4.3 Wave characteristics

Understanding

- Wavefronts and rays
- Amplitude and intensity
- Superposition
- Polarization

Applications and skills

- Sketching and interpreting diagrams involving wavefronts and rays
- Solving problems involving amplitude and intensity
- Sketching and interpreting the superposition of pulses and waves
- Describing methods of polarization
- Sketching and interpreting diagrams illustrating polarized reflected and transmitted beams
- Solving problems involving Malus's law

Equations

- Relationship between intensity and amplitude: $I \propto A^2$
- Malus's law: $I = I_0 \cos^2 \theta$

Nature of science

Imagination and physics

"Imagination ... is more important than knowledge. Knowledge is limited. Imagination encircles the world[1]."

Einstein was famous for his *gedanken* "thought" experiments and one of the qualities that makes a great physicist is surely a hunger to ask the question "what if ... ?". Mathematics is a crucial tool for the physicist and it is central to what a physicist does, to be able to quantify an argument. However, imagination can also play a major role in interpreting the results. Without imagination it is hard to believe that we would have Huygens' principle, Newton's law of gravitation, or Einstein's theory of special relativity. To visualize the abstract and apply theory to a practical situation is a flair that is fundamental to being an extraordinary physicist.

[1]Viereck, George Sylvester (October 26, 1929). 'What life means to Einstein: an interview'. *The Saturday Evening Post*.

Introduction

Wavefronts and rays are visualizations that help our understanding of how waves behave in different circumstances. By drawing ray or wave diagrams using simple rules we can predict how waves will behave when they encounter obstacles or a different material medium.

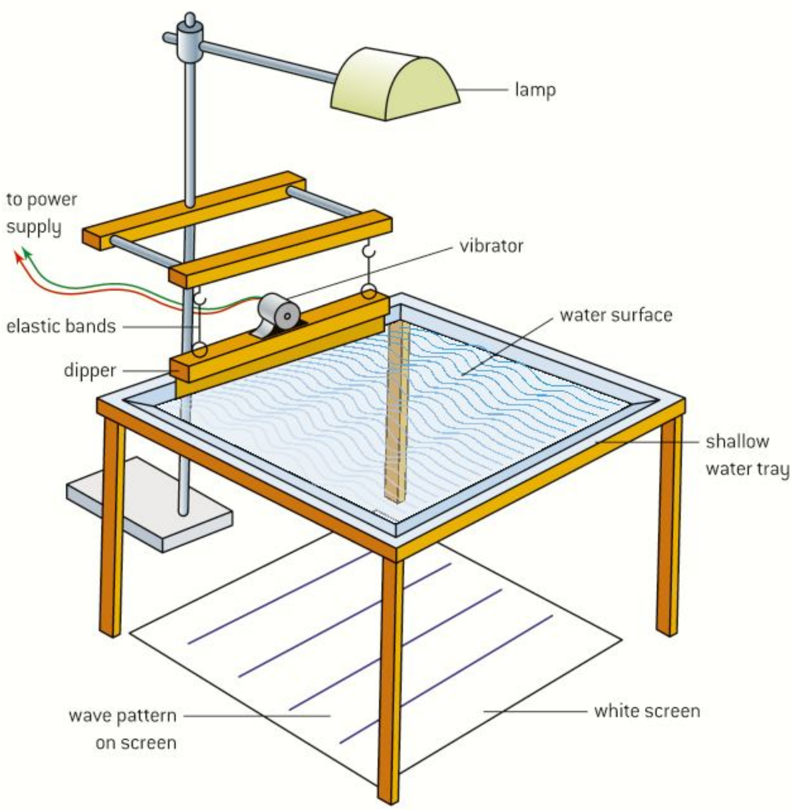
Definitions of these two quantities are:

- A **wavefront** is a surface that travels with a wave and is perpendicular to the direction in which the wave travels – the ray.
- A **ray** is a line showing the direction in which a wave transfers energy and is, of course, perpendicular to a wavefront.

The distance between two consecutive wavefronts is one wavelength (λ).

The motion of wavefronts

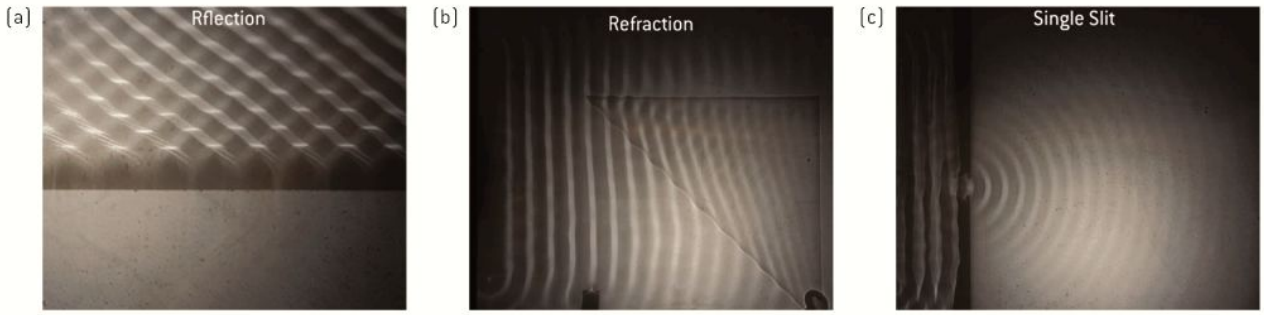
One of the simplest ways to demonstrate the motion of wavefronts is to use a ripple tank such as that shown in figure 1. This is simply a glass-bottomed tank that contains water, illuminated from above. Any waves on the water focus the light onto a screen often placed below the tank. The bright patches result from the crests focusing the light and the dark patches from the troughs defocusing the light. Using a vibrating dipper, plane or circular waves can be produced allowing us to see what happens to wavefronts in different situations.



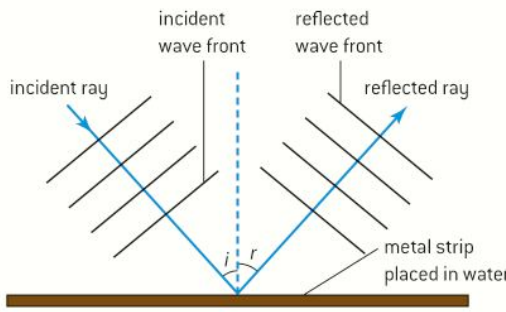
▲ Figure 1 A ripple tank.

The images shown in figure 2 show the effect on the wavefronts as they meet (a) a plane barrier, (b) a shallower region over a prism-shaped glass and (c) a single narrow slit.

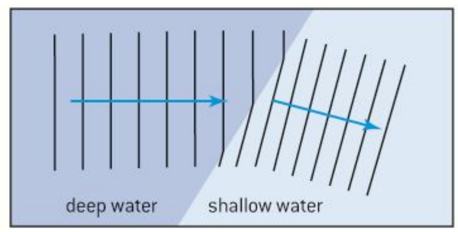
Using wavefront or ray diagrams we can illustrate how waves behave when they are reflected, refracted, and diffracted. We will refer to these diagrams in the sections of this topic following on from this.



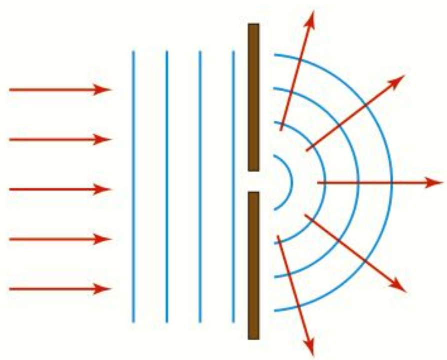
▲ Figure 2 Reflection, refraction, and diffraction of waves in a ripple tank.



▲ Figure 3a Wavefront and ray diagrams for reflection of waves.



▲ Figure 3b Wavefront and ray diagrams for refraction of waves.



▲ Figure 3c Wavefront and ray diagrams for diffraction of waves by a single slit.

In figure 3a we can see that there is no change of wavelength and that the angle of incidence (i) is equal to the angle of reflection (r).

In figure 3b we see the wave slowing down and bending as it enters the denser medium. The wavelength of the wave in the denser medium is shorter than in the less dense medium – but the frequency remains unchanged (although it is not possible to tell this from the the pattern shown in the ripple tank).

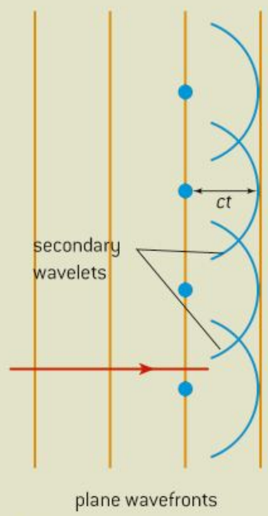
Figure 3c shows diffraction where the wave spreads out on passing through the slit but there is no change in the wavelength.

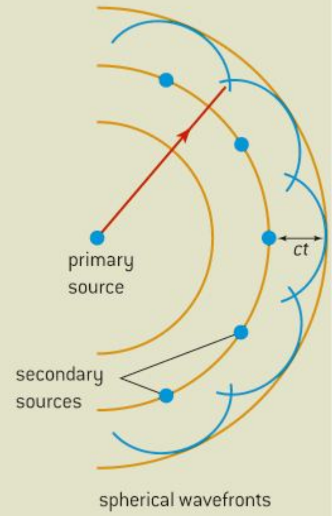
Nature of Science

Huygens' principle

One of the ways to predict what will happen to wavefronts under different circumstances is to use Huygens' principle. This was suggested in 1678 by the Dutch physicist Christiaan Huygens: the wavefront of a travelling wave at a particular instant consists of the tangent to circular wavelets given out by each point on the previous wavefront as shown in figure 4. In this way the wavefront travels forward with a velocity c.

Huygens was able to derive the laws of reflection and refraction from his principle. What the principle does not explain is why an expanding circular (but really spherical) wave continues to expand outwards from its source rather than travel back and focus on the source. The French physicist Augustin Fresnel (1788–1827) adapted Huygens' principle to explain diffraction by proposing the principle of superposition, discussed in Sub-topic 4.4. The Huygens–Fresnel principle (as the overall principle should more accurately be called) is useful in explaining many wave phenomena.





▲ Figure 4 Huygens' principle.

The intensity of waves

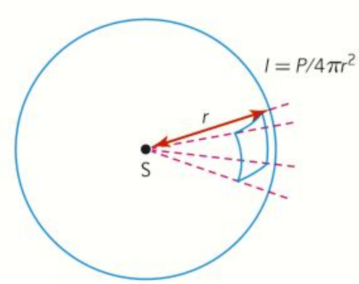
The loudness of a sound wave or the brightness of a light depends on the amount of energy that is received by an observer. For example, when a guitar string is plucked more forcefully the string does more work on the air and so there will be more energy in the sound wave. In a similar way, to make a filament lamp glow more brightly requires more electrical energy. The energy E is found to be proportional to the square of the amplitude A:

$$E \propto A^2$$

So doubling the amplitude increases the energy by a factor of four; tripling the amplitude increases the energy by a factor of nine, etc.

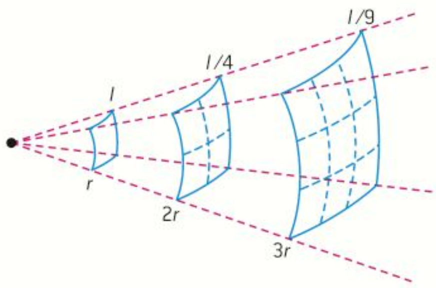
Loudness is the observer's perception of the intensity of a sound and brightness that of light; loudness and brightness are each affected by frequency.

If we picture waves being emitted by a point source, S, they will spread out in all directions. This will mean that the total energy emitted will be spread increasingly thinly the further we go from the source. Figure 5 shows how the energy spreads out over the surface area of a sphere.



▲ Figure 5 Energy spreading out from a point source.

In order to make intensity comparisons more straightforward it is usual to use the idea of the energy transferred per second – this is the *power* (P) of the source. This means that the intensity (I) at a distance (r)



▲ Figure 6 Inverse square law.

from a point source is given by the power divided by the surface area of the sphere at that radius:

$$I = \frac{P}{4\pi r^2}$$

This equation shows that intensity has an **inverse-square relationship** with distance from the point source; this means that, as the distance doubles, the intensity falls to a quarter of the previous value and when the distance the is tripled, the intensity falls to one nineth as shown in figure 6.

The SI unit for intensity is W m^{-2}.

Worked example

At a distance of 15 m from the source, the intensity of a loud sound is 2.0×10^{-4} W m^{-2}.

a) Show that the intensity at 120 m from the source is approximately 3×10^{-6} W m^{-2}.

b) Deduce how the amplitude of the wave changes.

Solution

a) Using the equation $I = \frac{P}{4\pi r^2}$, $\frac{P}{4\pi}$ remains constant so $I_1 r_1^2 = I_2 r_2^2$ this gives $2.0 \times 10^{-4} \times 15^2 = I_2 \times 120^2$

$I_2 = 2.0 \times 10^{-4} \times \frac{15^2}{120^2} = 3.1 \times 10^{-6}$ W $m^{-2} \approx$

3×10^{-6} W m^{-2}

Note

In "show that" questions there is an expectation that you will give a detailed answer, showing all your working and that you will give a final answer to more significant figures than the data in the question – actually this is good practice for any answer!

b) With the intensity changing there must be a change of amplitude. The intensity is proportional to the square of the amplitude so:

$$\frac{I_2}{I_2} = \frac{A_1^2}{A_2^2} \text{ or } \frac{A_1}{A_2} = \sqrt{\frac{I_2}{I_2}} = \sqrt{\frac{2.0 \times 10^{-4}}{3.1 \times 10^{-6}}} = 8.0$$

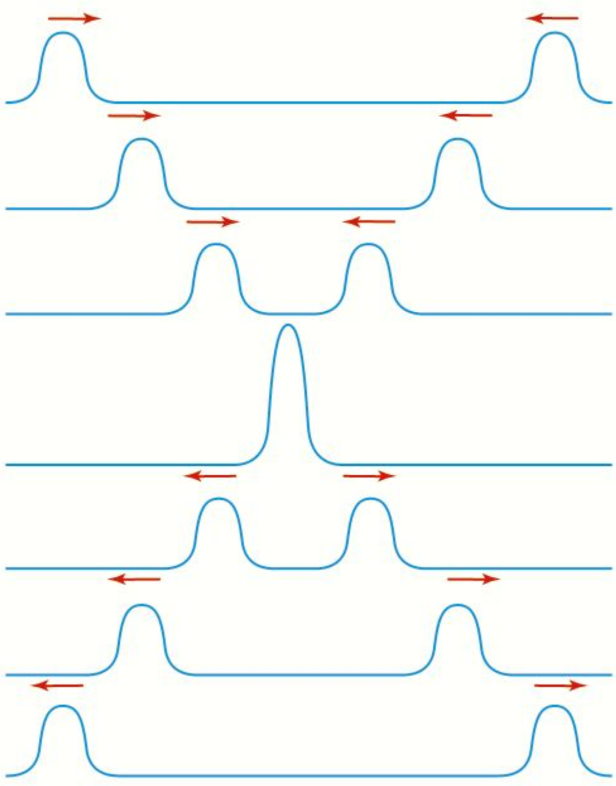
Thus the amplitude at 15 m from the source is 8.0 times that at 120 m from the source. Another way of looking at this is to say that A is proportional to $\frac{1}{r}$.

Note

Because the previous part was a "show that" question you had all the data needed to answer this question. In questions where you need to calculate data you will never be penalized for using incorrect data that you have calculated previously. In a question that has several parts you might fail to gain a sensible answer to one of the parts. When a subsequent part requires the use of your answer as data – don't give up, invent a sensible value (and say that is what you are doing). You should then gain any marks available for the correct method.

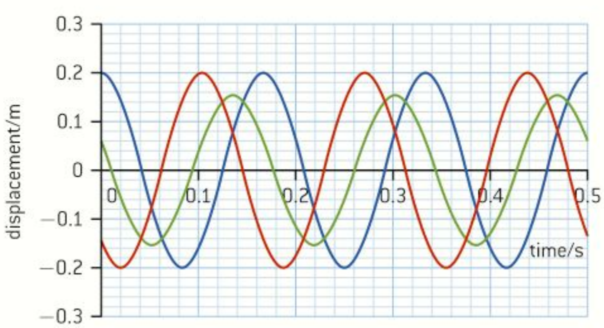
The principle of superposition

Unlike when solid objects collide, when two or more waves meet the total displacement is the vector sum of their individual displacements. Having interacted, the waves continue on their way as if they had never met at all. This principle is used to explain interference and standing waves in Sub-topics 4.4 and 4.5 respectively. For a consistent pattern, waves need to be of the same type and have the same frequency and speed; the best patterns are achieved when the waves have the same or very similar amplitudes. Figure 7 shows two pulses approaching and passing through each other. When they meet, the resultant amplitude is the algebraic sum of the two amplitudes of the individual pulses.



▲ Figure 7 Superposition of pulses.

This principle applies equally well to complete waves as to pulses – this is shown in figure 8. You should convince yourself that the green wave is the vector sum of the red and blue waves at every instant. It is equally valid to use the principle of superposition with displacement–distance graphs.



▲ Figure 8 Displacement–time graph showing the superposition of waves.

Worked examples

1 Two identical triangular pulses of amplitude *A* travel towards each other along a rubber cord. At the instant shown on the diagram below, point M is midway between the two pulses.

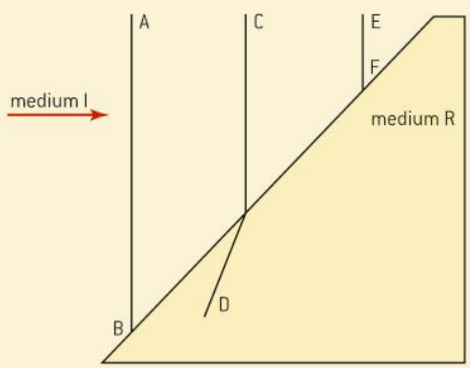
What is the amplitude of the disturbance in the string as the pulses move through M?

Solution

The pulses are symmetrical, so when they meet they completely cancel out giving zero amplitude at M.

2 **a)** For a travelling wave, distinguish between a *ray* and a *wavefront*.

The diagram below shows three wavefronts incident on a boundary between medium I and medium R. Wavefront CD is shown crossing the boundary. Wavefront EF is incomplete.



b) (i) On a copy of the diagram above, draw a line to complete the wavefront EF.

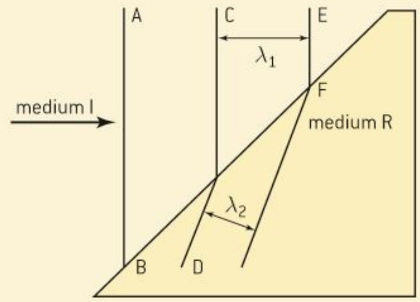
(ii) Explain in which medium, I or R, the wave has the higher speed.

(iii) By taking appropriate measurements from the diagram, determine the ratio of the speeds of the wave travelling from medium I to medium R.

Solution

a) A ray is a line that shows the direction of propagation of a wave. Wavefronts are lines connecting points on the wave that are in phase, such as a crest or a trough. The distance between wavefronts is one wavelength and wavefronts are always perpendicular to rays.

b) (i)



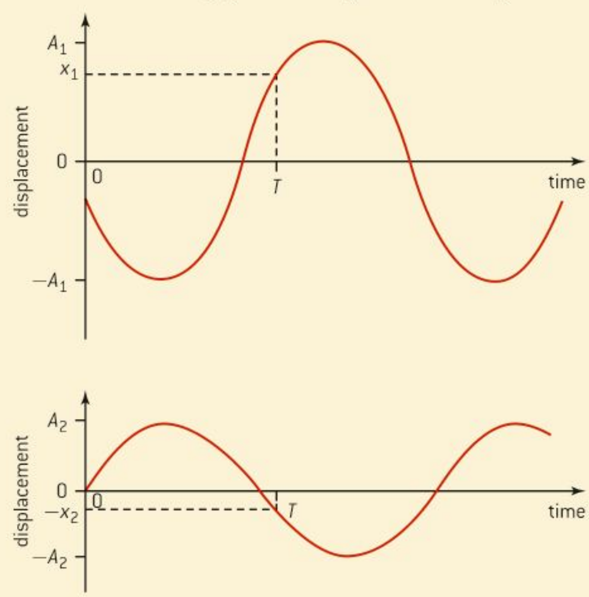
On entering the new medium the waves refract by the same amount to give parallel wavefronts.

(ii) As the wavefronts are closer in the medium R the waves are travelling more slowly.

(iii) The frequency of a wave does not change when the wave moves from one medium to another. As $c = f\lambda$ then $\frac{c}{\lambda}$ = constant and so:

$$\frac{c_1}{\lambda_1} = \frac{c_2}{\lambda_2} \therefore \frac{c_1}{c_2} = \frac{\lambda_1}{\lambda_2}$$

3 The graphs below show the variation with time of the individual displacements of two waves as they pass through the same point.



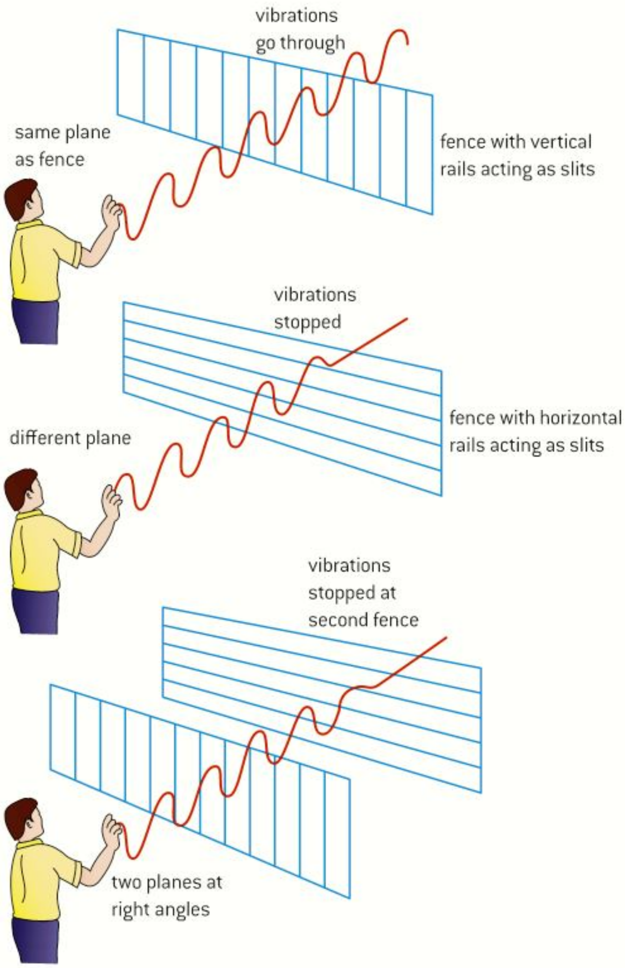
What is the total displacement of the resultant wave at the point at time *T*?

Solution

The total displacement will be the vector sum of the individual displacements. As they are in opposite directions the vector sum will be their difference, i.e. $x_1 - x_2$

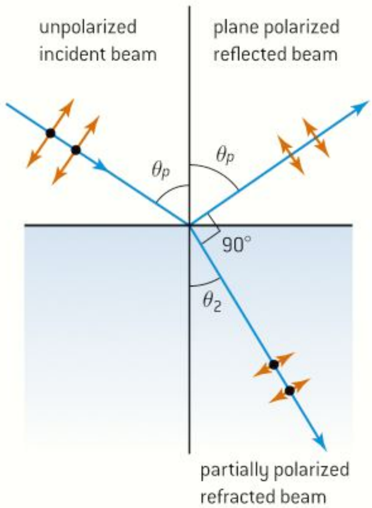
Polarization

Although transverse and longitudinal waves have common properties – they reflect, refract, diffract and superpose – the difference between them can be seen by the property of polarization. Polarization of a transverse wave restricts the direction of oscillation to a plane perpendicular to the direction of propagation. Longitudinal waves, such as sound waves, do not exhibit polarization because, for these waves, the direction of oscillation is parallel to the direction of propagation. Figure 9 shows a demonstration of the polarization of a transverse wave on a rubber tube.



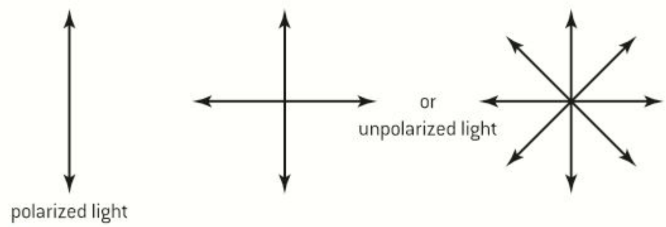
▲ Figure 9 Polarization demonstration.

Most naturally occurring electromagnetic waves are completely unpolarized; this means the electric field vector (and therefore the magnetic field vector perpendicular to it) vibrate in random directions but in a plane always at right angles to the direction of propagation of the wave. When the direction of vibration stays constant over time, the wave is said to be **plane polarized** in the direction of vibration – this is the case with many radio waves which are polarized as a result of the orientation of the transmitting aerial (antenna). **Partial polarization** is when there is some restriction to direction of vibration but not 100%. There is a further type of polarization when the direction of vibration rotates at the same



▲ Figure 11 Polarization of reflected light.

frequency as the wave – this called circular or elliptical polarization and is caused when a wave is in a strong magnetic field – but you will not be examined on this. Figure 10 shows how we represent polarized and unpolarized light diagramatically; the double-headed arrow represents a polarized wave, showing the plane of polarization of the wave. The crossed arrows show that the vibration has an electric field vector in all planes and these are resolved into the two perpendicular planes shown (you may see this marked as four double-headed arrows in some texts). These diagrams can become a little confusing when rays are added to show the direction in which the waves are travelling. Figure 11 shows some examples of this.



▲ Figure 10 Representing polarized and unpolarized waves.

Polarization of light

In 1809 the French experimenter Étienne-Louis Malus showed that when unpolarized light reflected off a glass plate it could be polarized depending upon the angle of incidence – the plane of polarization being that of the flat surface reflecting the light. In 1812 the Scottish physicist Sir David Brewster showed that when unpolarized light incident on the surface of an optically denser material (such as glass), at an angle called the polarizing angle, the reflected ray would be completely plane polarized. At this angle the reflected ray and refracted ray are at right angles as shown in figure 11.

Today the most common method of producing polarized light is to use a polarizing filter (usually called **Polaroid**). These filters are made from chains of microcrystals of iodoquinine sulfate embedded in a transparent cellulose nitrate film. The crystals are aligned during manufacture and electric field vibration components, parallel to the direction of alignment, become absorbed. The electric field vector causes the electrons in the crystal chains to oscillate and thus removes energy from the wave. The direction perpendicular to the chains allows the electric field to pass through. The reason for this is that the limited width of the molecules restricts the motion of the electrons, meaning that they cannot absorb the wave energy. When a pair of Polaroids are oriented to be at 90° to each other, or "crossed", no light is able to pass through. The first Polaroid restricts the electric field to the direction perpendicular to the crystal chains; the second Polaroid has its crystals aligned in this direction and so absorbs the remaining energy. The first of the two Polaroids is called the **polarizer** and the second is called the **analyser**.

Malus's law

When totally plane-polarized light (from a polarizer) is incident on an analyser, the intensity I of the light transmitted by the analyser is directly proportional to the square of the cosine of angle between the transmission axes of the analyser and the polarizer.

Figure 12 shows polarized light with the electric field vector of amplitude E_0 incident on an analyser. The axis of transmission of the analyser makes an angle θ with the incident light. The electric field vector E_0 can be resolved into two perpendicular components $E_0 \cos\theta$ and $E_0 \sin\theta$. The analyser transmits the component that is parallel to its transmission axis, which is $E_0 \cos\theta$.

We have seen that intensity is proportional to the square of the amplitude of a wave so $I_0 \propto E_0^2$ the transmitted intensity will be I which is proportional to $(E_0 \cos\theta)^2$ or $I \propto E_0^2 \cos^2\theta$

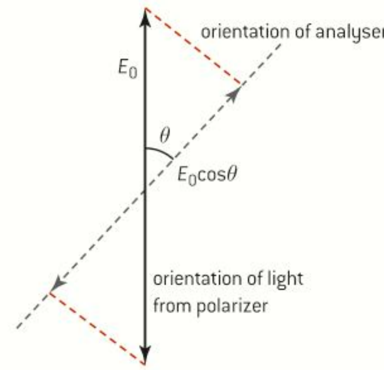
Taking ratios we have $\frac{I}{I_0} = \frac{E_0^2 \cos^2\theta}{E_0^2}$ cancelling E_0^2 gives

$$I = I_0 \cos^2\theta$$

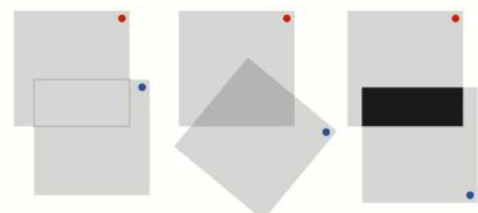
When $\theta = 0°$ (or 180°) $I = I_0$ (since $\cos 0° = 1$); this means that the intensity of light transmitted by the analyser is maximum when the transmission axes of the two Polaroids are parallel.

When $\theta = 90°$, $I = I_0 \cos^2 90° = 0$; this means that no light is transmitted by the analyser when the Polaroids are crossed.

Figure 13 shows a pair of Polaroids. The left-hand image shows that, when their transmission axes are aligned, the same proportion of light passing through one Polaroid passes through both. The central image shows that when one of the Polaroids is rotated slightly, less light passes through their region of overlap. The right-hand image shows that where the Polaroids are crossed no light is transmitted.



▲ Figure 12 Analysing polarized light.



▲ Figure 13 Two Polaroids.

Nature of science

Uses of polarization of light

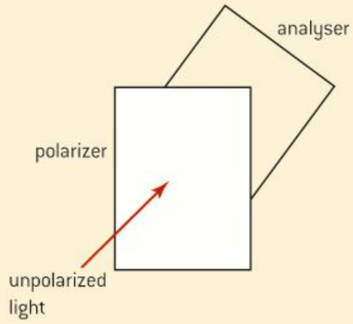
Polaroid sunglasses are used to reduce the glare coming from the light scattered by surfaces such as the sea or a swimming pool. In industry, stress analysis can be performed on models made of transparent plastic by placing the model in between a pair of crossed Polaroids. As white light passes through a plastic, each colour of the spectrum is polarized with a unique orientation. There is high stress in the regions where the colours are most concentrated – this is where the model (or real object being modelled) is most likely to break when it is put under stress.

Certain asymmetric molecules (called *chiral molecules*) are **optically active** – this is the ability to rotate the plane of plane-polarized light. The angle that the light is rotated through is measured using a **polarimeter**, which consists of a light source, and a pair of Polaroids. The light passes through the first Polaroid (the polarizer) and, initially with no sample present, the second Polaroid (the analyser) is aligned so that no light passes through. With a sample placed between the Polaroids, light does pass through because the sample has rotated the plane of polarization. The analyser is now rotated so that again no light passes through. The angle of rotation is measured and from this the concentration of a solution, for example, can be found. An easily produced, optically active substance is a sugar solution.

Polarization can also be used in the recording and projection of 3D films. These consist of two films projected at the same time. Each of the films is recorded from a slightly different camera angle and the projectors are also set up in this way. The two films are projected through polarizing filters – one with its axis of transmission horizontal and one with it vertical. By wearing polarized eye glasses with one lens horizontally polarized and one vertically polarized, the viewer's left eye only sees the light from the left projector and the right eye light from the right projector – thus giving the viewer the perception of depth. In cinemas the screen needs to be metallic as non-metallic flat surfaces have a polarizing effect.

Worked examples

1 Unpolarized light of intensity I_0 is incident on a polarizer. The transmitted light is then incident on an analyser. The axis of the analyser makes an angle of 60° to the axis of the polarizer.



Calculate the intensity emitted by the analyser.

Solution

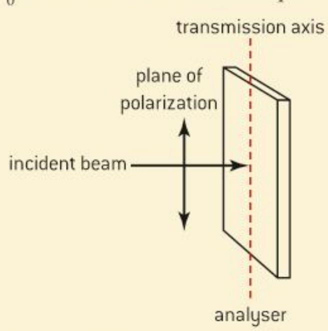
The first polarizer restricts the intensity to $\frac{I_0}{2}$.
Using Malus's law $I = I_0 \cos^2 \theta$

$\cos 60° = 0.5$ so $\cos^2 60° = 0.25$

thus $I = 0.25\frac{I_0}{2} = 0.125I_0$

2 a) Distinguish between polarized and unpolarized light.

b) A beam of plane-polarized light of intensity I_0 is incident on an analyser.



The angle between the transmission axis of the analyser and the plane of polarization of the light θ can be varied by rotating the analyser about an axis parallel to the direction of the incident beam. In the position shown, the transmission axis of the analyser is parallel to the plane of polarization of the light ($\theta = 0°$).

Sketch a graph to show how the intensity I of the transmitted light varies with θ as the analyser is rotated through 180°.

Solution

a) In unpolarized light the **electric field vector** vibrates randomly in any plane (perpendicular to the direction of propagation). In polarized light this vector is restricted to just one plane.

b) This is an application of Malus's law $I = I_0 \cos^2 \theta$

When $\theta = 0$ or 180°, $\cos \theta = 1$ and so $\cos^2 \theta = 1$ and $I = I_0$

When $\theta = 90°$, $\cos \theta = 0$ and so $\cos^2 \theta = 0$ and $I = 0$

These are the key points to focus on. Note that $\cos^2 \theta$ will never become negative. There is no need to include a unit for intensity as this is a sketch graph.