4.4 Wave behaviour

Understanding

- Reflection and refraction
- Snell's law, critical angle, and total internal reflection
- Diffraction through a single-slit and around objects
- Interference patterns
- Double-slit interference
- Path difference

Applications and Skills

- Sketching and interpreting incident, reflected and transmitted waves at boundaries between media
- Solving problems involving reflection at a plane interface
- Solving problems involving Snell's law, critical angle, and total internal reflection
- Determining refractive index experimentally
- Qualitatively describing the diffraction pattern formed when plane waves are incident normally on a single-slit
- Quantitatively describing double-slit interference intensity patterns

Equations

- Snell's law: $\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{v_1}{v_2}$

  Interference at a double slit: $s = \frac{\lambda D}{d}$

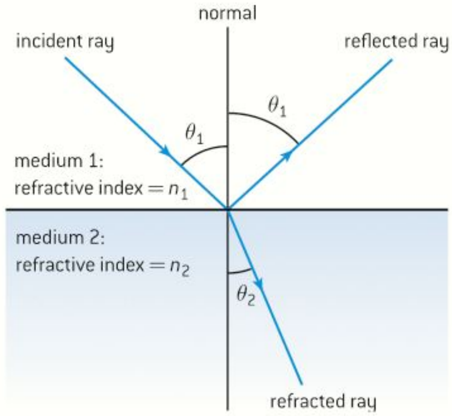
Nature of science

Wave or particle?

In the late seventeenth century two rival theories of the nature of light were proposed by Newton and Huygens. Newton believed light to be particulate and supported his view by the facts that it apparently travels in straight lines and can travel through a vacuum; at this time it was a strongly held belief that waves needed a medium through which to travel. Huygens' wave model was supported by the work of Grimaldi who had shown that light diffracts around small objects and through narrow openings. He was also able to argue that when a wave meets a boundary the total incident energy is shared by the reflected and transmitted waves; Newton's argument for this was based on the particles themselves deciding whether or not to reflect or transmit – this was not a strong argument and the wave theory of light became predominant. In the 21st century light is treated as both a wave and a particle in order to explain the full range of its properties.

Introduction

Now we have looked at how to describe waves, we are in the position to look at wave properties – or, as it is described in IB Physics, wave behaviour. You are likely to have come across some of this topic if you



▲ Figure 1 Reflection and refraction of waves.

have studied physics before starting the IB Diploma Programme and we have used some of the ideas in the previous sub-topic. In Sub-topic 4.3 we studied polarization – a property that is restricted to transverse waves alone. The ideas examined in this sub-topic apply to transverse waves (both mechanical and electromagnetic) and longitudinal waves. There are many demonstrations of wave properties utilizing sound and light; however, microwaves are commonly used for demonstrating these wave properties too.

Reflection and refraction of waves

We have looked at much of the content of figure 1 when we considered polarization. We will now focus on what is happening to the rays – remember we could always add wavefronts at right angles to the rays drawn on these diagrams. What the ray diagrams do not show is what is happening to the wavelength of the waves – we will return to this in due course.

The laws of reflection and refraction can be summarized in three laws as follows:

1 **The reflected and refracted rays are in the same plane as the incident ray and the normal.**
This means that the event of reflection or refraction does not alter the plane in which the light ray travels – this is not obvious because we draw ray diagrams in two dimensions but, when we use ray boxes to perform experiments with light beams, we can confirm that this is the case.

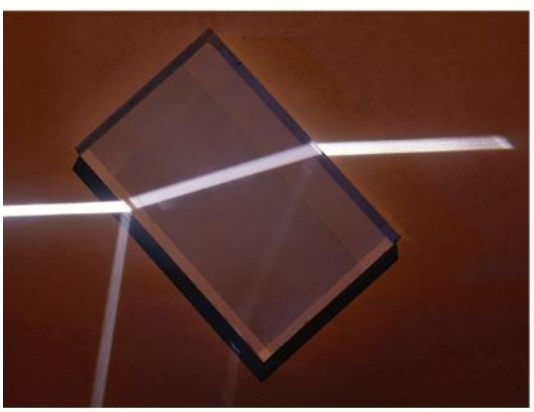
2 **The angle of incidence equals the angle of reflection.**
The angle of incidence is the angle between the incident ray and the normal and the angle of reflection is the angle between the reflected ray and the normal. The normal is a line perpendicular to a surface at any chosen point. The angle of incidence and reflection are both labelled as θ_1 on figure 1.

3 **For waves of a particular frequency and for a chosen pair of media the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant called the (relative) refractive index.**
This is called Snell's law (or Descartes's law in the French speaking world). The angle of refraction is the angle between the refracted ray and the normal. Snell's law can be written as

$$\frac{\sin \theta_1}{\sin \theta_2} = {}_1n_2$$

For light going from medium 1 to medium 2 – this way of writing Snell's law has several variants and the IB course uses one that we will look at soon.

When light is normal on a surface Snell's law breaks down because the light passes directly through the surface.



▲ Figure 2 Use of ray box to demonstrate the laws of reflection and refraction.

Nature of science

What happens to light at an interface between two media?

This is a complex process but in general terms, when charges are accelerated, for example when they are vibrated, they can emit energy as an electromagnetic wave. In moving through a vacuum the electromagnetic wave travels with a velocity of 3.00×10^8 m s^{-1}. When the wave reaches an atom, energy is absorbed and causes electrons within the atom to vibrate. All particles have frequencies at which they tend to vibrate most efficiently – called the **natural frequency**. When the frequency of the electromagnetic wave does not match the natural frequency of vibration of the electron, then the energy will be re-emitted as an electromagnetic wave. This new electromagnetic wave has the same frequency as the original wave and will travel at the usual speed in the vacuum between atoms. This process continues to be repeated as the new wave comes into contact with further atoms of unmatched natural frequency. With the wave travelling at 3.00×10^8 m s^{-1} in space but being delayed by the absorption–re-emission process, the overall speed of the wave will be reduced. In general, the more atoms per unit volume in the material, the slower the radiation will travel. When the frequency of the light does match that of the atom's electrons the re-emission process is occurs in all directions and the atom gains energy, increasing the internal energy of the material.

TOK

Conservation of energy and waves

The wave and ray diagram for reflection and refraction tells just part of the story. As with many areas of physics, returning to the conservation of energy is important. For the total energy incident on the interface between the two media the energy is shared between the reflected wave, the transmitted wave and the energy that is ***absorbed*** – the further the wave passes through the second medium, the more of the energy is likely to be absorbed. The conservation of mass/energy is a principle in physics which, to date, has not let physicists down. Does this mean that we have proved that the principle of conservation of energy is infallible?

Refractive index and Snell's law

The **absolute refractive index (*n*)** of a medium is defined in terms of the speed of electromagnetic waves as:

$$n = \frac{\text{speed of electromagnetic waves in a vacuum}}{\text{speed of electromagnetic waves in the medium}} = \frac{c}{v}$$

The refractive index depends on the frequency of the electromagnetic radiation and, since the speed of light in a vacuum is the limit of speed, the absolute refractive index is always greater than 1 (although there are circumstances when this is not true – but that is well beyond the expectation of your IB Physics course). For all practical purposes the absolute refractive index of air is 1 so it is not necessary to perform refractive index experiments in a vacuum.

Worked example

Calculate the angle of refraction when the angle of incidence at a glass surface is 55° (refractive index of the glass = 1.48).

Solution

As we are dealing with air and glass there is no difference between absolute refractive index and relative refractive index.

Snell's law gives $\frac{\sin\theta_1}{\sin\theta_2} = n_{\text{glass}}$

$$= \frac{\sin 55°}{\sin\theta_2} = 1.48$$

$$\sin\theta_2 = \frac{\sin 55°}{1.48} = \frac{0.819}{1.48} = 0.553$$

$$\theta = \sin^{-1}(0.553) = 33.6 \approx 34°$$

Reversibility of light

You should be able to prove to yourself that rays are reversible. Place a ray box and a glass block on a piece of paper. Mark, on the paper, the path of the beam of light emitted by the ray box as it approaches and leaves the glass block. Then place the ray box on the other side of the block and you will see that the light travels along the same path in the opposite direction.

We have seen that for light travelling from medium 1 to medium 2 Snell's law can be written as

$$\frac{\sin\theta_1}{\sin\theta_2} = {}_1n_2$$

here ${}_1n_2$ means the relative refractive index going from medium 1 to medium 2. For light travelling in the opposite direction and since light is reversible we have

$$\frac{\sin\theta_2}{\sin\theta_1} = {}_2n_1$$

It should be clear from this that ${}_1n_2 = \frac{1}{{}_2n_1}$

Worked example

The (absolute) refractive index of water is 1.3 and that of glass is 1.5.

a) Calculate the relative refractive index from glass to water.

b) Explain what this implies regarding the refraction of light rays.

c) Draw a wavefront diagram to show how light travels through a plane interface from glass to water.

Solution

a) $n_{water} = {}_{vac}n_{water} = 1.3$ and $n_{glass} = {}_{vac}n_{glass} = 1.5$

We are calculating ${}_{glass}n_{water}$

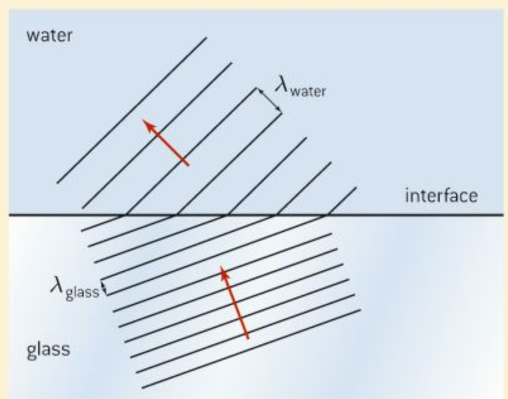
$$\frac{\sin\theta_{vac}}{\sin\theta_{water}} = {}_{vac}n_{water} \text{ and } \frac{\sin\theta_{vac}}{\sin\theta_{glass}} = {}_{vac}n_{glass}$$

$$\frac{\sin\theta_{glass}}{\sin\theta_{vac}} \times \frac{\sin\theta_{vac}}{\sin\theta_{water}} = \frac{\sin\theta_{glass}}{\sin\theta_{water}} = {}_{glass}n_{water}$$

This means that ${}_{glass}n_{water} = \frac{1}{1.5} \times 1.3 = 0.87$

b) With a relative refractive index less than one this means that the light travels faster in the water than the glass and therefore bends away from the normal.

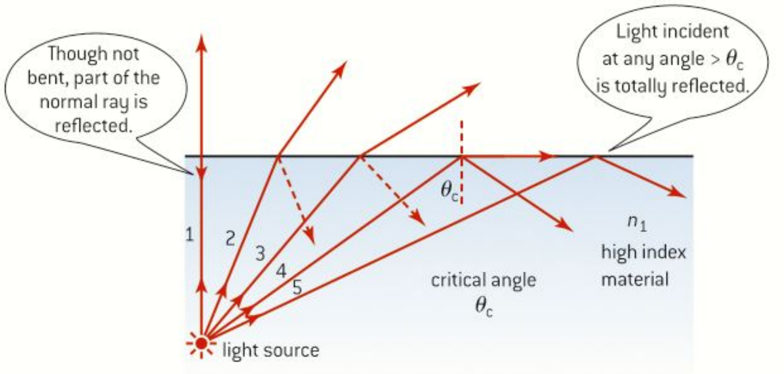
c)



As the refractive index of water is lower than that of glass the light wave speeds up on entering the water. The frequency is constant and, as $= f\lambda$, the wavelength will be greater in water – meaning that the wavefronts are further apart. The fact that the frequency remains constant is a consequence of Maxwell's electromagnetic equations – something not covered in IB Physics.

The critical angle and total internal reflection

When a light wave reaches an interface travelling from a higher optically dense medium to a lower one the wave speeds up. This means that the wavelength of the wave increases (frequency being constant) and the direction of the wave moves away from the normal – the angle of refraction being greater than the angle of incidence so as the angle of incidence increases the angle of refraction will approach 90°. **Optical density** is not the same as physical density, i.e. mass per unit volume, it is measured in terms of refractive index – higher refractive index material having higher optical density.

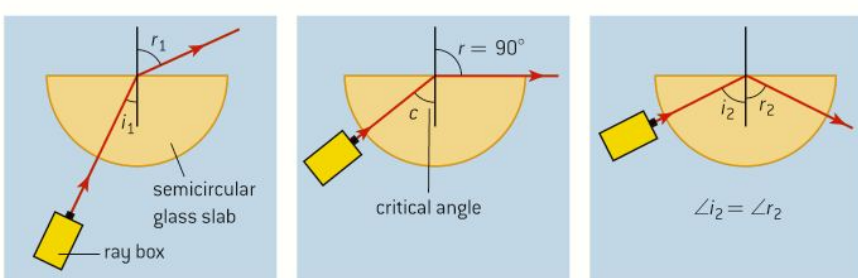


▲ Figure 3 Light passing from more optically dense medium to less optically dense medium.

Ray **1** in figure 3 shows a ray passing from a more optically dense medium to a less optically dense medium normal to an interface. Most of the light passes though the interface but a portion is reflected back into the original medium. Increasing the angle of incidence (as for rays **2** and **3**) will increase the angle of refraction and ray **4** shows an angle of incidence when the angle of refraction is 90°. The angle of incidence at this value is called the **critical angle (θ_c)**. Ray **5** shows that, when the angle of incidence is larger than the critical angle, the light wave does not move into the new medium at all but is reflected back into the original medium. This process is called **total internal reflection**.

It should be noted that for angles smaller than the critical angle there will always be a reflected ray; although this will carry only a small portion of the incident energy.

A ray box and semicircular glass block can be used to measure the critical angle for glass as shown in figure 4. When the beam is incident on the curved face of the block making it travel towards the centre of the flat face, it is acting along a radius and so enters the block normally and, therefore, without bending. By moving the ray box, different angles of incidence can be obtained. Both the critical angle and total internal reflection can be seen.



▲ Figure 4 Use of ray box to investigate critical angle and total internal reflection.

Worked example

Calculate the (average) critical angle for a material of (average) absolute refractive index 1.2.

Solution

The word "average" is included because the refractive index and critical angle would each be different for different colours. Don't be surprised if it is left out in some questions – it is implied by talking about single values.

$\sin \theta_c = \frac{1}{n_1} = \frac{1}{1.2} = 0.833$

$\theta_c = \sin^{-1} 0.833 = 56.4° \approx 56°$

When a ray box emitting white light is used, the light emerging through the glass block is seen to **disperse** into the colours of the rainbow. This is due to each of the colours, of which white light is comprised, having a different frequency. The refractive index for each of the colours is different.

Calculating the critical angle

Snell's law gives $\frac{\sin\theta_1}{\sin\theta_2} = {}_1n_2$.

In order to obtain a critical angle, medium 1 must be more optically dense than medium 2.

When $\theta_1 = \theta_c$ then $\theta_2 = 90°$ so $\sin \theta_2 = 1$

This gives $\sin \theta_c = {}_1n_2$

$${}_1n_2 = \frac{n_2}{n_1}$$

When the less dense medium (medium 2) is a vacuum or air then $n_2 = 1$

So $\sin \theta_c = \frac{1}{n_1}$

Investigate!

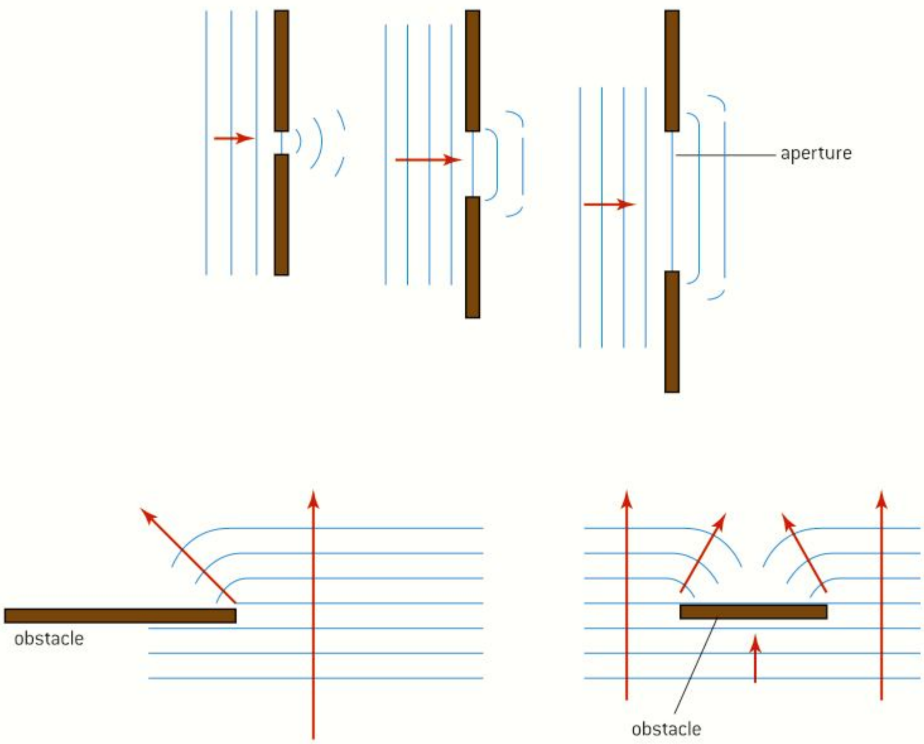
Measuring the refractive index

There are several possible experiments that you could do to measure the refractive index depending on whether a substance is a liquid or a solid (we assume the refractive index of gases is 1 – although mirages are a good example of how variations in air density can affect the refraction of light). You could research how to use real and apparent depth measurements to measure the refractive index of a liquid. The investigation outlined below will be to trace some rays (or beams of light, really) through a glass block of rectangular cross-section.

- The arrangement is similar to that shown in figure 2 on p146.
- Place the block on a piece of white paper and mark its position by drawing around the edges.
- Direct a beam of light to enter the block near the centre of a longer side and to leave by the opposite side.
- Mark the path of the beam entering and leaving the block – you will need at least two points on each beam to do this.
- Remove the block and use a ruler to mark the path of the beam on either side of the block and then inside. Add arrows to these to remind you that they represent rays and to indicate which is the incident beam and which is the refracted beam.
- Using a protractor, mark in and draw normals for the beam entering and leaving the block.
- Remembering that light is reversible and the beam is symmetrical, measure two values for each of θ_1 and θ_2.
- Calculate the refractive index of the block using Snell's law.
- Estimate the experimental uncertainty on your measurements of θ_1 and θ_2.
- Note that the uncertainty in θ_1 and θ_2 is not the same as that in $\sin \theta_1$ and $\sin \theta_2$ but you can calculate the uncertainty in $\frac{\sin\theta_1}{\sin\theta_2}$ (and hence n) by calculating half the difference between $\frac{\sin\theta_{1\max}}{\sin\theta_{2\min}}$ and $\frac{\sin\theta_{1\min}}{\sin\theta_{2\max}}$
- Repeat the experiment for a range of values of the angle of incidence.
- Which of your values is likely to be the most reliable?

Diffraction

The first detailed observation and description of the phenomenon that was named **diffraction** was made by the Italian priest Francesco Grimaldi. His work was published in 1665, two years after his death. He found that when waves pass through a narrow gap or slit (called an aperture), or when their path is partly blocked by an object, the waves spread out into what we would expect to be the shadow region. This is illustrated by figure 5 and can be demonstrated in a ripple tank. He noted that close to the edges the shadows were bordered by alternating bright and dark fringes. Given the limited apparatus available to Grimaldi his observations were quite extraordinary.



▲ Figure 5 Diffraction.

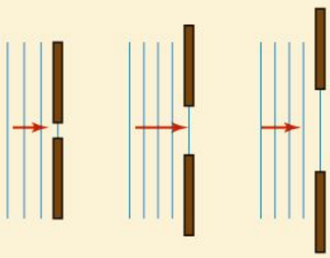
Further observation of diffraction has shown:

- the frequency, wavelength, and speed of the waves each remains the same after diffraction
- the direction of propagation and the pattern of the waves change
- the effect of diffraction is most obvious when the aperture width is approximately equal to the wavelength of the waves.
- the amplitude of the diffracted wave is less than that of the incident wave because the energy is distributed over a larger area.

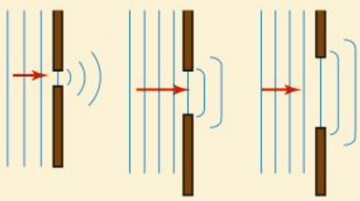
Explaining diffraction by a single slit is complex and you will only be asked for a qualitative description of single-slit diffraction at SL – however, as mentioned in Sub-topic 4.3, the Huygens–Fresnel principle gives a good insight into how the single-slit diffraction pattern comes about (see figure 6).

Worked example

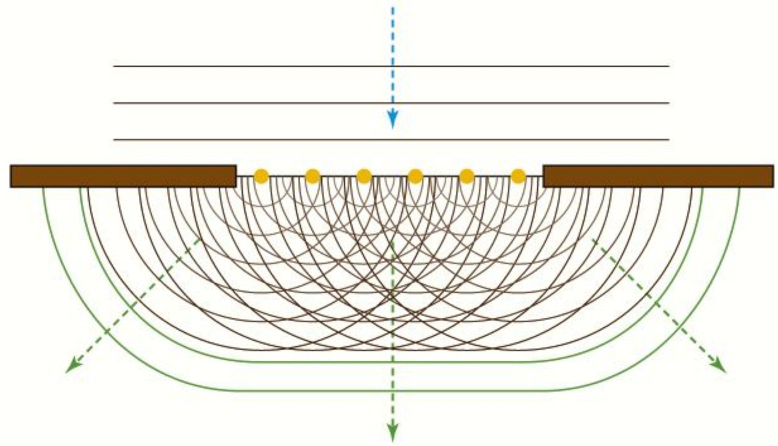
Complete the following diagrams to show the wavefronts after they have passed through the gaps.



Solution

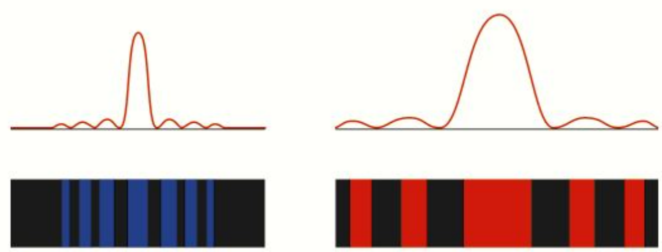


When the width of the slit is less than or equal to the wavelength λ of the wave, the waves emerge from the slit as circular wavefronts. As the slit width is increased, the spreading of the waves only occurs at the edges and the diffraction is less noticeable.



▲ Figure 6 Huygens–Fresnel explanation of diffraction.

Plane waves travelling towards the slit behave as if they were sources of secondary wavelets. The orange dots in figure 6 show these "secondary sources" within the slit. These "sources" each spread out as circular waves. The tangents to these waves will now become the new wavefront. The central image is bright and wide, beyond it are further narrower bright images separated by darkness. Single-slit diffraction is further explored for those studying HL Physics in Topic 9.



▲ Figure 7 Single-slit diffraction pattern.

Double-slit interference

We briefly discussed the principle of superposition in Sub-topic 4.3. Interference is one application of this principle.

When two or more waves meet they combine to produce a new wave – this is called **interference**. When the resultant wave has larger amplitude than any of the individual waves the interference is said to be **constructive**; when the resultant has smaller amplitude the interference is **destructive**. Interference can be achieved by using two similar sources of all types of wave. It is usually only observable if the two sources have a constant phase relationship – this means that although they need not emit the two sets of waves in phase, the phase of the waves cannot alter relative to one another. Such sources will have the same frequency and are said to be **coherent**.

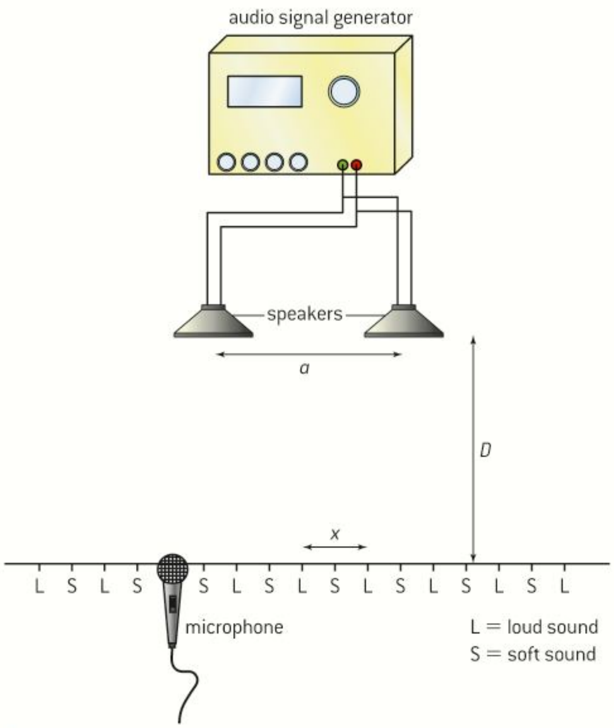


Figure 8 Interference of sound waves.

Interference of sound waves is easy to demonstrate using two loudspeakers connected to the same audio frequency oscillator as shown in figure 8. Moving the microphone (connected to an oscilloscope) in a line perpendicular to the direction in which the waves are travelling allows an equally spaced loud–soft sequence to be detected. More simply, the effect can be demonstrated by the observer walking along the loud–soft line while listening to the loudness.

When a coherent beam of light is incident on two narrow slits very close together the beam is diffracted at each slit and, in the region

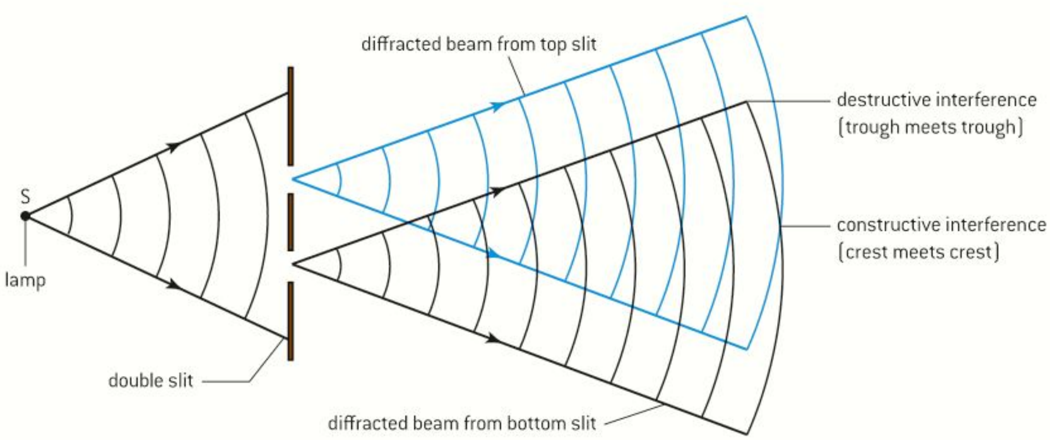
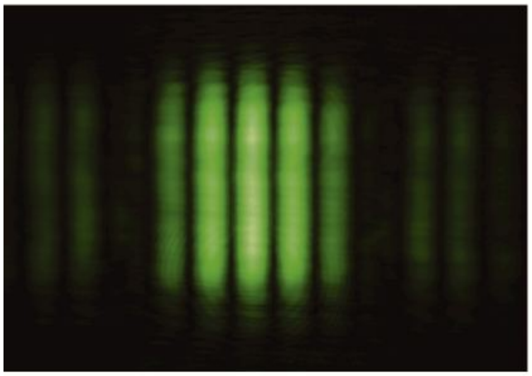


Figure 9 Interference of light waves.

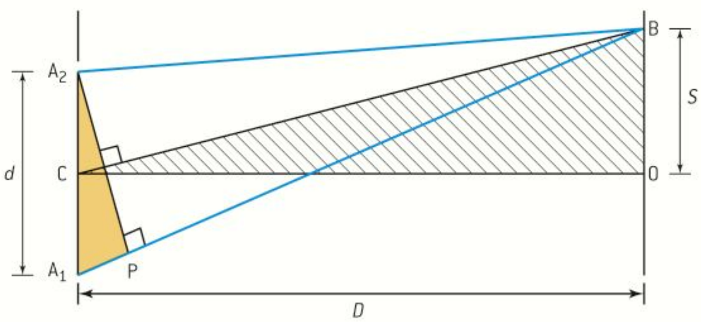


▲ Figure 10 Fringes produced by a double-slit.

where the two diffracted beams cross, interference occurs as seen in figure 9. A pattern of equally spaced bright and dark fringes (shown in figure 10) is obtained on a screen positioned in the region where the diffracted beams overlap. When a crest meets a crest (or a trough meets a trough) constructive interference occurs. When a crest meets a trough destructive interference occurs. A similar experiment to this was performed by the talented English physicist (and later physician) Thomas Young in 1801. The coherent beam is achieved by placing a single slit close to the source of light – this means that the wavefronts spreading from the single slit each reach the double slit with the same phase relationship and so the secondary waves coming from the double slit retain their constant phase relationship.

Path difference and the double-slit equation

You will never be asked to derive this equation but the ideas regarding path difference are vital to your understanding of interference.



▲ Figure 11 The double-slit geometry.

Figure 11 shows two slits (apertures) A_1 and A_2 distance d apart. The double slit is at distance D from a screen. O is the position of the central bright fringe (arising from constructive interference). B is the position of the next bright fringe above O; the distance OB is the fringe spacing s. There will be another bright fringe distance s below O. The beams from A_1 and A_2 to O will travel equal distances and so will meet with the same phase relationship that they had at A_1 and A_2 – they have zero path difference. At B the beam from A_1 will travel an extra wavelength compared with the beam from A_2 – the path distance (= A_1P) equals λ. Because of the short wavelength of light and the fact that D is very much larger than d, the line A_2P is effectively perpendicular to lines A_1B *and* CB (C being the midpoint of $A_1 A_2$). This means that the triangles A_1A_2P and CBO are similar triangles.

Taking ratios $\frac{BO}{CO} = \frac{A_1P}{A_1A_2}$ or $\frac{s}{D} = \frac{\lambda}{d}$

Rearranging gives $s = \frac{\lambda D}{d}$

This gives the separation of successive bright fringes (or bands of loud sound for a sound experiment).

In general for two coherent beams starting in phase, if the path difference is a whole number of wavelengths we get constructive interference and if it is an odd number of half wavelengths we get destructive interference. It must be an odd number of half wavelengths for destructive interference because an even number would give a whole number (integer) and that is constructive interference!

Summarizing this:

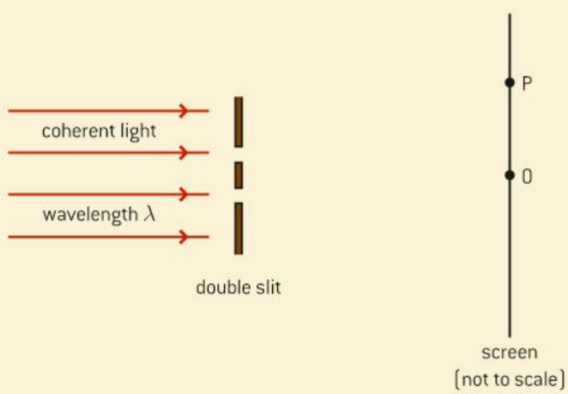
For constructive interference the path difference must $= n\lambda$ where $n = 0,1,2\ldots$

For destructive interference the path difference must $= \left(n + \frac{1}{2}\right)$ where $n = 0,1,2\ldots$

n is known as the order of the fringe, $n = 0$ being the zeroth order, $n = 1$ the first order, etc.

Worked examples

1 In a double-slit experiment using coherent light of wavelength λ, the central bright fringe is observed on a screen at point O, as shown below.



At point P, the path difference between light arriving at P from the two slits is 7λ.

a) Explain the nature of the fringe at P.

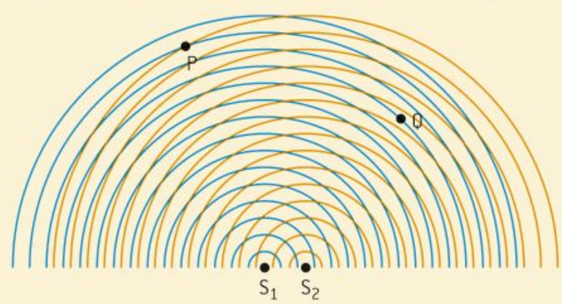
b) State and explain the number of dark fringes between O and P.

Solution

a) As the path difference is an integral number of wavelengths there will be a bright fringe at P.

b) For destructive interference the path difference must be an odd number of half wavelengths, so there will be dark fringes when the path difference is $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}, \frac{9\lambda}{2}, \frac{11\lambda}{2}, \frac{13\lambda}{2}$ giving a total of 7 dark fringes.

2 Two coherent point sources S_1 and S_2 oscillate in a ripple tank and send out a series of coherent wavefronts as shown in the diagram.



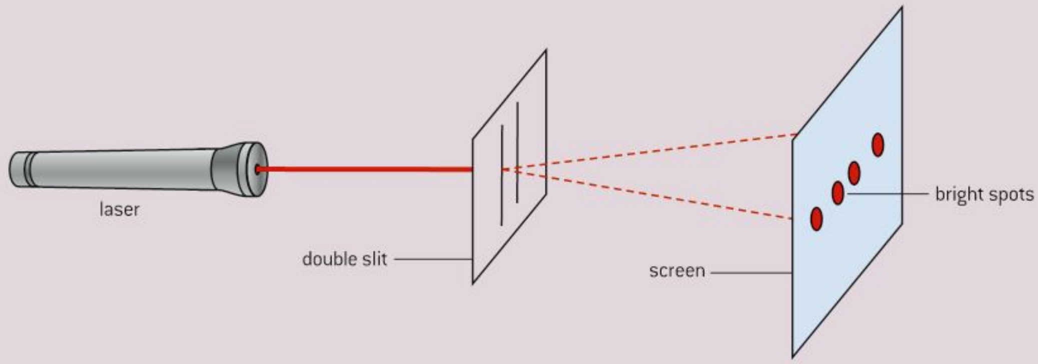
State and explain the intensity of the waves at P and Q?

Solution

Considering each wavefront to be a crest; at point P two crests meet and so superpose constructively giving an amplitude which is twice that of one wave (assuming the wavefronts each have the same amplitude) – the intensity is proportional to the square of this so will be four times the intensity of either of the waves alone. At Q a blue crest meets a red trough and so there is cancellation occurring and there will be zero intensity.

Investigate!

Measuring the wavelength of laser light using a double slit



▲ Figure 12 Measuring the wavelength of laser light using a double slit.

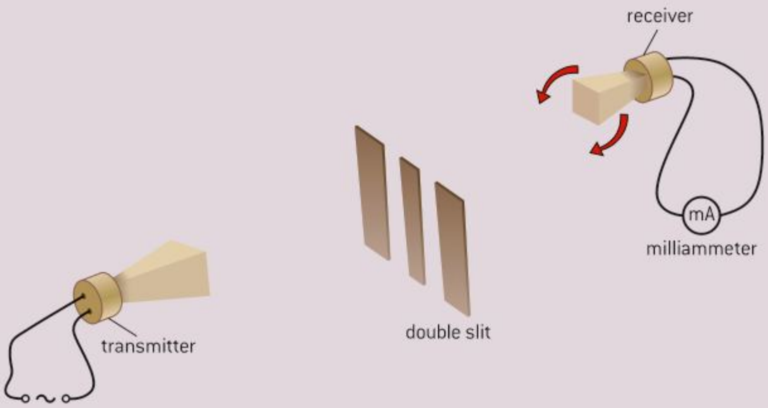
This experiment, using a gas laser (or laser pointer), is a modern version of the one performed by Young. A laser emits a highly coherent beam of light ideal for performing this experiment. **Care should be taken not to shine the laser beam or its reflection into your eye** – should this happen, look away immediately to avoid the risk of permanent damage to your eye. One way to minimise eye damage is to keep sufficient light in the room so that you can still do the experiment. This means that the iris of your eye will not be fully open.

- Set up the apparatus as shown in figure 12 – the screen should be a few metres from the double slit.
- The double slit can be homemade by scratching a pair of narrow lines on a piece of glass painted with a blackened material or it could be a ready prepared slide.
- The slit separation (d) can be measured using a travelling microscope (however, d is likely to be provided by a manufacturer).
- Light from the laser beam diffracts through the slits and emerges as two separate coherent waves. Both slits must be illuminated by the narrow laser beam; sometimes it may be necessary to use a diverging (concave) lens to achieve this.
- The interference pattern is then projected onto the screen and the separation of the spots (images of the laser aperture) is measured as accurately as possible using a metre ruler or tape measure. This is best done by measuring the distance between the furthest spots, remembering that nine spots would have a separation of $8S$.
- The distance from the double slit to the screen (D) should also be measured using metre rulers or a tape measure.
- Once your readings are taken the wavelength of the light can be calculated from $\lambda = \frac{Sd}{D}$.

When red light is used as the source, the bright fringes are all red. When blue light is used as a source, the bright fringes are all blue. As blue light has a shorter wavelength than red light the blue fringes are closer together than the red fringes. When the source is white light the zeroth order (central) bright fringe is white but the other fringes are coloured with the blue edges closer to the centre and the red edges furthest from the centre. Can you explain this?

Investigate!

Measuring the wavelength of microwaves using a double slit



▲ Figure 13 Microwave arrangement.

- The double slit is adjusted so that the slits are around 3 cm apart – for maximum diffraction.
- Arrange both transmitter and receiver about half a metre from the slit.
- Alter the position of the receiver until the received signal is at its strongest.
- Now slowly rotate the receiver until the signal is weakest.
- Cover up one of the slits with a book and explain the result.
- Remove the book so that two slits are again available and attempt to discover if "fringes" occur as in Young's double-slit interference experiment with light. Look for fringes on both sides of the maximum.
- Measure as accurately as you can the values for D, d, and S.
- Calculate the wavelength of the microwaves.
- Repeat for other distances of transmitter and receiver from the slits.

These investigations can be repeated with sound waves or radio waves given appropriate transmitters, receivers and slits of the correct dimension.

4.5 Standing waves

Understanding

- The nature of standing waves
- Boundary conditions
- Nodes and antinodes

Applications and Skills

- Describing the nature and formation of standing waves in terms of superposition
- Distinguishing between standing and travelling waves
- Observing, sketching, and interpreting standing wave patterns in strings and pipes
- Solving problems involving the frequency of a harmonic, length of the standing wave, and the speed of the wave

Nature of science

Fourier synthesis

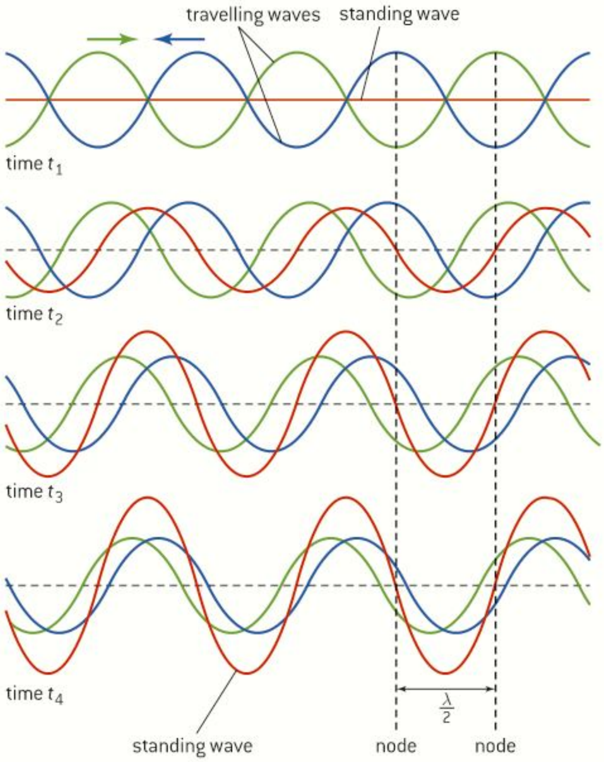
Synthesizers are used to generate a copy of the sounds naturally produced by a wide range of musical instruments. Such devices use the principle of superposition to join together a range of harmonics that are able to emulate the sound of the natural instrument. Fourier synthesis works by combining a sine-wave signal with sine-wave or cosine-wave harmonics of correctly chosen amplitude. The process is named after the French mathematician and physicist Jean Baptiste Joseph, Baron de Fourier who, in the early part of the nineteenth century, developed the mathematical principles on which synthesis of music is based. In many ways Fourier synthesis is a visualization of music.

Introduction

We have seen in Sub-topic 4.2 how travelling waves transfer energy from the source to the surroundings. In a travelling wave the position of the crests and troughs changes with time. Under the right circumstances, waves can be formed in which the positions of the crests and troughs do not change – in such a case the wave is called a **standing wave**. When two travelling waves of equal amplitude and equal frequency travelling with the same speed in opposite directions are superposed, a standing wave is formed.

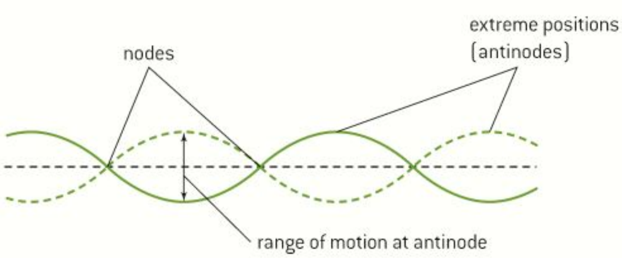
Standing waves on strings

Figure 1 shows two travelling waves (coloured green and blue) moving towards each other at four consecutive times t_1, t_2, t_3, and t_4. The green and blue waves superpose to give the red standing wave. As the green and blue waves move forward there are points where the total displacement (seen on the red wave) always remains zero – these are called **nodes**. At other places the displacement varies between a maximum in one direction and a maximum in the other direction – these are called **antinodes**.



▲ Figure 1 The formation of a standing wave.

At any instant the displacement of the standing wave will vary at all positions other than the nodes. Thus a single frame shot of a standing wave would look like a progressive wave – as can be seen from the red wave in figure 1. When representing the standing wave graphically it is usual to show the extremes of standing waves, but over a complete time period of the oscillation the wave will occupy a variety of positions as shown by the arrow in the loop of figure 2.



▲ Figure 2 Nodes and antinodes on a string.

Melde's string

The apparatus shown in figure 3 is useful in demonstrating standing waves on a string. A variant of this apparatus was first used in the late nineteenth century by the German physicist Franz Melde. A string is strung between a vibration generator and a fixed end. When the vibration generator is

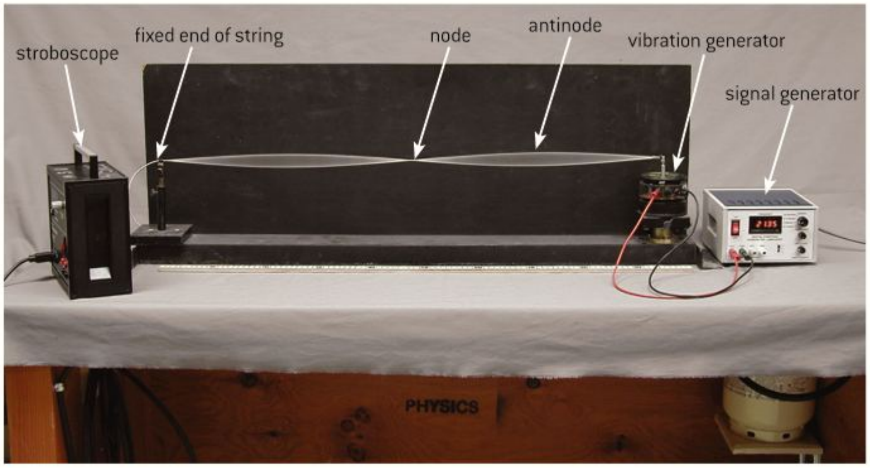


Figure 3 Melde's string.

connected to an audio frequency oscillator, the end of the string attached to the vibration generator oscillates vertically. A wave travels down the string before undergoing a phase change of 180° when it reflects at the fixed end. The reflected wave superposes with the incident wave and (at certain frequencies) a standing wave is formed. The frequency of the audio frequency generator is slowly increased from zero and eventually a frequency is reached at which the string vibrates with large amplitude in the form of a single loop – **the first harmonic**. If the frequency is further increased, the amplitude of the vibrations dies away until a frequency of twice the first harmonic frequency is achieved – in this case two loops are formed and we have the **second harmonic** frequency shown in figure 3. This is an example of resonance – the string vibrates with large amplitude only when the applied frequency is an integral multiple of the natural frequency of the string. We return to resonance in more detail in Option B. Using a stroboscope to freeze the string reveals detail about a standing wave. For example, when the flash frequency is slightly out of synch with the vibration frequency, it is possible to see the variation with time of the string's displacement; this will be zero at a node but a maximum at an antinode.

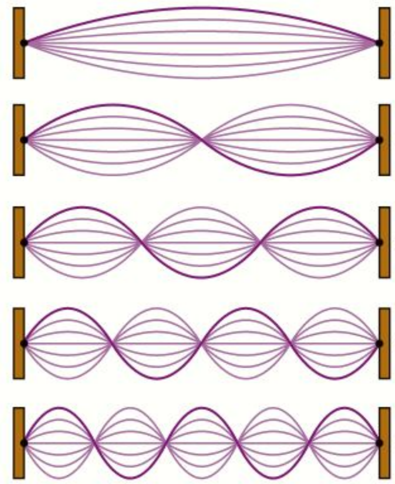
Note

- Although we often treat the point of attachment of the string to the vibration generator as being a node, this is not really correct. The generator vibrates the string to set up the wave and therefore the nearest node to the generator will be a short distance from the vibrator. We call this inaccuracy an "end correction" but often draw a diagram showing the node at the vibrator.
- Because some of the travelling wave energy is transmitted or absorbed by whatever is clamping the fixed end, the reflected waves will be slightly "weaker" than the incident waves. This means that cancellation is not complete and there will be some slight displacement at the nodes.
- Within each loop all parts of the string vibrate together in phase, but loops next to each other are constistently 180° out of phase.
- Although strings are often used to produce sound waves, it is not a sound wave travelling along the string – it is a transverse wave. This wave travels at a speed which is determined by the characteristics of the string.

Harmonics on strings

We have seen from Melde's string that a string has a number of frequencies at which it will naturally vibrate. These natural frequencies are known as the **harmonics of string**. The natural frequency at which a string vibrates depends upon the tension of the string, the mass per unit length and the length of the string. With a stringed musical instrument each end of a string is fixed, meaning there will be a node at either end. The first harmonic is the lowest frequency by which a standing wave will be set up and consists of a single loop. Doubling the frequency of vibration halves the wavelength and means that two loops are formed – this is called the second harmonic; three times the fundamental frequency gives the third harmonic (see figure 4).

You will see from figure 1 that the distance between two consecutive nodes is equal to half a wavelength – in other words each loop in a standing wave is equivalent to $\frac{\lambda}{2}$. Figure 4 shows the first five harmonics of a wave on a string of a fixed length. With the speed of the wave along the string being constant, halving the wavelength doubles the frequency; reducing the wavelength by a factor of three triples the frequency, etc. (the wave equation $c = f\lambda$ applies to all travelling waves). The different harmonics can be achieved either by vibrating the string at the appropriate frequency or by plucking, striking or bowing the string at a different position – although plucking at the centre is likely to produce the first, third and fifth harmonics. By pinching the string at different places a node is produced so, for example, pinching at the centre of the string would produce the even harmonics. In a musical instrument several harmonics occur at the same time, giving the instrument its rich sound.



▲ Figure 4 Harmonics on a string.

Worked example

A string is attached between two rigid supports and is made to vibrate at its first harmonic frequency f.

The diagram shows the displacement of the string at $t = 0$.

(a) Draw the displacement of the string at time

(i) $t = \frac{1}{4f}$ (ii) $t = \frac{1}{2f}$

(b) The distance between the supports is 1.0 m. A wave in the string travels at a speed of 240 m s^{-1}. Calculate the frequency of the vibration of the string.

Solution

(a) (i) We must remember the relationship between period T and frequency f is $T = \frac{1}{f}$.

This means that $t = \frac{T}{4} = \frac{1}{4f}$ so a quarter of a period has elapsed and the string has gone through quarter of a cycle to give:

(ii) In this case the wave has gone through half a period $\left(t = \frac{T}{2} = \frac{1}{2f}\right)$ and so will have moved from a crest to a trough:

(b) The string is vibrating in first harmonic mode and so the distance between the fixed ends is half a wavelength $\left(\frac{\lambda}{2}\right)$.

So $\frac{\lambda}{2} = 1.0$ m and $\lambda = 2.0$ m.

Using $c = f\lambda$ $\quad f = \frac{c}{\lambda} = \frac{240}{2.0}$

$= 120$ Hz

Standing waves in pipes

Standing waves in a pipe differ from standing waves on a string. In pipes the wave medium is (usually) air and the waves themselves are longitudinal. Pipes can have two closed ends, two open ends, or one open and one closed; the latter two being shown in figure 5. The sound waves are reflected at both ends of the pipe irrespective of whether they are open or closed. The variation of displacement of the air molecules in the pipe determines how we graphically represent standing waves in pipes. There is a displacement antinode at an open end (since the molecules can be displaced by the largest amount here) and there will be a displacement node at the closed end (because the molecules next to a closed end are unable to be displaced). Similar ideas to strings apply to the harmonics in pipes. Strictly speaking, the displacement antinode forms just beyond an open end of the pipe but this end effect can be ignored for most purposes.

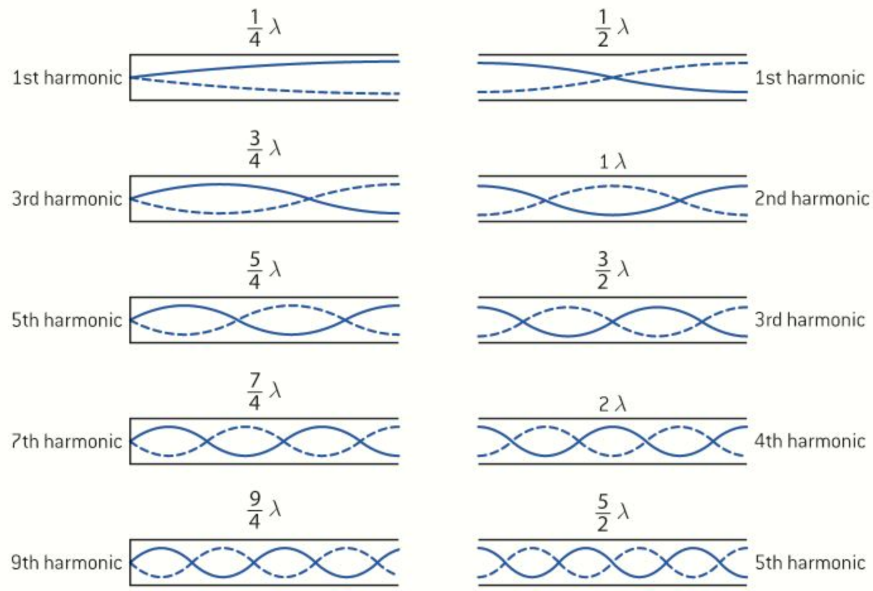


Figure 5 Harmonics in a pipe.

Harmonics in pipes

The harmonic in a pipe depends on whether or not the ends of a pipe are open or closed. For a pipe of fixed length with one open and one closed end there must always be a node at the closed end and an antinode at the open end. This means that only odd harmonics are available (the number of the harmonic is the number of half loops in this type of pipe). For a pipe with two open ends there must always be an antinode at each end and this means that all harmonics are achieveable (in this case the number of loops gives the number of the harmonic).

Let's compare the frequencies of the harmonics for the "one open end" pipe. Suppose the pipe has a length L. The wavelength (λ) of the first harmonic would be $4L$ and, from $c = f\lambda$, the frequency would be $\frac{c}{4L}$. c is the speed of sound in the pipe.

For the third harmonic $L = \frac{3}{4}\lambda$, so $\lambda = \frac{4}{3}L$ and the frequency $= \frac{3c}{4L}$ (or three times that of the first harmonic).

A harmonic is named by the ratio of its frequency to that of the first harmonic.

Worked examples

1 The first harmonic frequency for a particular organ pipe is 330 Hz. The pipe is closed at one end but open at the other. What is the frequency of its second harmonic?

Solution

A named harmonic is the ratio of its frequency to that of the first harmonic – so in this case the second harmonic will be 660 Hz since the second harmonic has twice the frequency of the first harmonic.

2 The first harmonic frequency of the note emitted by an organ pipe which is closed at one end is f. What is the first harmonic frequency of the note emitted by an organ pipe of the same length that is open at both ends?

Solution

The length of a pipe closed at one end in first harmonic mode is $\frac{\lambda}{4}$ $(= L)$ so $\lambda = 4L$

This length will be half the wavelength of a pipe open at both ends so $L = \frac{\lambda}{2}$ so $\lambda = 2L$

Since the wavelength has halved the frequency must double so the new frequency will be $2f$.

Boundary conditions

In considering both pipes and strings we have assumed reflections at the ends or boundaries. In meeting the boundary of a string the wave reflects (or at least partially reflects) – this is known as a **fixed boundary**; there will be the usual phase change of 180° at a fixed boundary meaning that the reflected wave cancels the incident wave and so forms a node. The closed ends of pipes and edges of a drumhead also have fixed boundaries. In the case of an open-ended pipe there is still a reflection of the wave at the boundary but no phase change, so the reflected wave does not cancel the incident wave and there is an antinode formed – the same idea applies to strips of metal vibrated at the centre, xylophones, and vibrating tuning forks. This type of boundary is called a **free boundary**.

Comparison of travelling waves and stationary waves

The following table summarizes the similarities and differences between travelling waves and standing waves:

Property	Travelling wave	Standing wave
energy transfer	energy is transferred in the direction of propagation	no energy is transferred by the wave although there is interchange of kinetic and potential energy within the standing wave
amplitude	all particles have the same amplitude	amplitude varies within a loop – maximum occurs at an antinode and zero at a node
phase	within a wavelength the phase is different for each particle	all particles within a "loop" are in phase and are antiphase (180° out of phase) with the particles in adjacent "loops"
wave profile (shape)	propagates in the direction of the wave at the speed of the wave	stays in the same position
wavelength	the distance between adjacent particles which are in phase	twice the distance between adjacent nodes (or adjacent antinodes)
frequency	all particles vibrate with same frequency.	all particles vibrate with same frequency except at nodes (which are stationary)

TOK

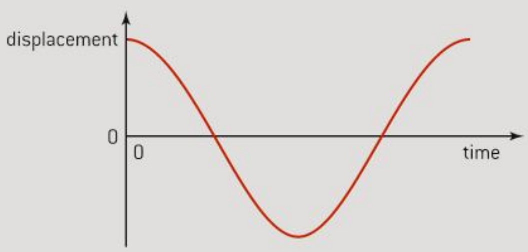
Pitch and frequency

Musical pitch is closely linked to frequency but also has a psychological component in relation to music. We think of pitch as being someone's perception of frequency. Musical notes of certain pitches, when heard together, will produce a pleasant sensation and are said to be *consonant* or *harmonic*. These sound waves form the basis of a *musical interval*. For example, any two musical notes of frequency ratio 2:1 are said to be separated by an *octave* and result in a particularly pleasing sensation when heard. Similarly, two notes of frequency ratio of 5:4 are said to be separated by an interval of a *third* (or strictly a *pure* third); again this interval sounds pleasing. Has music always been thought of in this way? Is the concept of consonance accepted by all societies?

Questions

1 *(IB)*

a) A pendulum consists of a bob suspended by a light inextensible string from a rigid support. The pendulum bob is moved to one side and then released. The sketch graph shows how the displacement of the pendulum bob undergoing simple harmonic motion varies with time over one time period.



On a copy of the sketch graph:

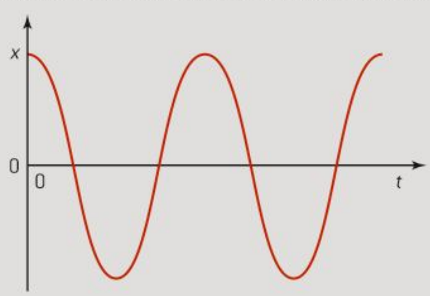
(i) Label, with the letter A, a point at which the acceleration of the pendulum bob is a maximum.

(ii) Label, with the letter V, a point at which the speed of the pendulum bob is a maximum.

b) Explain why the magnitude of the tension in the string at the midpoint of the oscillation is greater than the weight of the pendulum bob.

2 *(IB)*

The graph below shows how the displacement x of a particle undergoing simple harmonic motion varies with time t. The motion is undamped.



a) Sketch a graph showing how the velocity v of the particle varies with time.

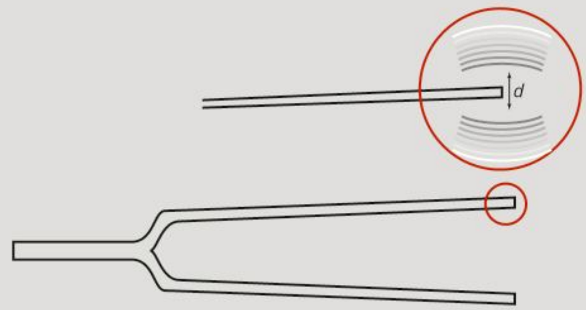
b) Explain why the graph takes this form.

(4 marks)

3 *(IB)*

a) In terms of the acceleration, state *two* conditions necessary for a system to perform simple harmonic motion.

b) A tuning fork is sounded and it is assumed that each tip vibrates with simple harmonic motion.



The extreme positions of the oscillating tip of one fork are separated by a distance d.

(i) State, in terms of d, the amplitude of vibration.

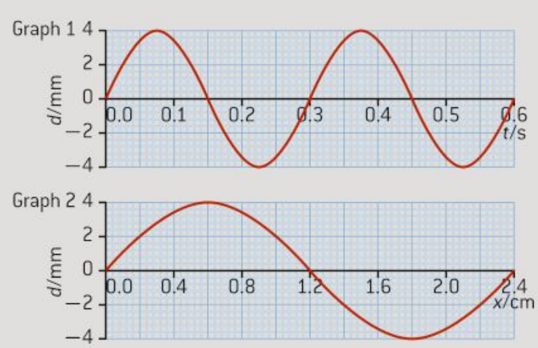
(ii) Sketch a graph to show how the displacement of one tip of the tuning fork varies with time.

(iii) On your graph, label the time period T and the amplitude A.

(8 marks)

4 *(IB)*

a) Graph 1 below shows the variation with time t of the displacement d of a travelling (progressive) wave. Graph 2 shows the variation with distance x along the same wave of its displacement d.



a) State what is meant by a *travelling wave*.

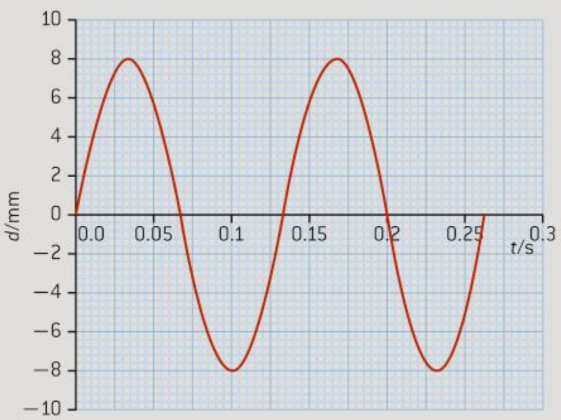
b) Use the graphs to determine the amplitude, wavelength, frequency and speed of the wave.

(5 marks)

5 (*IB*)

a) With reference to the direction of energy transfer through a medium, distinguish between a transverse wave and a longitudinal wave.

b) A wave is travelling along the surface of some shallow water in the *x*-direction. The graph shows the variation with time *t* of the displacement *d* of a particle of water.



Use the graph to determine the frequency and the amplitude of the wave.

c) The speed of the wave in b) is 15 cm s^{-1}. Deduce that the wavelength of this wave is 2.0 cm.

d) The graph in b) shows the displacement of a particle at the position $x = 0$.

Draw a graph to show the variation with distance *x* along the water surface of the displacement *d* of the water surface at time $t = 0.070$ s.

(11 marks)

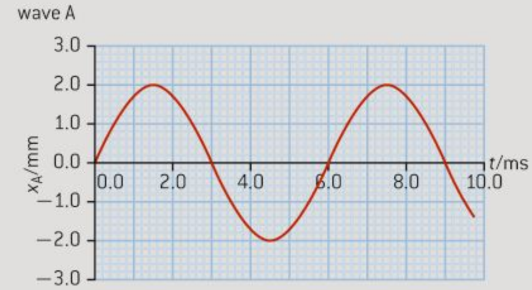
6 (*IB*)

a) By referring to the energy of a travelling wave, explain what is meant by:

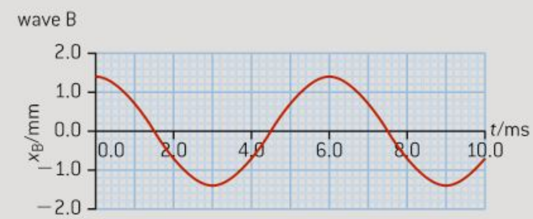
(i) a ray

(ii) wave speed.

b) The following graph shows the variation with time *t* of the displacement x_A of wave A as it passes through a point P.



The graph below shows the variation with time *t* of the displacement x_B of wave B as it passes through point P. The waves have equal frequencies.



(i) Calculate the frequency of the waves.

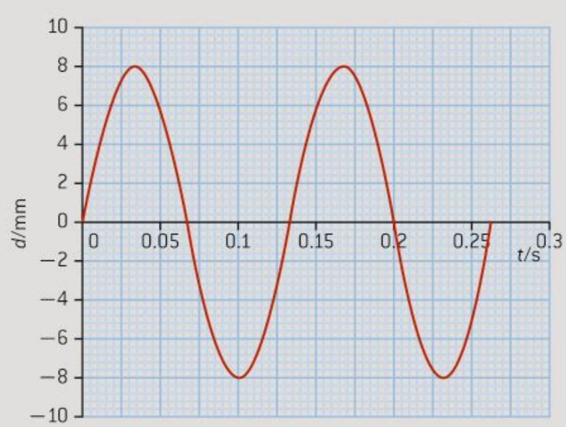
(ii) The waves pass simultaneously through point P. Use the graphs to determine the resultant displacement at point P of the two waves at time $t = 1.0$ ms and at time $t = 8.0$ ms.

(6 marks)

7 (*IB*)

a) With reference to the direction of energy transfer through a medium, distinguish between a transverse wave and a longitudinal wave.

b) The graph shows the variation with time *t* of the displacement *d* of a particular water particle as a surface water wave travels through it.



Use the graph to determine for the wave:

(i) the frequency

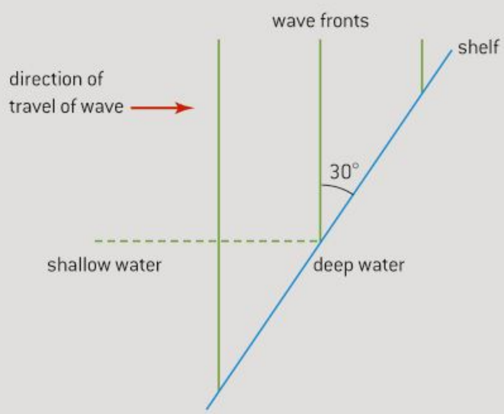
(ii) the amplitude.

c) The speed of the water wave is 12 cm s^{-1}. Calculate the wavelength of the wave.

d) The graph in b) shows the displacement of a particle at the position $x = 0$.

Sketch a graph to show the variation with distance x along the water surface of the displacement d of the water surface at time $t = 0.20$ s.

e) The wave meets a shelf that reduces the depth of the water.

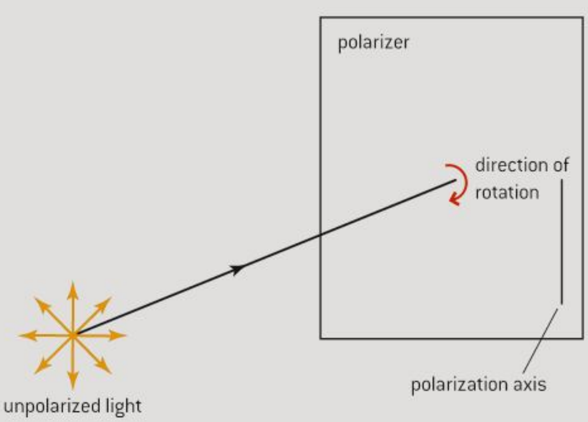


The angle between the wavefronts in the shallow water and the shelf is 30°. The speed of the wave in the shallow water is 12 cm s^{-1} and in the deeper water is 18 cm s^{-1}. For the wave in the deeper water, determine the angle between the normal to the wavefronts and the shelf.

(12 marks)

8 *(IB)*

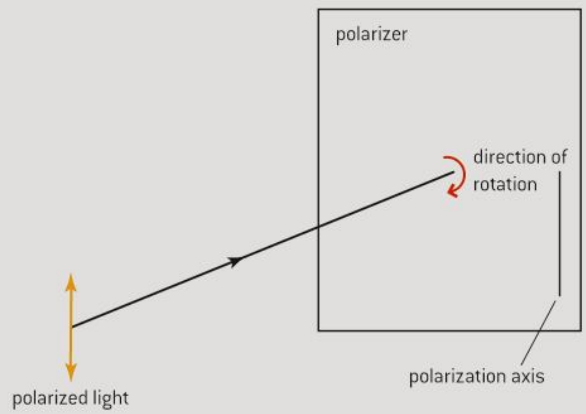
a) A beam of unpolarized light of intensity I_0 is incident on a polarizer. The polarization axis of the polarizer is initially vertical as shown.



The polarizer is then rotated by 180° in the direction shown. Sketch a graph to show the variation with the rotation angle θ, of the transmitted light intensity I, as θ varies from 0° to 180°. Label your sketch-graph with the letter U.

b) The beam in a) is now replaced with a polarized beam of light of the same intensity.

The plane of polarization of the light is initially parallel to the polarization axis of the polarizer.

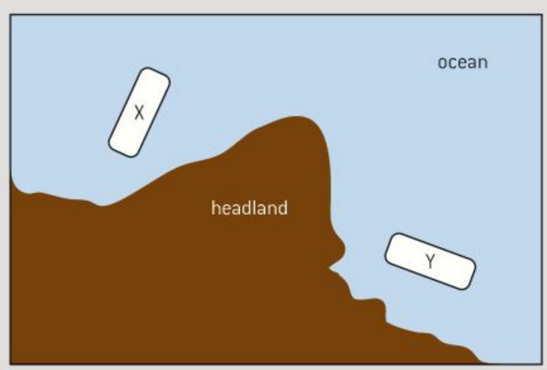


The polarizer is then rotated by 180° in the direction shown. On the same axes in a), sketch a graph to show the variation with the rotation angle θ, of the transmitted light intensity I, as θ varies from 0° to 180°.

(5 marks)

9 *(IB)*

An orchestra playing on boat X can be heard by tourists on boat Y, which is situated out of sight of boat X around a headland.



The sound from X can be heard on Y due to

A. refraction

B. reflection

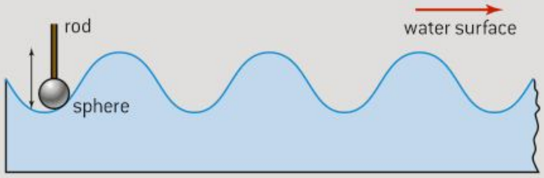
C. diffraction

D. transmission.

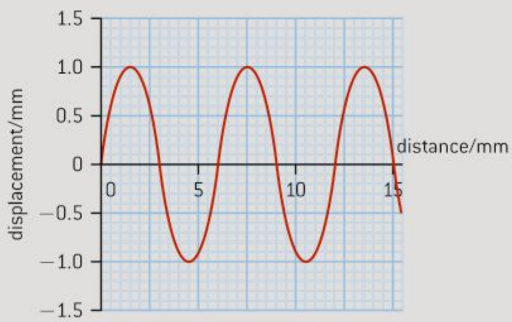
10 *(IB)*

A small sphere, mounted at the end of a vertical rod, dips below the surface of shallow water in a tray. The sphere is driven vertically up and down by a motor attached to the rod.

The oscillations of the sphere produce travelling waves on the surface of the water.



a) The diagram shows how the displacement of the water surface at a particular instant in time varies with distance from the sphere. The period of oscillation of the sphere is 0.027 s.



Use the diagram to calculate, for the wave:

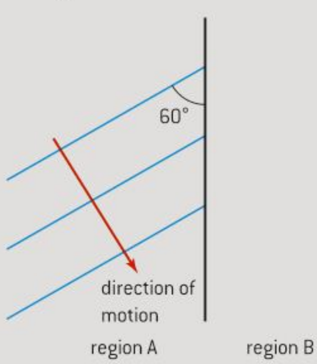
(i) the amplitude

(ii) the wavelength

(iii) the frequency

(iv) the speed.

b) The wave moves from region A into a region B of shallower water. The waves move more slowly in region B. The diagram (not to scale) shows some of the wavefronts in region A.

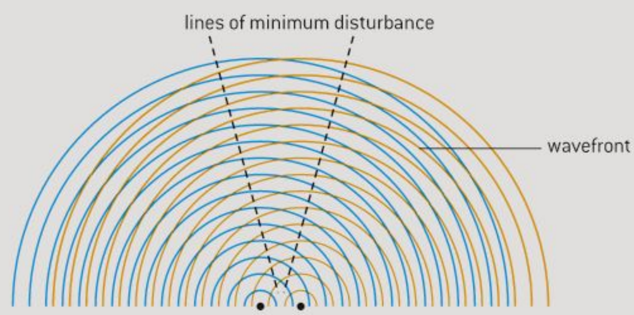


(i) With reference to a wave, distinguish between a ray and a wavefront.

(ii) The angle between the wavefronts and the interface in region A is 60°. The refractive index $_{A}n_{B}$ is 1.4.
Determine the angle between the wavefronts and the interface in region B.

(iii) On the diagram above, construct *three* lines to show the position of three wavefronts in region B.

c) Another sphere is dipped into the water. The spheres oscillate in phase. The diagram shows some lines in region A along which the disturbance of the water surface is a minimum.



(i) Outline how the regions of minimum disturbance occur on the surface.

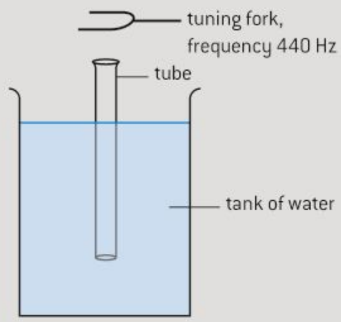
(ii) The frequency of oscillation of the spheres is increased.
State *and* explain how this will affect the positions of minimum disturbance.

(15 marks)

11 *(IB)*

a) Describe *two* ways in which standing waves differ from travelling waves.

b) An experiment is carried out to measure the speed of sound in air, using the apparatus shown below.



A tube that is open at both ends is placed vertically in a tank of water until the top of the tube is just at the surface of the water. A tuning fork of frequency 440 Hz is sounded above the tube. The tube is slowly raised out of the water until the loudness of the sound reaches a maximum for the first time, due to the formation of a standing wave.

(i) Explain the formation of a standing wave in the tube.

(ii) State the position where a node will always be produced.

(iii) The tube is raised a little further. Explain why the loudness of the sound is no longer at a maximum.

c) The tube is raised until the loudness of the sound reaches a maximum for a second time.

Between the two positions of maximum loudness the tube has been raised by 36.8 cm.

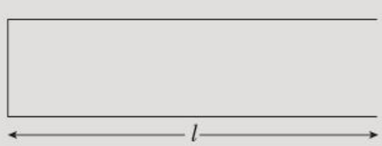
The frequency of the sound is 440 Hz. Estimate the speed of sound in air.

(10 marks)

12 *(IB)*

a) State *two* properties of a standing (stationary) wave.

b) The diagram shows an organ pipe that is open at one end.



The length of the pipe is *l*. The frequency of the fundamental (first harmonic) note emitted by the pipe is 16 Hz.

(i) On a copy of the diagram, label with the letter P the position along the pipe where the amplitude of oscillation of the air molecules is the largest.

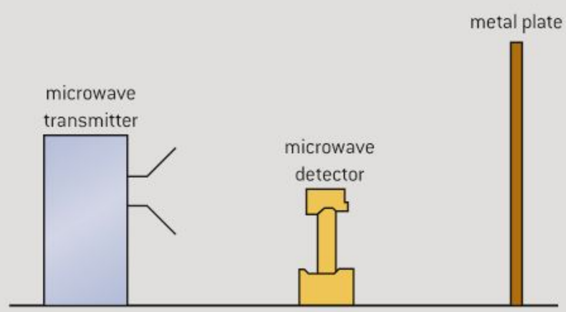
(ii) The speed of sound in the air in the pipe is 330 m s^{-1}. Calculate the length *l*.

c) Use your answer to b) (ii) to suggest why it is better to use organ pipes that are closed at one end for producing low frequency notes rather than pipes that are open at both ends.

(8 marks)

13 *(IB)*

A microwave transmitter emits radiation of a single wavelength towards a metal plate along a line normal to the plate. The radiation is reflected back towards the transmitter.



A microwave detector is moved along a line normal to the microwave transmitter and the metal plate. The detector records a sequence of equally spaced maxima and minima of intensity.

a) Explain how these maxima and minima are formed.

b) The microwave detector is moved through 130 mm from one point of minimum intensity to another point of minimum intensity. On the way it passes through nine points of maximum intensity. Calculate the

(i) wavelength of the microwaves.

(ii) frequency of the microwaves.

c) Describe and explain how it could be demonstrated that the microwaves are polarized.

(11 marks)