} - X_n = \frac{\lambda D}{s}$

$\therefore \quad s = \frac{\lambda D}{d}$

This equation only applies when the angle is small.

Example

Laser light of wavelength 450 nm is shone on two slits that are 0.1 mm apart. How far apart are the fringes on a screen placed 5.0 m away?

$$d = \frac{\lambda\,D}{s} = \frac{4.5 \times 10^{-7} \times 5}{1.0 \times 10^{-4}} = 0.0225 \text{ m} = 2.25 \text{ cm}$$

Nature and production of standing (stationary) aves

STANDING WAVES

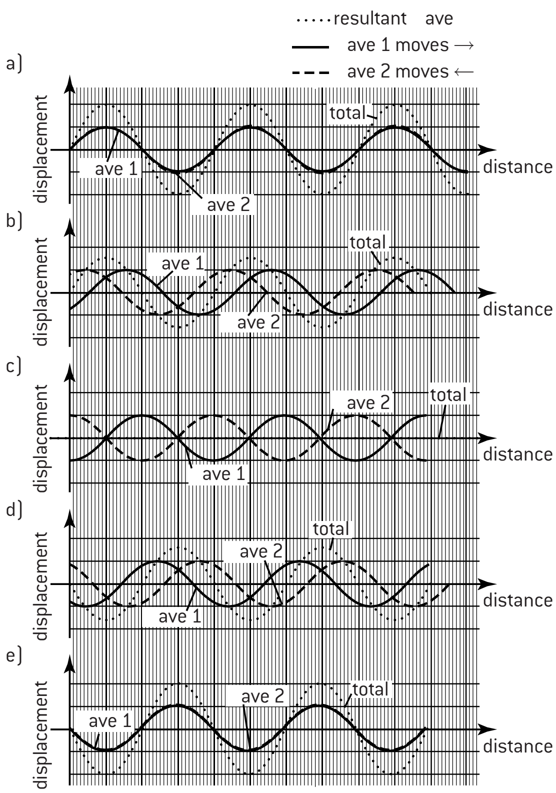
A special case of interference occurs when two waves meet that are:

- of the same amplitude
- of the same frequency
- travelling in opposite directions.

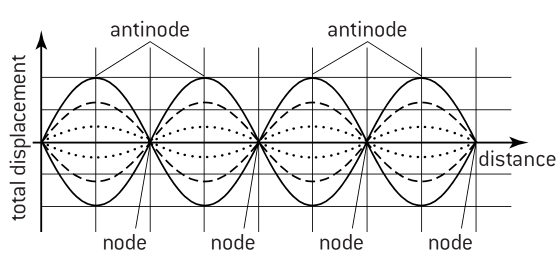
In these conditions a **standing wave** will be formed.

The conditions needed to form standing waves seem quite specialized, but standing waves are in fact quite common. They often occur when a wave reflects back from a boundary along the route that it came. Since the reflected wave and the incident wave are of (nearly) equal amplitude, these two waves can interfere and produce a standing wave.

Perhaps the simplest way of picturing a standing wave would be to consider two transverse waves travelling in opposite directions along a stretched rope. The series of diagrams below shows what happens.



Production of standing waves



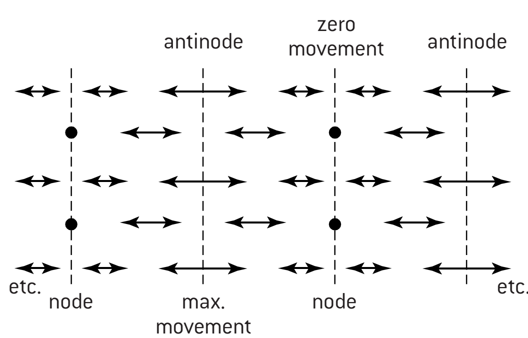
A standing wave – the pattern remains fixed

There are some points on the rope that are always at rest. These are called the **nodes**. The points where the maximum movement takes place are called **antinodes**. The resulting standing wave is so called because the wave pattern remains fixed in space – it is its amplitude that changes over time. A comparison with a normal (travelling) wave is given below.

	Stationary wave	Normal (travelling) wave
Amplitude	All points on the wave have different amplitudes. The maximum amplitude is $2A$ at the antinodes. It is zero at the nodes.	All points on the wave have the same amplitude.
Frequency	All points oscillate with the same frequency.	All points oscillate with the same frequency.
Wavelength	This is **twice** the distance from one node (or antinode) to the next node (or antinode).	This is the shortest distance (in metres) along the wave between two points that are in phase with one another.
Phase	All points between one node and the next node are moving in phase.	All points along a wavelength have different phases.
Energy	Energy is not transmitted by the wave, but it does have an energy associated with it.	Energy is transmitted by the wave.

Although the example left involved transverse waves on a rope, a standing wave can also be created using sound or light waves. All musical instruments involve the creation of a standing sound wave inside the instrument. The production of laser light involves a standing light wave. Even electrons in hydrogen atoms can be explained in terms of standing waves.

A standing longitudinal wave can be particularly hard to imagine. The diagram below attempts to represent one example – a standing sound wave.



A longitudinal standing wave

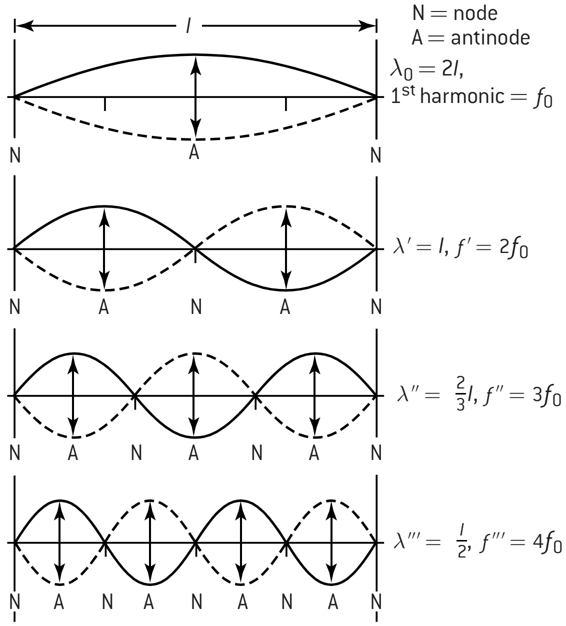
Boundar   conditions

BOUNDARY CONDITIONS

The boundary conditions o the system speci y the conditions that must be met at the edges (the boundaries) o the system when standing waves are taking place. Any standing wave that meets these boundary conditions will be a possible resonant mode o the system.

1. Transverse   aves on a string

I the string is fixed at each end, the ends o the string cannot oscillate. Both ends o the string would reflect a travelling wave and thus a standing wave is possible. The only standing waves that fit these boundary conditions are ones that have nodes at each end. The diagrams below show the possible resonant modes.



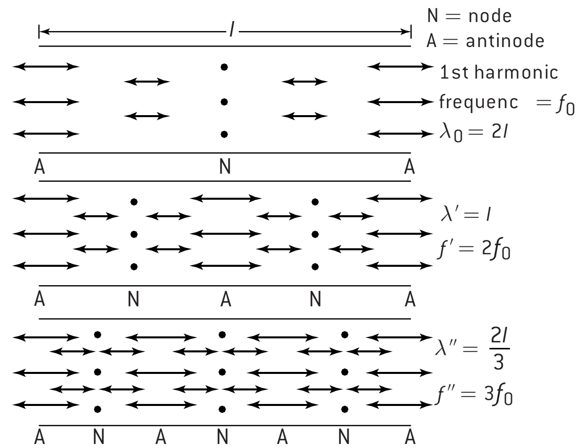
Harmonic modes or a string

The resonant mode that has the lowest requency is called the undamental or the **first harmonic**. Higher resonant modes are called **harmonics**. Many musical instruments (e.g. piano, violin, guitar etc.) involve similar oscillations o metal 'strings'.

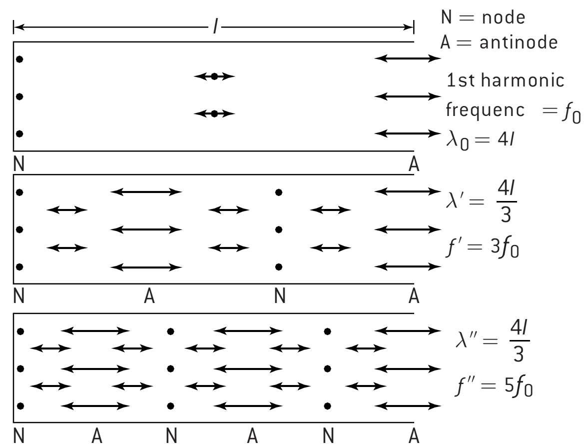
2. Longitudinal sound   aves in a pipe

A longitudinal standing wave can be set up in the column o air enclosed in a pipe. As in the example above, this results rom the reflections that take place at both ends.

As be ore, the boundary conditions determine the standing waves that can exist in the tube. A closed end must be a displacement node. An open end must be an antinode. Possible standing waves are shown or a pipe open at both ends and a pipe closed at one end.



Harmonic modes or a pipe open at both ends



Harmonic modes or a pipe closed at one end

Musical instruments that involve a standing wave in a column o air include the flute, the trumpet, the recorder and organ pipes.

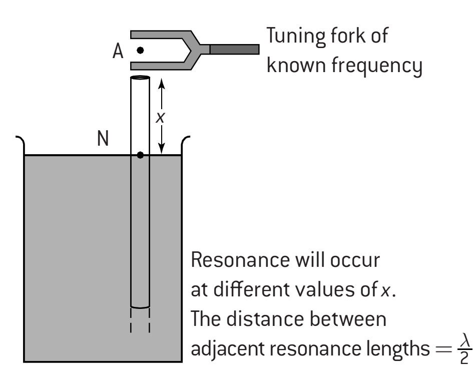
EXAMPLE

An organ pipe (open at one end) is 1.2 m long.

Calculate its undamental requency.
The speed o sound is 330 m s⁻¹.

$$l = 1.2 \text{ m} \quad \therefore \quad \frac{\lambda}{4} = 1.2 \text{ m (first harmonic)}$$
$$\therefore \quad \lambda = 4.8 \text{ m}$$
$$v = f\lambda$$
$$f = \frac{330}{4.8} \simeq 69 \text{ Hz}$$

RESONANCE TUBE



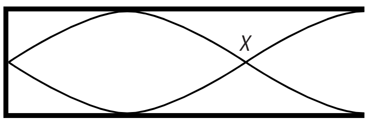
IB Quest ons – waves

1. A surfer is out beyond the breaking surf in a deep-water region where the ocean waves are sinusoidal in shape. The crests are 20 m apart and the surfer rises a vertical distance of 4.0 m from wave **trough** to **crest**, in a time of 2.0 s. What is the speed of the waves?

   A. 1.0 m s^{-1} B. 2.0 m s^{-1}

   C. 5.0 m s^{-1} D. 10.0 m s^{-1}

2. A standing wave is established in air in a pipe with one closed and one open end.



   The air molecules near X are

   A. always at the centre of a compression.

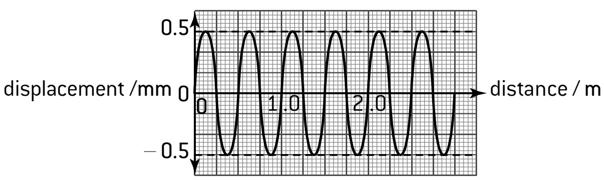
   B. always at the centre of a rarefaction.

   C. sometimes at the centre of a compression and sometimes at the centre of a rarefaction.

   D. never at the centre of a compression or a rarefaction.

3. This question is about sound waves.

   A sound wave of frequency 660 H passes through air. The variation of particle displacement withdistance along the wave at one instant of time is shown below.



   a) State whether this wave is an example of a longitudinal or a transverse wave. [1]

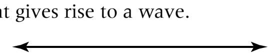
   b) Using data from the above graph, deduce for this sound wave,

      (i) the wavelength. [1]

      (ii) the amplitude. [1]

      (iii) the speed. [2]

4. The diagram below represents the direction of oscillation of a disturbance that gives rise to a wave.

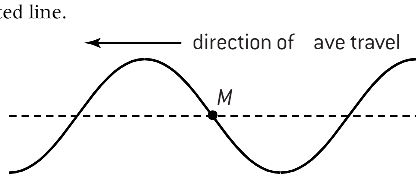


   a) By redrawing the diagram, add arrows to show the direction of wave energy transfer to illustrate the difference between

      (i) a transverse wave and [1]

      (ii) a longitudinal wave. [1]

   A wave travels along a stretched string. The diagram below shows the variation with distance along the string of the displacement of the string at a particular instant in time. A small marker is attached to the string at the point labelled M. The undisturbed position of the string is shown as a dotted line.



   b) On the diagram above

      (i) draw an arrow to indicate the direction in which the marker is moving. [1]

      (ii) indicate, with the letter A, the amplitude of the wave. [1]

      (iii) indicate, with the letter λ, the wavelength of the wave. [1]

      (iv) draw the displacement of the string a time $\frac{T}{4}$ later, where T is the period of oscillation of the wave. Indicate, with the letter N, the new position of the marker. [2]

   The wavelength of the wave is 5.0 cm and its speed is 10 cm s^{-1}.

   c) Determine

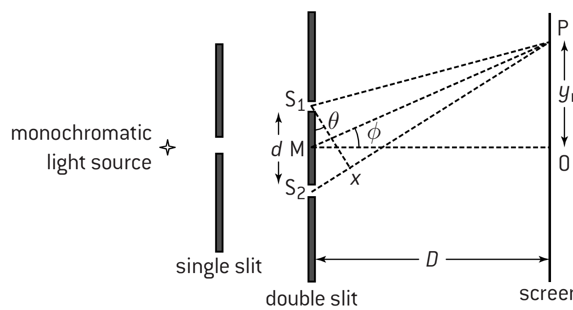
      (i) the frequency of the wave. [1]

      (ii) how far the wave has moved in $\frac{T}{4}$ s. [2]

   Interference of waves

   d) By reference to the principle of superposition, explain what is meant by constructive interference. [4]

   The diagram below (not drawn to scale) shows an arrangement for observing the interference pattern produced by the light from two narrow slits S_1 and S_2.



   The distance S_1S_2 is d, the distance between the double slit and screen is D and $D \gg d$ such that the angles θ and ϕ shown on the diagram are small. M is the mid-point of S_1S_2 and it is observed that there is a bright fringe at point P on the screen, a distance $_n$ from point O on the screen. Light from S_2 travels a distance S_2X further to point P than light from S_1.

   e) (i) State the condition in terms of the distance S_2X and the wavelength of the light λ, for there to be a bright fringe at P. [2]

      (ii) Deduce an expression for θ in terms of S_2X and d. [2]

      (iii) Deduce an expression for ϕ in terms of D and $_n$. [1]

   For a particular arrangement, the separation of the slits is 1.40 mm and the distance from the slits to the screen is 1.50 m. The distance $_n$ is the distance of the eighth bright fringe from O and the angle $\theta = 2.70 \times 10^{-3}$ rad.

   f) Using your answers to (e) to determine

      (i) the wavelength of the light. [2]

      (ii) the separation of the fringes on the screen. [3]

5. A bright source of light is viewed through two polarisers whose preferred directions are initially parallel. Calculate the angle through which one sheet should be turned to reduce the transmitted intensity to half its original value.