Worked example

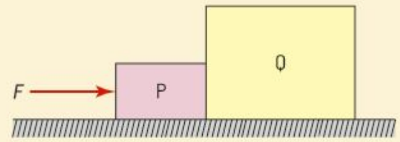
1 Calculate the upward force acting on a skydiver of mass 80 kg who is falling at a constant speed.

Solution

The weight of the skydiver is $80 \times 9.8 = 784$ N. Because the skydiver is falling at constant speed (i.e. terminal speed) the upwards drag force is equal to the downwards weight.

The upward force is 784 N.

2 P and Q are two boxes that are pushed across a rough surface at a constant velocity with a horizontal force of 30 N. The mass of P is 2.0 kg and the mass of Q is 4.0 kg.



State the resultant force on box Q.

Solution

Q is moving at constant speed. So the resultant force on this box must be 0 N.

2.3 Work, energy, and power

Understanding

- Principle of conservation of energy
- Kinetic energy
- Gravitational potential energy
- Elastic potential energy
- Work done as energy transfer
- Power as rate of energy transfer
- Efficiency

Nature of science

The theory of conservation of energy allows many areas of science to be understood at a fundamental level. It allows the explanation of natural phenomena but also means that scientists can predict the outcome of a previously unknown effect. The conservation of energy also demonstrates that paradigm shifts occur in science: the interchangeability of mass and energy as predicted by Einstein is an example of this.

Applications and skills

- Discussing the conservation of total energy within energy transformations
- Sketching and interpreting force – distance graphs
- Determining work done including cases where a resistive force acts
- Solving problems involving power
- Quantitatively describing efficiency in energy transfers

Equations

- work: $W = Fs\cos\theta$
- kinetic energy: $E_K = \frac{1}{2}mv^2$
- elastic potential energy: $E_p = \frac{1}{2}k(\Delta x)^2$
- change in gravitational potential energy: $\Delta E_p = mg\Delta h$
- power $= Fv$
- efficiency $= \frac{\text{useful workout}}{\text{total work in}} = \frac{\text{useful power out}}{\text{total power in}}$

Introduction

This sub-topic looks at the physics of energy transfers. The importance of energy is best appreciated when it moves or transfers between different forms. Then we can make it do a useful job for us.

What is important is that you learn to look below the surface of the bald statement: "electrical energy is converted into internal energy" and ensure that you can talk about the detailed changes that are taking place.

Energy forms and transfers

Energy can be stored in many different forms. Some of the important ones are listed in the table below.

Energy	Nature of energy associated with...	Notes
kinetic	the motion of a mass	
(gravitational) potential	the position of a mass in a gravitational field	sometimes the word "gravitational" is not used
electric/magnetic	charge flowing	
chemical	atoms and their molecular arrangements	
nuclear	the nucleus of an atom	related to a mass change by $\Delta E = \Delta mc^2$
elastic (potential)	an object being deformed	The word "potential" is not always used
thermal (heat)	a change in temperature or a change of state	A change of state is a change of a substance between phases, i.e. solid to liquid, or liquid to gas. This is referred to as "energy transferred as a result of temperature difference" in line with the IB Guide. The colloquial term "heat" is usually acceptable when referring to situations involving conservation of energy situations.
mass	conversion to binding (nuclear) energy when nuclear changes occur	
vibration (sound)	mechanical waves in solids, liquids, or gases	the amount of sound energy transferred is almost always negligible when compared with other energy forms
light	photons of light	sometimes called "radiant energy" another form of electric/magnetic

Energy can be transferred between any of its forms and it is during such transfers we see the effects of energy. For example, water can fall vertically to turn the turbine of a hydroelectric power station and drive a generator. Lots of things are happening here. The water molecules are attracted by the Earth and accelerate downwards through the pipe. Their momentum is transferred to the blades of the turbine which rotates and turns the coils in the generator. As a result of the coils turning, electrons are forced to move and there is an electric current. Overall, this chain of physical processes can be summed up as the conversion of gravitational potential energy of the water into an electrical form. Another example of an energy transfer is that of an animal converting stored chemical energy in the muscles leading to kinetic and gravitational potential energy forms with some of the chemical energy also appearing as frictional losses.

Learn to recognise the physical (and sometimes chemical) processes that are going on in the system. It is easy to describe the changes in fairly broad energy transfer terms. Always try, however, to explain the effects in terms of microscopic or macroscopic interactions. Whatever the form of the energy transfer, we use a unit of energy called the joule (J). This is in honour of James Joule (1818–1889) the English scientist who devoted his scientific efforts to studying energy and its transfers.

One joule is the energy required when a force of one newton acts through a distance of one metre.

When energy changes from one form to another we find that nothing is lost (providing that we take care to include every single form of energy that is included in the changes).

This is known as the **principle of conservation of energy** which says that energy cannot be created or destroyed.

Since Einstein's work at the turn of the twentieth century, we now recognise that mass must be included in our table of energy forms. For most changes the mass difference is insignificant, but in nuclear changes it makes a major contribution.

In some applications, such as when discussing the output of power stations, the joule is too small a unit so you will frequently see energies expressed in megajoules (MJ or 10^6 J) or even gigajoules (GJ or 10^9 J). Get used to working in large powers of ten and with prefixes when dealing with energies.

In some parts of physics, different energy units are used. These have usually arisen historically. An example of this is the electronvolt (eV), which is the energy gained by an electron when it is accelerated through a potential difference of one volt. Another example is the calorie used by dieticians; this is an old unit that has lingered in the public domain for perhaps longer than it should! One calorie (cal) is 4.2 J. You will learn the detail about any special units used in the course in the appropriate place in this book.

Worked example

Describe the mechanisms associated with the energy changes that occur when a balloon is blown up.

Solution

Air molecules gain kinetic energy that is used to store elastic potential energy in the skin of the balloon and to make changes in the energy of the air. The air inside is able to exert more force (pressure) outwards on the skin of the balloon until a new equilibrium is established between the tension in the skin and the atmospheric pressure.

Doing work

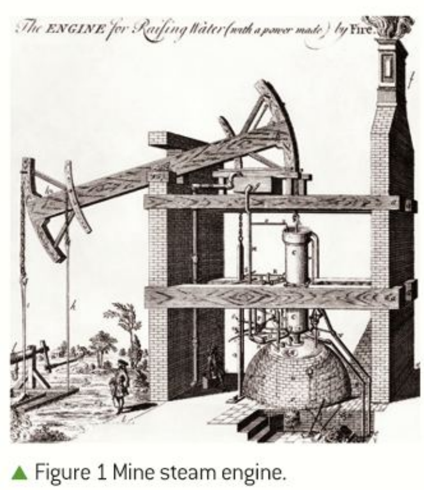
In 1826, Gaspard-Gustave Coriolis was studying the engineering involved in raising water from a flooded underground mine. He realized that energy was being transferred when the steam engines were pumping the water vertically. He described this energy transfer as "work done", and he recognized that the energy transferred when the pumping engines exerted a force on a particular mass of water and lifted it from the bottom of the mine to the surface.

In other words

$$\textit{work done (in J)} = \textit{force exerted (in N)} \times \textit{distance moved in the direction of the force (in m)}$$

So, when a weight of 5 N of water is lifted vertically through a height of 150 m, then the work done by the engine on the water is $5 \times 150 = 750$ J.

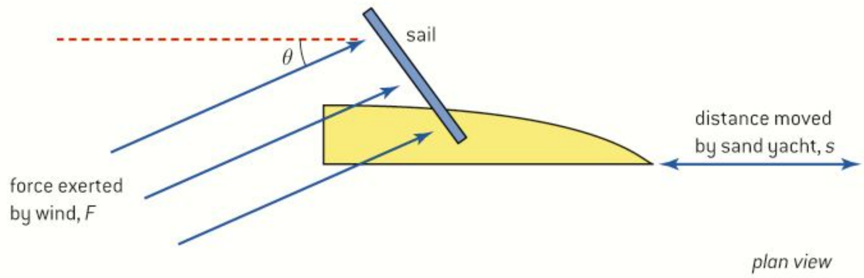
In the example of the mine, the force and the distance moved are in the same direction (vertically upwards), but in many cases this will not



▲ Figure 1 Mine steam engine.

be the case. A good example of this is a sand yacht, where the force F from the wind acts in one direction and the sail is set so that the yacht moves through a displacement s that is at an angle θ to the wind.

In this case, the distances moved in the direction of the force and by the yacht are not the same. You must use the component of force in the direction of movement.



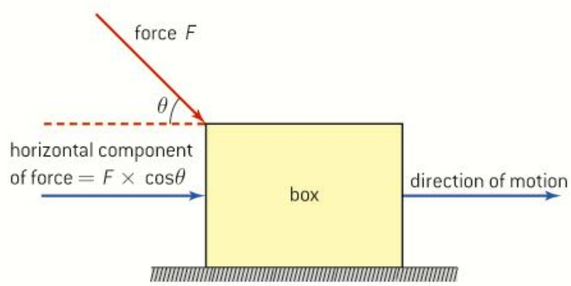
▲ Figure 2 Force on sail.

In this case,

$work\ done = F \cos\theta \times s$

Work done against a resistive force

Work is done when a resistive force is operating too. Consider a box being pushed at a constant speed in a horizontal straight line. For the speed to be constant, friction forces must be overcome. The force that overcomes the friction may not act in the direction of movement. Again, the work that will be done by the force (or the force provider) is *force acting* × *distance travelled* × cos θ (figure 3).



▲ Figure 3 Resistive force.

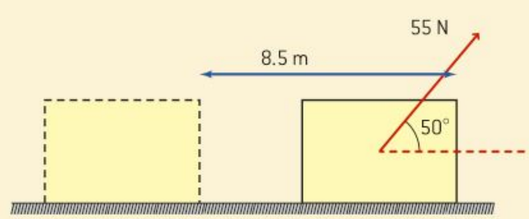
Worked examples

1 The thrust (driving force) of a microlight aircraft engine is 3.5×10^3 N. Calculate the work done by the thrust when the aircraft travels a distance of 15 km.

Solution

Work done = force × distance = 3500 × 15 000 = 5.3×10^7 J ≡ 53 MJ.

2 A large box is pulled a distance of 8.5 m along a rough horizontal surface by a force of 55 N that acts at 50° to the horizontal. Calculate the work done in moving the box 8.5 m.



Solution

The component of force in the direction of travel is $55 \times \cos 50 = 35.4$ N.

The work done = this force component × distance travelled $= 35.4 \times 8.5 = 301$ J.

3 An object moving in a straight line has an initial kinetic energy of 24 J. Calculate the distance in which the object will come to rest if a net force of 4.0 N opposes the motion.

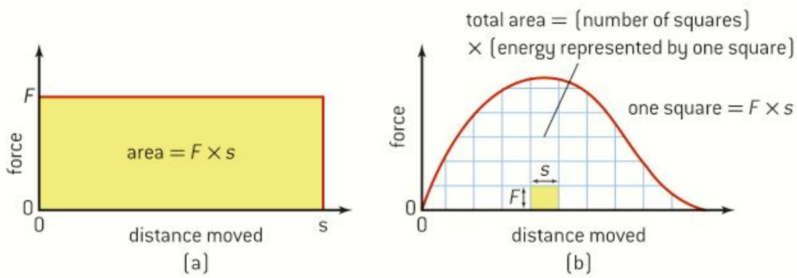
Solution

There must be 24 J of work done to stop the motion of the object. The force acting is 4.0 so $\frac{24}{4} = 6$ m.

Force–distance graphs

In practice, it is rare for the force acting on a moving object to be constant. Real railway trucks or sand yachts have air resistance and other forces that lead to energy losses that vary with the speed of the object or the surface over which it runs.

We can deal with this if we know how the force varies with distance. Some examples are shown in figure 4.



▲ Figure 4 Force–distance graphs.

For a constant force (figure 4(a)), the graph of force against distance will be a straight line parallel to the *x*-axis.

The *work done* is the product of *force* × *distance* (we are assuming that $\theta = 90°$ in this case); this corresponds to the area under the graph of force against distance.

When the force is not constant with distance moved (figure 4(b)) the work done is still the area under the line, but this time you have to work a little harder by estimating the number of squares under the graph and equating each square to the energy that it represents. The product of (*energy for one square*) × (*number of squares*) will then give you the overall work done.

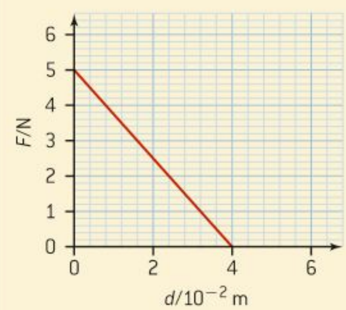
There is a further example of this type of calculation later in the topic on elastic potential energy.

Power

Imagine two boys, Jean and Phillipe with the same weight (let's say 650 N) who climb the same hill (70 m high). Because they have the same weight and climb the same vertical distance they both gain the same amount of gravitational potential energy. This is at the expense of the chemical energy reserves in their bodies. However, suppose Jean climbs the hill in 150 s whereas Phillipe takes 300 s.

Worked example

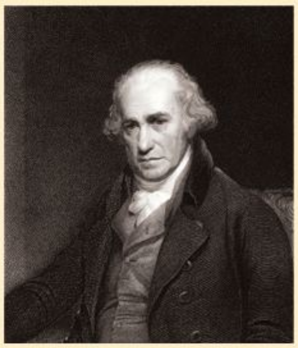
The graph shows the variation with displacement *d* of a force *F* that is applied to a toy car. Calculate the work done by *F* in moving the toy through a distance of 4.0 cm.



Solution

The work done is equal to the area of the triangle enclosed by the graph and the axes. This is $\frac{1}{2}$ × base of the triangle × height of triangle $= \frac{1}{2} \times 4.0 \times 10^{-2} \times 5.0 = 0.10$ J

James Watt was a mechanical engineer who worked at the end of the 18th century and the beginning of the 19th. He did major work in improving steam engine design and even developed a way of making copies of paper documents that was used in offices until early in the 20th century.



The obvious difference here is that Jean is gaining potential energy twice as fast as Phillipe because Jean's time to make the climb is half that of Phillipe. This difference is important when we want to compare two machines taking different times to carry out the same amount of work.

The quantity **power** is used to measure the **rate of doing work**, in other words it is the number of joules that can be converted every second. So, power is defined as:

$$\text{power} = \frac{\textit{energy transferred}}{\textit{time taken for transfer}}$$

Using the correct quantities is important here. If the energy change is in joules, and the time for the transfer measured in seconds then the power is in watts (W).

$$1\ \text{W} \equiv 1\ \text{J s}^{-1}$$

In the example of Jean and Phillipe above, both did 45.5 kJ of work, but Jean's power in climbing the hill was $\frac{650 \times 70}{150} = 303$ W and Phillipe's was 152 W because he took twice as long in the climb.

The equation *work done* = *force* × *distance* can be rearranged to give another useful expression for power.

$$\textit{power} = \frac{\textit{work done}}{\textit{time}} = \textit{force} \times \frac{\textit{distance moved by force}}{\textit{time}}$$

It is easy to see that this is the same as *power* = *force* × *speed*.

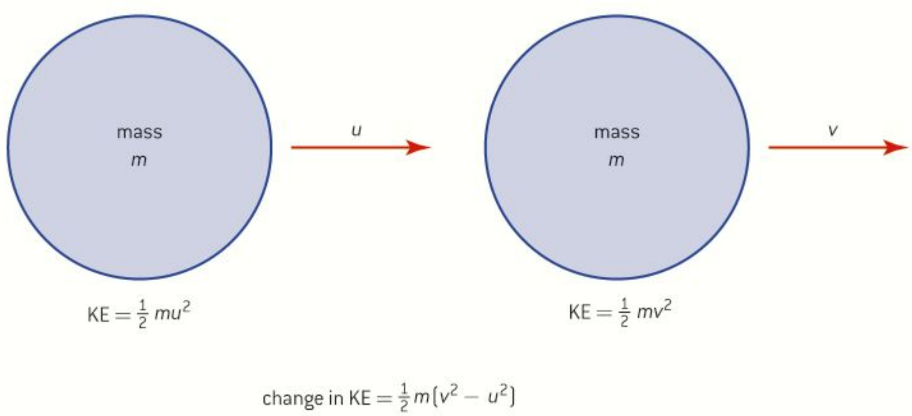
So the power required to move an object travelling at a speed v with a force F is Fv.

Nature of science

You will see other units for power used in various areas of everyday life. As an example, the horsepower is one measure used by car manufacturers to provide customers with data about car engines. This is a unit that dates back to the beginning of the 18th century and was used to compare the power of an average horse with steam engines. (1 horsepower is equal to about 750 W in modern units.)

Kinetic energy, KE

Kinetic energy is the energy an object has because of its motion; kinetic energy (sometimes abbreviated to KE) has its own symbol, E_K. Objects gain kinetic energy when their speed increases.



▲ Figure 5 Kinetic energy of mass.

Imagine an object of mass m which is at rest at time $t = 0$ and which is accelerated by a force F for a time T. The kinematic equations and Newton's second law allow us to work out the speed of the object v at time $t = T$.

The acceleration a is $\frac{F}{m}$ (using Newton's second law of motion).

Therefore $v = 0 + \frac{F}{m}T$ (because the initial speed is zero). So $F = \frac{mv}{T}$

The work done on the mass is the gain in its kinetic energy, E_K, and is $F \times s$ where s is the distance travelled and therefore

$$E_K = F \times s = \frac{mv}{T} \times \frac{vT}{2}$$

because $s = \frac{(v+0)T}{2}$

The work done by the force is equal to the gain in kinetic energy and is

$$E_K = \frac{1}{2}mv^2$$

Remember that this is the case where the initial speed was 0. If the object is already moving at an initial speed u then the change in kinetic energy ΔE_K will be $\frac{1}{2}m(v^2 - u^2)$.

There is a subtle piece of notation here. When we talk about a value of kinetic energy, we write E_K, but when we are talking about a change in kinetic energy from one value to another then we should write ΔE_K where Δ, as usual, means "the change in".

Worked examples

1 A vehicle is being designed to capture the world land speed record. It has a maximum design speed of 1700 km h^{-1} and a fully fuelled mass of 7800 kg.

Calculate the maximum kinetic energy of the vehicle.

Solution

1700 km h^{-1} $\equiv$ 470 m s^{-1} (this is greater than the speed of sound in air!)

$E_K = \frac{1}{2}mv^2 = 0.5 \times 7800 \times 470^2 = 8.6 \times 10^8$ J

$\equiv 0.86$ GJ

2 A car of mass 1.3×10^3 kg accelerates from a speed of 12 m s^{-1} to a speed of 20 m s^{-1}. Calculate the change in kinetic energy of the car.

Solution

$\Delta E_K = \frac{1}{2}m(v^2 - u^2) = 0.5 \times 1300 \times (20^2 - 12^2)$

$= 650 \times (400 - 144) = 1.7 \times 10^5$ J

Worked example

A car is travelling at a constant speed of 25 m s^{-1} and its engine is producing a useful power output of 20 kW. Calculate the driving force required to maintain this speed.

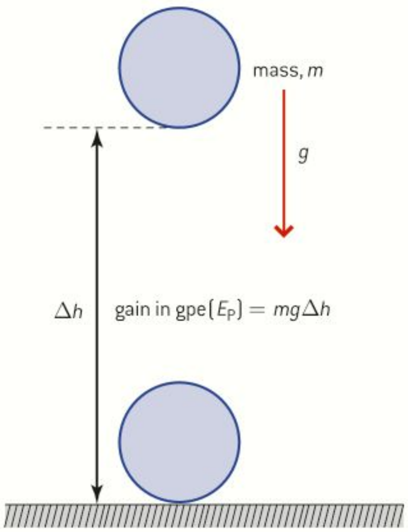
Solution

driving force $= \frac{\text{power}}{\text{speed}}$

$= \frac{20\,000}{25} = 800$ N

Examiner's tip

This equation for ΔE_K is one that needs a little care. Notice where the squares are; they are attached to each individual speed. This equation is not the same as $\frac{1}{2}m(v - u)^2$.



▲ Figure 6 Gravitational potential energy.

Gravitational potential energy, GPE

Gravitational potential energy is the energy an object has because of its position in a gravitational field. When a mass is moved vertically up or down in the gravity field of the Earth, it gains or loses gravitational potential energy (GPE). The symbol assigned to GPE is E_P. Only the initial and final positions relative to the surface determine the change of GPE (assuming that there is no air resistance on the way). For this reason, gravitational force is among the group of forces said to be **conservative**, because they conserve energy (see the Nature of Science box below).

The work done when an object is raised at constant speed through a change in height Δh is, as usual, equal to *force* × *distance* moved. In this case the force required is mg and work done, $E_P = mg \times \Delta h$.

As usual, the value of g is 9.8 m s^{-2} close to the Earth's surface, but this value becomes smaller when we move away from the Earth.

Nature of science

Conservative forces

There is an important difference between forces such as gravity and frictional forces. When only gravity acts, the energy change depends on the start height and end height of the motion but not on the route taken by an object to get from start to finish. Horizontal movement does not have to be counted if there is no friction acting. This type of force is said to be *conservative;* in other words, it conserves energy. We could recover all the energy by moving the object back to the start. Contrast this with the friction force that acts between a book and a table as the book is moved around the table's surface from one point to another. If the book goes directly from start to finish a certain amount of energy will be used up to overcome the friction. But if the book goes by a longer route, more energy is needed (*work done = friction force × distance travelled*). When it is necessary to know the exact route before we can calculate the total energy conversion, the force is said to be *non-conservative*. If we move the book back again, we cannot recover the energy in the way that we could when only gravity acted.

You will not need to write about the meaning of the term "conservative force" in the IB Diploma Programme physics examination.

Energy moving between GPE and KE

Sometimes the use of gravitational potential energy and kinetic energy together provides a neat way to solve a problem.

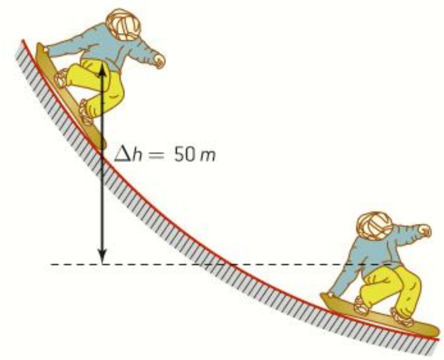
A snowboarder is moving down a curved slope starting from rest ($u = 0$). The vertical change in height of the slope is $\Delta h = 50$ m.

What is the speed of the snowboarder at the bottom of the slope? Assume we can ignore friction at the base of the board and the air resistance.

What we must *not* do in this example is to use the kinematic (*suvat*) equations. They cannot be applied in this case because the acceleration of the snowboarder will not be constant – *suvat* only works when we are certain that a does not change.

Although the *suvat* equations give the correct answer here, they should not be used because the physics is incorrect. The final answer happens to be correct only because we use the start and end points and also as a consequence of the conservative nature of gravity. We are also using an average value for acceleration down the slope by assuming that the angle to the horizontal is constant. The equations would not give the correct answer if we brought friction forces into the calculation.

Conservation of energy comes to our aid because (as the friction losses are negligible) we know that the loss of gravitational potential energy as the boarder goes down the slope is equal to the gain in kinetic energy over the length of the slope. Because we know that the GPE change only depends on the initial and final positions then we do not need to worry at all about what is going on at the base of a board during the ski run.



▲ Figure 7 Mechanics of snowboarding.

So $\Delta E_p = mg\Delta h = \frac{1}{2}mv^2$ and $v = \sqrt{2g\Delta h}$,

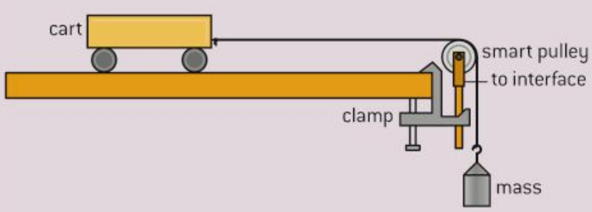
in this case $v = \sqrt{2 \times 9.8 \times 50} = 31 \text{ m s}^{-1}$

(This is a speed of about $110\,\text{km h}^{-1}$ which tells you that the assumption about no air resistance and no friction is a poor one, as any snowboarder will tell you!)

Notice that the answer does not depend on the mass of the boarder; the mass term cancels out in the equations.

Investigate!

Converting GPE to KE



This experiment will help you to understand the conversion between KE and GPE.

- Arrange a cart on a track and compensate the track for friction. This is done by raising the left-hand end of the track through a small distance so that the cart neither gains nor loses speed when travelling down the track without the string attached.
- Have a string passing over a pulley at the end of the track and tie a weight of known mass to the other end of the track.
- Measure the mass of the cart too.
- Devise a way to measure the speed of the cart. You could use a "smart pulley" that can measure the speed as the string turns the pulley wheel, or an ultrasound sensor, or a data logger with light gates.
- When the mass is released, the cart will gain speed, as the gravitational potential energy of the weight changes.
- Make measurements to assess the gravitational potential energy lost (you will need to know the vertical height through which the mass falls) and the kinetic energy gained (you will need the final speed). Notice that only the falling mass is losing GPE but both the mass and the cart are gaining kinetic energy.
- Compare the two energies in the light of the likely errors in the experiment. Is the energy conserved? Where do you expect energy losses in the experiment to occur?

Worked examples

1 A ball of mass 0.35 kg is thrown vertically upwards at a speed of 8.0 m s^{-1}. Calculate

a) the initial kinetic energy

b) the maximum gravitational potential energy

c) the maximum height reached.

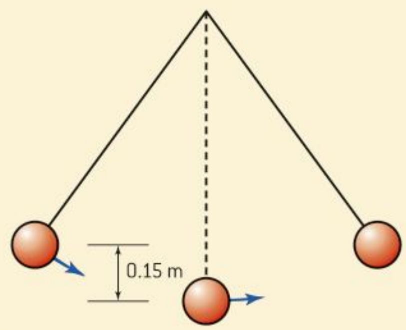
Solution

a) $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.35 \times 8^2 = 11.2\,\text{J}$

b) At the maximum height all the initial kinetic energy will have been converted to gravitational potential energy so the maximum value of the GPE is also 11.2 J.

c) The maximum GPE is 11.2 J and this is equal to $mg\Delta h$, so $\Delta h = \frac{11.2}{mg} = \frac{11.2}{0.35 \times 9.8} = 3.3$ m.

2 A pendulum bob is released from rest 0.15 m above its rest position. Calculate the speed as it passes through the rest position.



Solution

E_K at the rest position $= E_P$ at the release position;

$\frac{1}{2}mv^2 = mg\Delta h$ which rearranges to

$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.15} = 1.7\,\text{m s}^{-1}$.

Elastic potential energy

The shape of a solid can be changed by applying a force to it. Different materials will respond to a given force in different ways; some materials will be able to return the energy that has been stored in them when the force is removed. A metal spring is a good example of this; most springs are designed to store energy in this way in many different contexts. The materials that can return energy in this way have stored **elastic potential energy**.

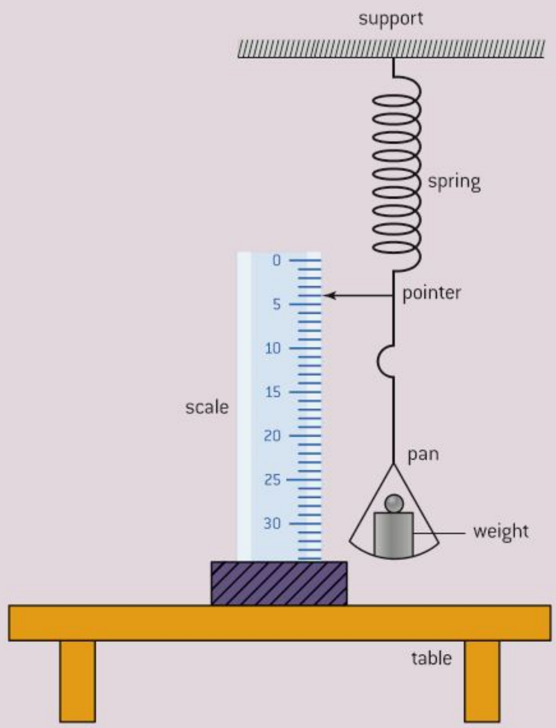
To discuss elastic potential energy we need to know something about the properties of springs. This is an area of physics that has been studied for a long time. Robert Hooke, a contemporary of Newton, published a rule about springs that has become known as **Hooke's law**.

Investigate!

Investigating Hooke's law

Robert Hooke realized that there was a relationship between the load on a spring and the extension of the spring.

- Arrange a spring of known unstretched length with a weight hanging on the end of the spring.
- You will need to devise a way to measure the **extension** (change in length from the original unstretched length) of the string for each of a number of different increasing weights hanging on the end of the spring.
- Repeat the measurements as you remove the weights as a check.
- Plot a graph of force (weight) acting on the spring (y-axis) against the extension (x-axis). This is not the obvious way to draw the graph, but it is the way normally used. Normally, we would plot the dependent variable (the extension in this case) on the y-axis.

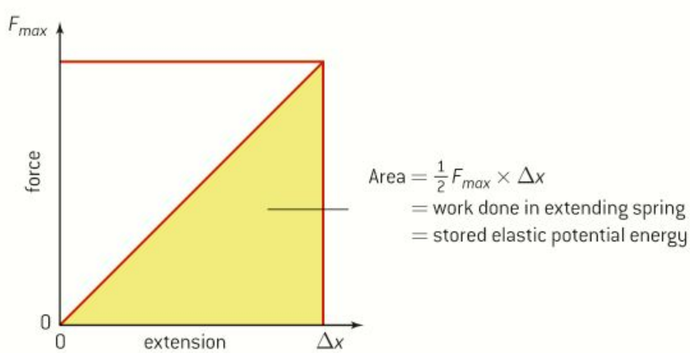


For small loads acting on the spring, Hooke showed that the extension of the spring is directly proportional to the load. (For larger loads, this relationship breaks down as the microscopic arrangement of atoms in the spring changes. We shall not consider this part of the deformation, however.)

Hooke's rule means that the graph of force F against extension (Δx) is a straight line going through the origin.

In symbols the rule is $F \propto \Delta x$, or $F = k\Delta x$ where k is a constant known as the **spring constant**, which has units of N m^{-1}. The gradient of the graph is equal to k (this is why the graph is plotted the "wrong way round").

We can now relate this graph to the work done in stretching the spring. The force is not constant (the bigger the extension, the bigger the force



▲ Figure 8 Work done in stretching spring.

Worked examples

1 A spring, of spring constant $48\,N\,m^{-1}$, is extended by 0.40 m. Calculate the elastic potential energy stored in the spring.

Solution

Energy stored $= \frac{1}{2}kx^2 = 0.5 \times 48 \times 0.40^2 = 0.38$ J

2 An object of mass 0.78 kg is attached to a spring of unstretched length 560 mm. When the object has come to rest the new length of the spring is 620 mm. Calculate the energy stored in the spring as a result of this extension.

Solution

The change in length of the spring $\Delta x = 620 - 560 = 60$ mm

The tension in the spring will be equal to the weight of the object $= mg = 0.78 \times 9.8 = 7.64$ N

The energy stored in the spring $= \frac{1}{2}F\Delta x = 0.5 \times 7.64 \times 0.06 = 0.23$ J

required) but we now know how to deal with this. The work done on the spring is the area under the graph of force against extension.

The area is a right-angled triangle and is equal to

$\frac{1}{2} \times$ *force to stretch spring* $\times$ *extension*,

in symbols $E_p = \frac{1}{2} F_{max}\Delta x$.

We know from Hooke's law that $F = k\Delta x$, so $k = \frac{F}{\Delta x}$ and therefore E_p is also equal to $\frac{1}{2} k(\Delta x)^2$.

Efficiency

In the *Investigate!* experiment where gravitational potential energy was transferred to kinetic energy, it is likely that some gravitational potential energy did not appear in the motion of the cart. Some energy will have been lost to internal energy as a consequence of friction and to elastic potential energy stored when the string changes its shape. We need to have a way to quantify these losses. One way to do this is to compare the total energy put into a system with the useful energy that can be taken out. This is known as the **efficiency** of the transfer and can be applied to all energy transfers, whether carried out in a mechanical system, electrical system or other type of transfer. You can expect to meet efficiency calculations in any area of physics where energy transfers occur. The definition can also be applied to power transfers, because the energy change in these cases takes place in the same time for the total energy in and the useful work out.

$$\text{efficiency} = \frac{\text{useful work out}}{\text{total energy in}} = \frac{\text{useful power out}}{\text{total power}}$$

Worked example

1 An electric motor raises a weight of 150 N through a height of 7.2 m. The energy supplied to the motor during this process is 3.5×10^4 J. Calculate:

a) the increase in gravitational potential energy

b) the efficiency of the process.

Solution

a) $\Delta E_p = 150 \times 7.2 = 1080$ J

b) Efficiency $= \frac{\text{useful work out}}{\text{energy in}} = \frac{1080}{3500} = 0.31$ or 31%

2.4 Momentum

Understanding

- Newton's second law expressed as a rate of change of momentum
- Impulse and force–time graphs
- Conservation of linear momentum
- Elastic collisions, inelastic collisions, and explosions

Nature of science

The concept of momentum has arisen as a result of evidence from observations carried out over many centuries. The principle of conservation of momentum is an example of a law that has universal applicability. It allows the prediction of the outcomes of physical interactions at both the macroscopic and the microscopic level. Many areas of physics are informed by it, from the kinetic theory of gases to nuclear interactions.

Applications and skills

- Applying conservation of momentum in simple isolated systems including (but not limited to) collisions, explosions, or water jets
- Using Newton's second law quantitatively and qualitatively in cases where mass is not constant
- Sketching and interpreting force–time graphs
- Determining impulse in various contexts including (but not limited to) car safety and sports
- Qualitatively and quantitatively comparing situations involving elastic collisions, inelastic collisions, and explosions

Equations

- momentum: $p = mv$
- Newton's second law (momentum version): $F = \frac{\Delta p}{\Delta t}$
- kinetic energy: $E_K = \frac{p^2}{2m}$
- impulse: $= F\Delta t = \Delta p$

Introduction

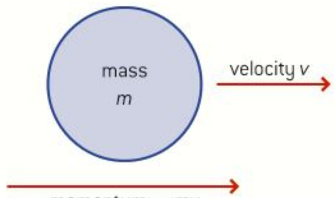
Many sports involve throwing or catching a ball. Compare catching a table-tennis (ping-pong) ball with catching a baseball travelling at the same speed. One of these may be a more painful experience than the other! What is different in these two cases is the mass of the object. The velocity may be the same in both cases but the combination of velocity and mass makes a substantial difference. Equally, comparing the experience of catching a baseball when gently tossed from one person to another with catching a firm hit from a good player should tell you that changes in velocity make a difference too.

Momentum

We call the product of the mass m of an object and its instantaneous velocity v the **momentum p of the object** ($p = m \times v$) and this quantity turns out to have far-reaching importance in physics.

First, here are some basics ideas about momentum.

- Momentum is mass × velocity *never* mass × speed.
- Momentum has direction. The mass is a scalar but velocity is a vector. When mass and velocity are multiplied together, the momentum is also a vector with the same direction as the velocity. Think of the mass as "scaling" the velocity – in other words, just making it bigger by a factor equal to the mass of the object.
- It follows that the unit of momentum is the product of the units of mass and velocity, in other words **kg m s^{-1}**. There is a shorter alternative to this that we shall see later.
- If velocity or mass is changing then the momentum must also be changing. We shall be looking at the two cases where one quantity changes while the other is held constant.
- When a net resultant force acts on an object, the object accelerates and the velocity must change. This means a change in momentum too. So a net force leads to a change in momentum.



▲ Figure 1 Momentum and velocity.

Worked examples

1 A ball of mass 0.25 kg is moving to the right at a speed of 7.4 m s^{-1}. Calculate the momentum of the ball.

Solution

$p = mv = 0.25 \times 7.4 = 1.85$ kg m s^{-1} to the right.

2 A ball of mass 0.25 kg is moving to the right at a speed of 7.4 m s^{-1}. It strikes a wall at 90° and rebounds from the wall leaving it with a speed of 5.8 m s^{-1} moving to the left. Calculate the change in momentum.

Solution

From the previous example the initial momentum is 1.85 kg m s^{-1} to the right.

The final momentum is $0.25 \times 5.8 = 1.45$ kg m s^{-1} to the left.

So taking the direction to the right as being positive, the change in momentum $= -1.45 - (+1.85) = -3.3$ kg m s^{-1} to the right (or alternatively $+3.3$ kg m s^{-1} to the left).


▲ Figure 2 Newton's cradle.

Collisions and changing momentum

You may have seen a "Newton's cradle". Newton did not invent this device (it was developed in the twentieth century as an executive toy), but it helps us to visualize some important rules relating to his laws of motion, and it certainly seems appropriate to associate his name with it.

One of the balls (the right-hand one in figure 2) is moved up and to the right, away from the remaining four. When released the ball falls back and hits the second ball from the right. The right-hand ball then stops moving and the left-most ball moves off to the left. It is as though the motion of the original ball transfers through the middle three – which remain stationary – and appears at the left-hand end.

You can analyse this in terms of physics that you already know.

The right-hand ball gains gravitational potential energy when it is raised. When it is released the potential energy is converted into kinetic energy as the ball gains speed. Eventually the ball strikes the next stationary one along, and a pair of action–reaction forces acts at the surfaces of the two balls (you might like to consider what they are). The stationary ball is compressed slightly by the force acting on it and this compression moves as a wave through the middle three balls. When the compression reaches the left-hand ball the elastic potential energy associated with the compression wave is converted into kinetic energy (because there is a net force on this sphere) and work begins to be done against the Earth's gravity. The left-hand ball starts to move to the left. When the speed of the ball becomes zero, it is at its highest point on the left-hand side and the motion repeats in the reverse direction. The overall result is that the end spheres appear to perform an alternate series of half oscillations.

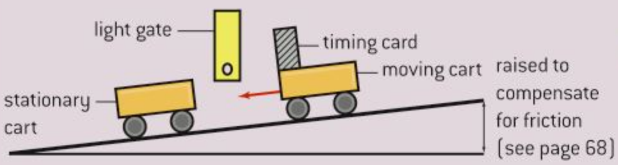
Another way to view this sequence is as a transfer of momentum. Think about a simpler case where only two spheres are in contact (in the toy, three balls can be lifted out of the way). The right-hand sphere gains momentum as it falls from the top of its swing. When it collides with the other sphere, the momentum appears to be transferred to the second sphere. The first sphere now has zero momentum (it is stationary) and the second has gained momentum. What rules govern this transfer of momentum?

These interactions between the balls in the Newton's cradle are called **collisions**. This is the term given to any interaction where momentum transfers. Examples of collisions include firing a gun, hitting a ball with a bat in sport, two toy cars running into each other, and a pile driver sinking vertical cylinders into the ground on a construction site. There are many more.

Investigate!

Is momentum conserved?

The exact details of this experiment will depend on the apparatus you have in your school.



- The experiment consists of measuring the speed of a cart of known mass and then launching it at another cart also of known mass that is initially stationary. Often there will be carts available to you of almost identical mass, as this is a particularly easy case to begin with.
- You will need a way to measure the velocity of the carts just before and just after the collision. This could be done in various ways:
  - using a data logger with motion sensors
  - using a paper tape system where a tape attached to the cart is pulled through a device that makes dots at regular time intervals on the tape
  - using a video camera and computer software
  - using a stop watch to measure the time taken to cover a short, known distance before and after the collision.
- If your carts run on a track, you can allow for the friction at the cart axles and the air resistance. This is done by raising the end of the track a little so that, when pushed, a cart runs at a constant speed. The friction at the bearings and the air resistance will be exactly compensated by the component of the cart weight down the track.
- For the first part of the experiment, begin by arranging that the two carts will stick together after colliding. This can be done in a number of ways, including using modelling clay, or a pin attached to one cart entering a piece of cork in the other, or two magnets (one on each cart) that attract.
- Make the first (moving) cart collide with and stick to the second (stationary) cart.
- Measure the speed of the first cart before the collision and the combined speed of the carts after the collision.
- Repeat the experiment a number of times and think carefully about the likely errors in the results.

The initial momentum is *mass of first cart* × *velocity of first cart*. The final momentum is (*mass of first cart* + *mass of second cart*) × *combined velocity of both carts*.

- What can you say about the total momentum of the system before the collision compared with the total momentum after the collision? You should consider the experimental errors in the experiment before making your judgement.
- If you have done the experiment carefully, you should find that the momentum before and the momentum after the collision are approximately equal.
- Now extend your experiment to different cases:
  - where the carts do not stick together
  - where they are both moving before the collision
  - where the masses are not the same, and so on.
- You may need to alter how you measure the velocity to cope with the different cases.

If you carry out an experiment that compares the momentum before with the momentum after a collision then you should find (within the experimental uncertainty of your measurements) that the total momentum in the system does not change.

An important point here is that there must be no force from outside the system acting on the objects taking part in the collision. As we saw earlier, external forces produce accelerations, and this changes the velocity and hence the momentum.

You might argue that gravitational force is acting – and gravity certainly is acting on the carts in the experiment – but because the gravitational force is not being allowed to do any work, the force does not contribute to the interaction because the carts are not moving vertically.

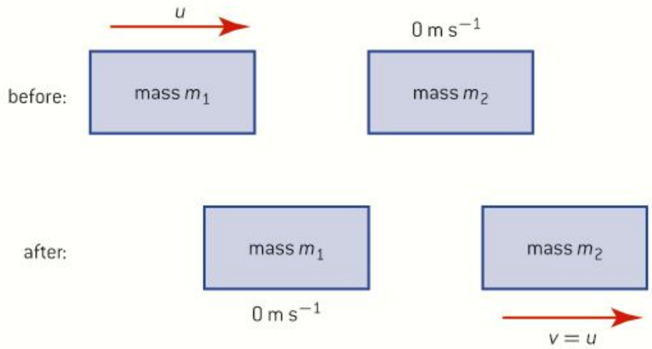
Momentum is always constant if no external force acts on the system. This is known as the **principle of conservation of linear momentum**, the word "linear" is here because this is momentum when the objects concerned are moving in straight lines. This conservation rule has never been observed to be broken, and is one of the important conservation rules that (as far as we know) are true throughout the universe. In nuclear physics, particles have been proposed in order to conserve momentum in cases where it was apparently not being conserved. These particles were subsequently found to exist.

Momentum conservation is such an important rule that it is worth us considering a few different situations to see how momentum conservation works.

In each of these cases we will assume that the centres of the objects lie on a straight line so that the collision happens in one dimension.

Two objects with the same mass, one initially stationary, when no energy is lost

This is known as an **elastic collision** and for it to happen no permanent deformation must occur in the colliding objects and no energy can be released as internal energy (through friction), sound, or any other way. The spheres in the Newton's cradle lose only a little energy every time they collide and this is why this is a reasonable demonstration of momentum effects.



▲ Figure 3 Elastic collision between two identical masses.

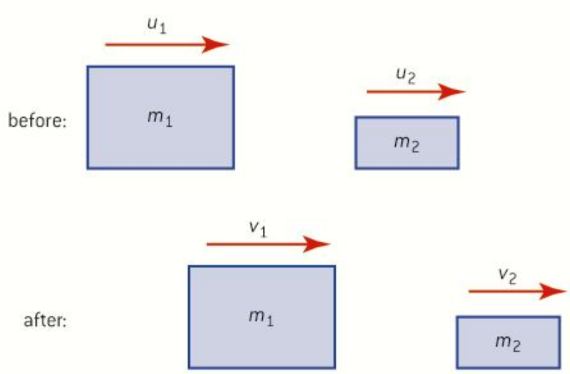
The first object collides with the second stationary object. The first object stops and remains at rest while the second moves off at the speed that the first object had before the collision. (Try flicking a coin across a smooth table to hit an identical coin head-on.) In this case momentum is conserved because (using an obvious set of symbols for the mass m of the objects and their velocities u and v)

$$m_1 u = m_2 v$$

Because $m_1 = m_2$ then $u = v$ so the velocity of one mass before the collision is equal to the velocity of the second mass afterwards. The kinetic energy of the moving mass (whichever mass is moving) is $\frac{1}{2}mu^2$ and it does not change either.

Two objects with different masses when no energy is lost

This time (as you may have seen in an experiment or demonstration) the situation is more complicated.



▲ Figure 4 Two moving objects with different mass in an elastic collision.

Again

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

but this time we cannot eliminate the mass terms so easily. What we do know, though, is that kinetic energy is conserved. So the kinetic energy before the collision must equal the kinetic energy after the collision (because no energy is lost).

This means that (summing the kinetic energies before and after the collision)

$$\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

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Popper and falsifiability

No-one has yet observed a case where the momentum in an isolated system is not conserved, but we should continue to look! Karl Popper, a philosopher of the twentieth century, argued that the test of whether a theory was truly scientific was that it was capable of being falsified. By this he meant that there has to be an experiment that could in principle contradict the hypothesis being tested. Popper argued that psychoanalysis was not a science because it could not be falsified by experiment.

Popper also applied his ideas to scientific induction. He said that, although we cannot prove that the Sun will rise tomorrow, because it always has we can use the theory that the Sun rises in the morning until the day when it fails to do so. At that point we must revise our theory. What should happen if momentum appears not to be conserved in an experiment? Is it more sensible to look for a new theory or to suggest that something has been overlooked?

The momentum and kinetic energy equations can be solved to show that

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \left(\frac{2m_2}{m_1 + m_2}\right) u_2$$

and

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) u_2 + \left(\frac{2m_1}{m_1 + m_2}\right) u_1$$

(You will not be expected to prove these in, or memorize them for, an examination.)

There are some interesting cases when mass m_2 is stationary and is struck by m_1:

- Case 1: m_1 is much smaller than m_2. Look at the v_1 equation. If $m_1 \ll m_2$ then the first term becomes roughly $\left(\frac{-m_2}{m_2}\right) u_1$ which is u_1; the second term is zero because $u_2 = 0$. So the small mass "bounces" off the large mass, reversing its direction (shown by the minus sign). The large mass gains speed in the forward direction (the original direction of the small mass). The magnitude of the speed of the larger mass is roughly $\left(\frac{2m_1}{m_2}\right) u_1$ and this is a small fraction of the original speed of the small mass because $\left(\frac{2m_1}{m_2}\right)$ is much smaller than 1.
- Case 2: m_1 is much greater than m_2. Here the original mass loses hardly any speed (though it must lose a little). The momentum lost by m_1 is given to m_2 which moves off in the same direction, but at about twice the original speed of m_1. Look at the v_1 and v_2 equations and satisfy yourself that this is true.

Two objects colliding when energy is lost

When a moving object collides with a stationary one and the two objects stick together, then some of the initial kinetic energy is lost. After the collision, there is a single object with an increased (combined) mass and a single common velocity. This is known as an **inelastic collision** (figure 5).

This is a case you may have studied experimentally in the *Investigate!*

The momentum equation this time is

$$m_1 u_1 = (m_1 + m_2) v_1$$

A rearrangement shows that $v_1 = \frac{m_1}{(m_1 + m_2)} u_1$ and, as we might expect, the final velocity is in the same direction as before but is always smaller than the initial velocity.

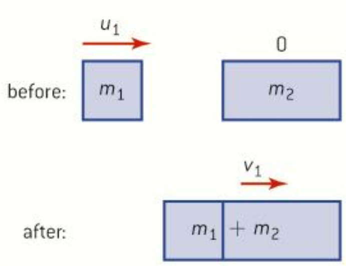
As for energy loss, the incoming kinetic energy is $\frac{1}{2} m_1 u_1^2$ and the final kinetic energy is (substituting for v_1)

$$\frac{1}{2}(m_1 + m_2)\frac{m_1^2}{(m_1 + m_2)^2} u_1^2$$

This is

$$\frac{1}{2}\frac{m_1^2}{(m_1 + m_2)} u_1^2.$$

The ratio $\frac{\text{initial kinetic energy}}{\text{final kinetic energy}}$ is $\frac{(m_1 + m_2)}{m_1}$



▲ Figure 5 Inelastic collision between two masses.

Two objects when energy is gained

There are many occasions when two initially stationary masses gain energy in some way. Some laboratory dynamics carts have a way to show this. An easy way is to attach two small strong magnets to the front of the carts so that when the carts are released after being held together with like poles of the magnets facing each other, the magnets repel and drive the carts apart.

The analysis is quite straightforward in this case.

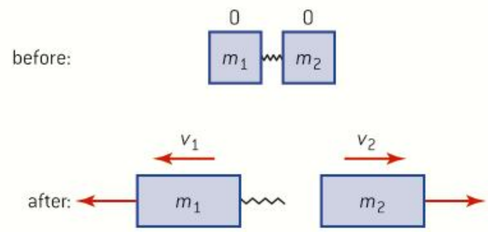
The initial momentum is zero as neither object is moving.

After the collision the momentum is $m_1v_1 + m_2v_2 = 0$.

So $m_1v_1 = -m_2v_2$

The objects move apart in opposite directions. If the masses are equal the speeds are the same, with one velocity the negative of the other. If the masses are not equal then

$$\frac{m_1}{m_2} = \frac{-v_2}{v_1}$$



▲ Figure 6 Energy gained in a collision.

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Helpful or not?

Are conservation laws helpful to scientists? On the one hand they allow the prediction of as yet untested cases, but on the other hand they may restrict the progress of science. This can happen if scientists are not prepared to challenge the status quo.

In 2012 the results of an experiment suggested that neutrinos could travel faster than the speed of light. This flew in the face of the accepted science originally proposed by Einstein. Later investigations showed that small errors in the timings had occurred in the experiments, and that there was no evidence for faster-than-light travel. Were the scientists right to publish their results so that others could test the new proposals?

Worked examples

1 A rail truck of mass 4500 kg moving at a speed of 1.8 m s^{-1} collides with a stationary truck of mass 1500 kg. The two trucks couple together. Calculate the speed of the trucks immediately after the collision.

Solution

The intial momentum = 4500×1.8 kg m s^{-1}.

The final momentum = $(4500 + 1500) \times v$, where v is the final speed.

Momentum is conserved and so $v = \frac{4500 \times 1.8}{(4500 + 1500)} = \frac{4.5 \times 1.8}{6.0} = 1.3(5) = 1.4$ m s^{-1}.

2 Stone A of mass 0.5 kg travelling at 3.8 m s^{-1} across the surface of a frozen pond collides with stationary stone B of mass 3.0 kg. Stone B moves off at a speed of 0.65 m s^{-1} in the same original direction as stone A.

Calculate the final velocity of stone A.

Solution

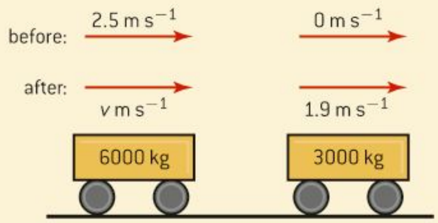
The initial momentum is $0.5 \times 3.8 = 1.9$ kg m s^{-1}

The final momentum is $3.0 \times 0.65 + 0.5v_A$ and this is equal to 1.9 kg m s^{-1}

$$v_A = \frac{1.9 - (3.0 \times 0.65)}{0.5} = -\frac{0.05}{0.5} = -0.1 \text{ kg m s}^{-1}$$

The final velocity of stone A is 0.1 kg m s^{-1} in the opposite direction to its original motion.

3



A railway truck of mass 6000 kg collides with a stationary truck of mass 3000 kg. The first truck moves with an initial speed of 2.5 m s^{-1} and the second truck moves off with a speed of 1.9 m s^{-1} in the same direction.

Calculate:

a) the velocity of the first truck immediately after the collision

b) the loss in kinetic energy as a result of the collision.

Solution

a) The initial momentum is $6000 \times 2.5 = 15\,000$ kg m s^{-1}.

The final momentum is $6000v + 3000 \times 1.9$

Equating these values: $15\,000 = 6000v + 5700$, so $6000v = 9300$ and $v = 1.5(5)$ kg m s^{-1}. This is positive so the first truck continues to move to the right after the collision at a speed of 1.6 kg m s^{-1}.

b) The initial kinetic energy $= \frac{1}{2} \times 6000 \times 2.5^2 = 18\,750$ J.

After the collision the total kinetic energy $= \frac{1}{2} \times 6000 \times 1.55^2 + \frac{1}{2} \times 3000 \times 1.9^2$

$= 7208 + 5415 = 12\,623$ or to 2 s.f. 13 000 J.

So the loss in kinetic energy is $18\,750 - 13\,000 = 5800$ J.

Energy and momentum

There is a convenient link between kinetic energy and momentum.

Kinetic energy, $E_K = \frac{1}{2}mu^2$; momentum, $p = mu$ and therefore $p^2 = m^2u^2$. So

$$E_K = \frac{p^2}{2m}$$

This is often of use in calculations.

Applications of momentum conservation

Recoil of a gun

Figure 7 shows a gun being fired to trigger a snow fall. This prevents a more dangerous avalanche. When the gun fires its shell, the gun moves backwards in the opposite direction to that in which the shell goes. You should be able to explain this in terms of **momentum conservation**.

▲ Figure 7 Firing a gun to cause an avalanche.

Initially, both gun and shell are stationary; the initial total momentum is zero. The shell is propelled in the forwards direction through the gun by the expansion of gas following the detonation of the explosive in the shell. So this is one of the cases discussed earlier, one in which energy is gained. The explosion is a force internal to the system. Gas at high pressure is generated by the explosion in the chamber behind the shell. The gas exerts a force on the interior of the shell chamber and hence a force on the gun as well. The explosive releases energy and this is transferred into kinetic energies of both the shell and the gun.

The initial linear momentum was zero and no *external* force has acted on the system. The momentum must continue to be zero and this can only be so if the gun and the shell move in opposite directions with the same magnitude of momentum. The shell will go fast because it has a small mass compared to the gun; the gun moves relatively slowly.

Water hoses

Watch a fire being extinguished by firemen using a high-pressure hose and you will see the effect of water leaving the system. Often two or more firemen are needed to keep the hose on target because there is a large force on the hose in the opposite direction to that of the water. This can be seen in a more modest form when a garden hose connected to a tap starts to shoot backwards in unpredictable directions if it is not held when the tap is turned on.

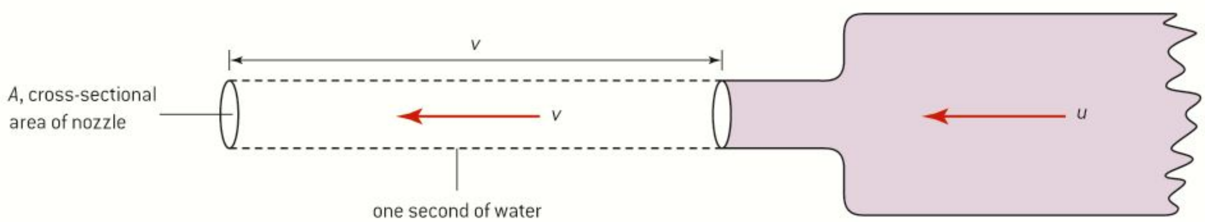
▲ Figure 8 Fire fighting.

The cross-sectional area of the hose is greater than that of the nozzle through which the water emerges. The mass of water flowing past a point in the hose every second is the same as the mass that emerges from the nozzle every second. So the speed of the water emerging from the nozzle must be greater than the water speed along the hose itself. The water gains momentum as it leaves the hose because of this increase in exit speed compared with the flow speed in the hose.

The momentum of the system has to be constant and so there must be a force backwards on the end of the hose which needs to be countered by the efforts of the firemen. The kinetic energy and the momentum are being supplied by the water pump that feeds water to the fire hose or whatever originally created the pressure in the supply to the garden tap.

The momentum lost by the system per second is *(mass of water leaving per second)* × *(speed at which water leaves the nozzle – speed in the hose)*.

The mass of water lost per second is $\frac{\Delta m}{\Delta t}$ and so the momentum lost per second $\dot{p}$ is $\frac{\Delta m}{\Delta t}(v - u)$ where v is the speed of water as it leaves the nozzle and u is the speed of the water in the hose.



▲ Figure 9 Water leaving the hose.

If we know the cross-sectional area A of the nozzle of the hose and the density of the water, ρ, then $\frac{\Delta m}{\Delta t}$ can be determined. Figure 9 shows what happens inside the hose during a one-second time interval. Every second you can imagine a cylinder of water leaving the hose; this cylinder is v long and has an area A. The volume leaving per second is therefore Av

The mass of the water leaving in one second $\frac{\Delta m}{\Delta t}$

$$= (\textit{density of water}) \times (\textit{volume of cylinder}) = \rho Av.$$

Because the mass entering and leaving the nozzle per second must be the same, this means that the change of momentum in one second $= \frac{\Delta m}{\Delta t} \times (v - u) = \rho Av(v - u)$. If $u \ll v$ then the expression simplifies to ρAv^2.

The hose is an example of where care is needed when looking at the whole system. Consider what happens if the water is directed at a vertical wall. The water strikes the wall, loses all its horizontal momentum, and trickles vertically down the wall. The momentum must

Worked example

A mass of 0.48 kg of water leaves a garden hose every second. The nozzle of the hose has a cross-sectional area of 8.4×10^{-5} m^2. The water flows in the hose at a speed of 0.71 m s^{-1}. The density of water is 1000 kg m^{-3}.

Calculate:

a) the speed at which water leaves the hose

b) the force on the hose.

Solution

a) 0.48 kg of water in one second corresponds to a volume of $\frac{0.48}{1000}$ m^{-3} leaving the hose per second. This leaves through a nozzle of area 8.4×10^{-5} m^2 so the speed v must be $\frac{0.48}{1000 \times 8.4 \times 10^{-5}} =$ 5.71 m s^{-1}.

b) The force on the hose = mass lost per second × change in speed
$= 0.48 \times (5.71 - 0.71)$
$= 2.4$ N

have gone into the wall, its foundations and, therefore, the ground. So we might conclude that the Earth itself has gained momentum and that we can speed up the Earth's rotation by using a garden hose. But this is not true, because the water had to be given momentum originally by a pump. This gain in momentum at the pump must have given some momentum to the Earth too. The amount of momentum the Earth gained at the pump is equal and opposite to the momentum gained by the Earth when the water strikes the wall.

Rocketry

Earlier in this topic we discussed the acceleration of a rocket and looked at the situation from the perspective of Newton's second and third laws. A similar analysis is possible in terms of momentum conservation.

Rockets operate effectively in the absence of an atmosphere because momentum is conserved. All rockets release a liquid or gas at high speed. The fluid can be a very hot gas generated in the combustion of a solid chemical (as in a domestic rocket) or from the chemical reaction when two gases are mixed and burnt. Or it can be a fluid stored inside the rocket under pressure. In each case, the fluid escapes from the combustion/storage chamber through nozzles at the base of the rocket. As a result the rocket accelerates in the opposite direction to the direction in which the fluid is ejected. Momentum is conserved. The rate of loss of momentum from the rocket in the form of high-speed fluid must be equal to the rate of gain in momentum of the rocket. We shall look again at the mathematics of the rocket later in this topic.

Helicopters

Helicopters are aircraft that can take off and land vertically and also can hover motionless above a point on the ground. Vertical flight is said to have been invented in China about 2500 years before the present, but anyone who has seen the seeds from certain trees spiral down to the ground will realize where the original design came from. There were many attempts to build flying machines on the helicopter principle over the centuries, but the first commercial aircraft flew in the 1930s.

A helicopter uses the principle of conservation of linear momentum in order to hover. The rotating blades exert a force on originally stationary air causing it to move downwards towards the ground gaining momentum in the process. No external force acts and as a result there is an upward force on the helicopter through the rotors.

▲ Figure 10 Helicopter.

Impulse

Earlier we used Newton's second law of motion: $F = ma$ where the symbols have their usual meaning.

We can rearrange this equation using one of the kinematic equations:

$$a = \frac{(v-u)}{t}$$

Eliminating a gives $F = \dfrac{m(v-u)}{t}$

which means that $force = \dfrac{change\ in\ momentum}{time\ taken\ for\ change}$

or

$$\textit{force} \times \textit{time} = \textit{change in momentum}$$

This equation gives a relationship between force and momentum and provides a further clue to the real meaning of momentum itself. The equation shows that we can change momentum (in other words, accelerate an object) by exerting a large force for a short time or by exerting a small force for a long time. A small number of people can get a heavy vehicle moving at a reasonable speed, but they have to push for a much longer time than the vehicle itself would take if powered by its own engine (which produces a larger force).

The product of ***force*** and ***time*** is given the name **impulse**, its units are newton seconds (N s). Impulse is the same as change in momentum, and N s gives us an alternative to kg m s^{-1} as a unit for momentum. You can use whichever you prefer.

There is a short-hand way to write the $force = \frac{change\ in\ momentum}{time}$ equation. We have already used the convention that "Δ" means "change in". So in symbols the equation becomes

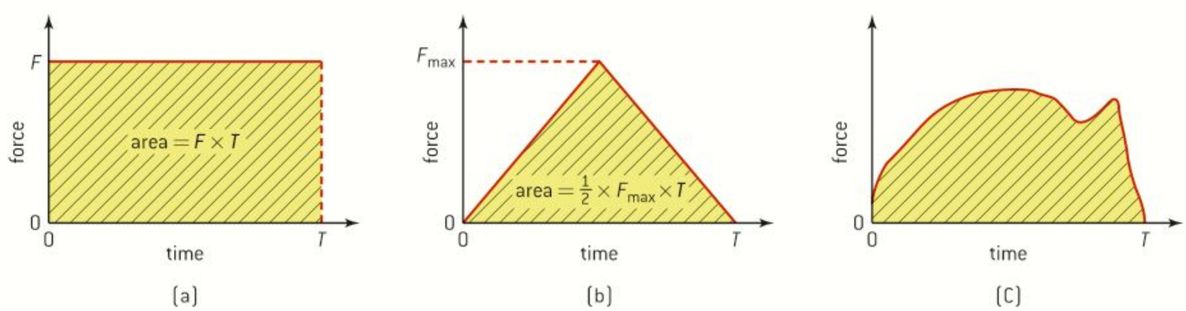
$$F = \frac{\Delta p}{\Delta t}$$

where p is the symbol for momentum and t (as usual) means time.

(If you are familiar with differential calculus you may also come across this expression in the form $F = \frac{dp}{dt}$, but using the equation in this form will take us too far away from IB Diploma Programme physics.)

Force–time graphs

Up till now we have usually assumed that forces are constant and do not change with time. This is rarely the case in real life and we need a way to cope with changes in momentum when the force is not constant. The equation $F = \frac{\Delta p}{\Delta t}$ helps here because it suggests that we use a force–time graph.



▲ Figure 11 Force–time graphs.

If a constant force acts on a mass, then the graph of force, F, against time, t, will look like figure 11(a). The change in momentum is $F \times T$ and this is the shaded area below the line. The area below the line in a force–time graph is equal to the change in momentum.

Another straightforward case that is more plausible than a constant force is the graph shown in figure 11(b) where the force rises to a maximum $\left(F_{max}\right)$ and then falls back to zero in a total time T. The area under the graph this time is $\frac{1}{2}F_{max}T$ and, again, this is the change in momentum.

The final case (figure 11(c)) is one where there is no obvious mathematical relationship between F and t, but nevertheless we have

Worked examples

1 An impulse of 85 N s acts on a body of mass 5.0 kg that is initially at rest. Calculate the distance moved by the body in 2.0 s after the impulse has been delivered.

Solution

The change in momentum is 85 kg m s^{-1} so that the final speed is $\frac{85}{5} = 17$ m s^{-1}. In 2.0 s the distance travelled is 34 m.

2 An impulse I acts on an object of mass m initially at rest. Determine the kinetic energy gained by the object.

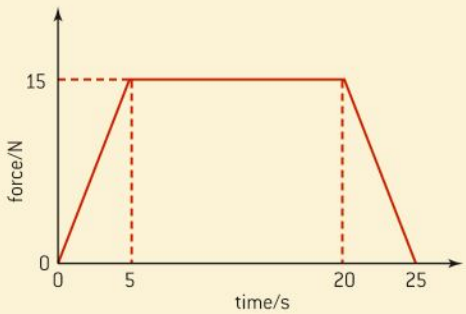
Solution

The change in speed of the object is $\frac{I}{m}$. The object is initially at rest and the gain in kinetic energy is $\frac{1}{2}mv^2$ which is $\frac{1}{2}m\left(\frac{I}{m}\right)^2 = \frac{I^2}{2m}$

a graph of how F varies with t. This time you will need to estimate the number of squares under the graph and use the area of one square to evaluate the momentum change.

Worked examples

1 The sketch graph shows how the force acting on an object varies with time.



The mass of the object is 50 kg and its initial speed is zero.

Calculate the final speed of the object.

Solution

The total area under the $F-t$ graph is
$2 \times \left(\frac{1}{2} \times 15 \times 5\right) + (15 \times 15) = 75 + 225$
$= 300$ N s

This is the change in momentum.

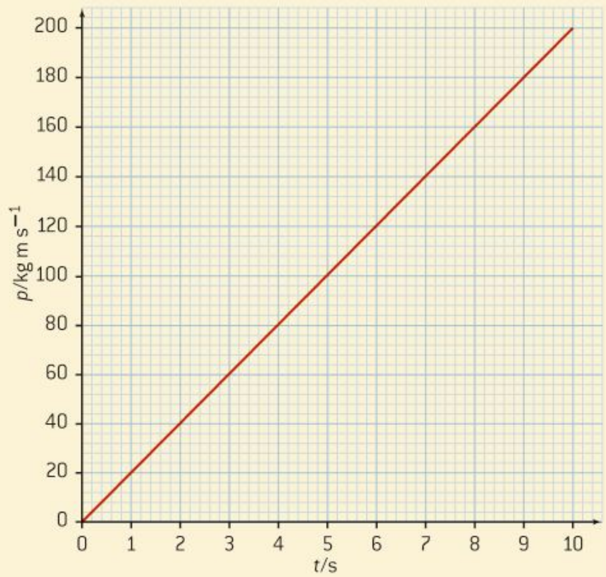
So the final speed is $\frac{300}{50} = 6.0$ m s^{-1}

2 The graph shows how the momentum of an object of mass 40 kg varies with time.

Calculate, for the object:

a) the force acting on it

b) the change in kinetic energy over the 10 s of the motion.



Solution

a) The gradient of the graph is $\frac{\Delta p}{\Delta t}$ and this is equal to the force.

The magnitude of the gradient is $\frac{200}{10}$ and this gives a value for force of 20 N.

b) $\Delta E_K = \frac{p^2}{2m} = \frac{200^2}{2 \times 40} = 500$ J

Revisiting Newton's second law

In the last sub-topic we used $F = ma$ to show that F also equals $\frac{\Delta p}{\Delta t}$. Using the full expression for momentum p gives

$$F = \frac{\Delta(mv)}{\Delta t}$$

This can be written as (using the product rule)

$$F = m\frac{\Delta v}{\Delta t} + v\frac{\Delta m}{\Delta t}$$

You may have to take this algebra on trust but, by thinking through what the two terms stand for, you should be able to understand the physics that they represent.

The first term on the left-hand side is just *mass* $\times$ $\frac{\textit{change in velocity}}{\textit{change in time}}$ which we know as *mass* $\times$ *acceleration* – our original form of Newton's second law of motion. The second term on the right-hand side is something new, it is the *instantaneous velocity* $\times$ $\frac{\textit{change in mass}}{\textit{change in time}}$.

Our first version of Newton's second law was a simpler form of the law than the full "change in momentum" version. The new extra term takes account of what happens when the mass of the accelerating object is changing.

Rockets and helicopters again

Rockets

We showed that rockets are an excellent example of momentum conservation in action. The difficulty in analysing the rocket example is that the rocket is always losing mass (in the form of gas or liquid propellant), so m is not constant.

$F = \frac{m\Delta v}{\Delta t} + \frac{v\Delta m}{\Delta t}$ is our new version of Newton's second law of motion and, in this case, $F = 0$ because there is no external force acting on the system.

So $\frac{m\Delta v}{\Delta t} = -\frac{v\Delta m}{\Delta t}$. The two terms have quite separate meanings: $\frac{m\Delta v}{\Delta t}$ refers to the *instantaneous* mass of the rocket (including the remaining fuel) and to the acceleration of this total mass $\frac{\Delta v}{\Delta t}$. The other term is the ejection speed of the fuel and the rate at which mass is lost from the system $\frac{\Delta m}{\Delta t}$. So at one instant in time, the acceleration of the rocket

$$a = \frac{\Delta v}{\Delta t} = -\frac{v\Delta m}{m\Delta t}.$$

The negative sign reminds us that the rocket is *losing* mass while *gaining* speed.

Helicopters

With the helicopter hovering, there is a weight force downwards and so the equation now becomes

$$Mg = \frac{m\Delta v}{\Delta t} + \frac{v\Delta m}{\Delta t} \text{ where } M \text{ is the mass of the helicopter}$$

There is no change in the speed of the helicopter (it is hovering) so $\Delta v = 0$, there is however momentum gained by the air that moves downwards. This is the speed, v, the air gains multiplied by the mass of the air accelerated every second so $Mg = v\frac{\Delta m}{\Delta t}$ or

weight of helicopter (in N) = *mass of air pushed downwards per second* (in kg s^{-1}) × *speed of air downwards* (in m s^{-1})

Momentum and safety

Many countries have made it compulsory to wear seat belts in a moving vehicle. Likewise, many modern cars have airbags that inflate very quickly if the car is involved in a collision.

Obviously, both these devices restrain the occupants of the car, preventing them from striking the windscreen or the hard areas around it. But there is more to the physics of the air bag and the seat belt than this.

On the face of it, someone in a car has to lose the same amount of kinetic energy and momentum whether they are stopped by the windscreen or restrained by the seat belt. What differs in these two

Worked example

A small firework rocket has a mass of 35 g. The initial rate at which hot gas is lost from the firework has been lit is $3.5\ \text{g s}^{-1}$ and the speed of release of this gas from the rear of the rocket is $130\ \text{m s}^{-1}$.

Calculate the initial acceleration of the rocket.

Solution

$a = \frac{\Delta v}{\Delta t} = -\frac{v\Delta m}{m\Delta t}$ where v is the release speed of the gas, $\frac{\Delta m}{\Delta t}$ is the rate of loss of gas, and m is the mass of the rocket

$a = \frac{130 \times 3.5}{65} = 7.0\ \text{m s}^{-2}$

▲ Figure 12 Car safety: seat belts and air bags.

cases is the time during which the loss of energy and momentum occur. Unrestrained, the time to stop will be very short and the deceleration will therefore be very large. A large deceleration means a large force and it is the magnitude of the force that determines the amount of damage.

Seat belts and air bags dramatically increase the time taken by the occupants of the car to stop and as *force* $\times$ *time* $=$ *momentum change*, for a constant change in momentum, a long stopping time will imply a smaller, and less damaging, force.

Momentum and sport

This sub-topic began with a suggestion that it was less painful to catch a table-tennis ball than a baseball. You should now be able to understand the reason for the difference. You should also realize why good technique in many sports hinges on the application of momentum change.

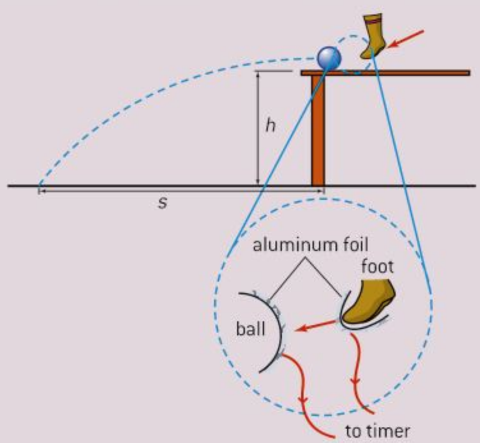
Many sports in which an object – usually a ball – is struck by hand, foot or bat rely on the efficient transfer of momentum. This transfer is often enhanced by a "follow through", which increases the contact time between bat and ball. The player maintains the same force but for a longer time, so the impulse on the ball will increase, increasing the momentum change as well.

Think about your sport and how effective use of momentum change can help you.

Investigate!

Estimating the force on a soccer ball

This experiment will allow you to estimate the force used to kick a soccer ball. It uses many of the ideas contained in this topic and is a good place to conclude our look at IB mechanics!

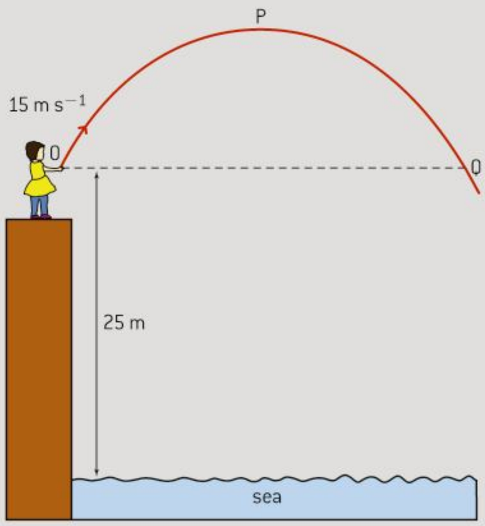


- The basis of the method is to measure the contact time between the foot and the ball and the subsequent change in momentum of the ball. The use of *force* $\times$ *contact time* $=$ *change in momentum* allows the force to be calculated.
- To measure the contact time: stick some aluminium foil to the shoe of the kicker and to the soccer ball. Set up a data logger or fast-timer so that it will measure the time T for which the two pieces of foil are in contact.
- To measure the change in momentum: the ball starts from rest so all you need is the magnitude of the final momentum. Get the kicker to kick the ball horizontally from a lab bench. Measure the distance s from where the ball is kicked to where it lands. Measure the distance h from the bottom of the ball on the bench to the floor. Use the ideas of projectile motion to calculate (i) the time t taken for the ball to reach the floor $\left[h = \frac{1}{2}gt^2 \text{ and so } t = \sqrt{\frac{2h}{g}}\right]$. Then using this t, the initial speed u of the ball can be estimated as $\frac{s}{t}$. Measure the mass of the ball M and therefore the change in momentum is Mu which is equal to the *force on the ball* $\times$ T.
- This method can be modified for many sports including hockey, baseball, and golf.

Questions

1 *(IB)*

Christina stands close to the edge of a vertical cliff and throws a stone at 15 m s^{-1} at an angle of 45° to the horizontal. Air resistance is negligible.



Point P on the diagram is the highest point reached by the stone and point Q is at the same height above sea level as point O. Christina's hand is at a height of 25 m above sea level.

a) At point P on a copy of the diagram above draw arrows to represent:

(i) the acceleration of the stone (label this A)

(ii) the velocity of the stone (label this V).

b) Determine the speed with which the stone hits the sea. (8 marks)

2 *(IB)*

Antonia stands at the edge of a vertical cliff and throws a stone vertically upwards.

The stone leaves Antonia's hand with a speed $v = 8.0$ m s^{-1}. The time between the stone leaving Antonia's hand and hitting the sea is 3.0 s. Assume air resistance is negligible.

a) Calculate:

(i) the maximum height reached by the stone

(ii) the time taken by the stone to reach its maximum height.

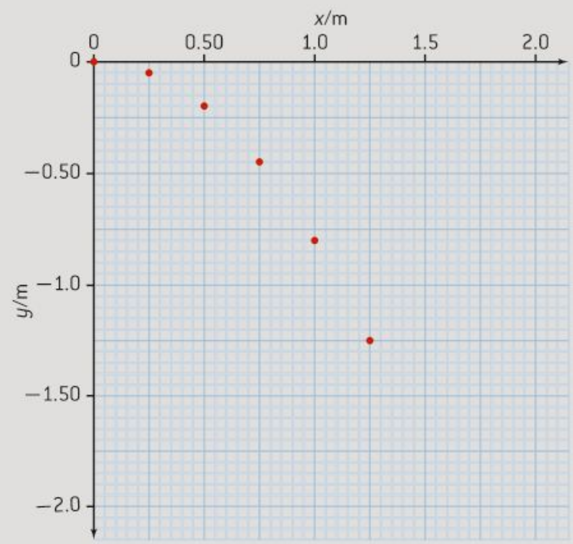
b) Determine the height of the cliff. (6 marks)

3 *(IB)*

A marble is projected horizontally from the edge of a wall 1.8 m high with an initial speed *V*.

A series of flash photographs are taken of the marble and combined as shown below. The images of the marble are superimposed on a grid that shows the horizontal distance *x* and vertical distance *y* travelled by the marble.

The time interval between each image of the marble is 0.10 s.



Use data from the photograph to calculate a value of the acceleration of free fall. (3 marks)

4 *(IB)*

A cyclist and his bicycle travel at a constant velocity along a horizontal road.

a) (i) State the value of the resultant force acting on the cyclist.

(ii) Copy the diagram and draw labelled arrows to represent the vertical forces acting on the bicycle.

(iii) Explain why the cyclist and bicycle are travelling at constant velocity.

b) The total mass of the cyclist and bicycle is 70 kg and the total resistive force acting on them is 40 N. The initial speed of the cycle is 8.0 m s^{-1}. The cyclist stops pedalling and the bicycle comes to rest.

(i) Calculate the magnitude of the initial acceleration of the bicycle and rider.

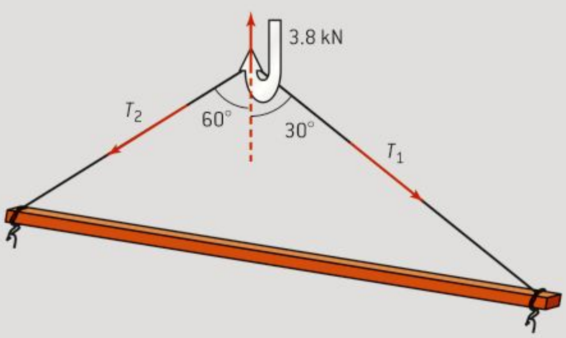
(ii) Estimate the distance taken by the bicycle to come to rest from the time the cyclist stops pedalling.

(iii) State and explain *one* reason why your answer to b)(ii) is an estimate.

(13 marks)

5 A car of mass 1000 kg accelerates on a straight flat horizontal road with an acceleration $a = 0.30$ m s^{-2}. The driving force T on the car is opposed by a resistive force of 500 N. Calculate T. (3 marks)

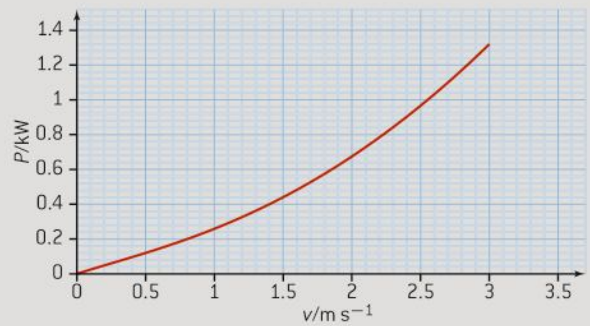
6 A crane hook is in equilibibrium under the action of three forces as shown in the diagram.



Calculate T_1 and T_2. (4 marks)

7 *(IB)*

A small boat is powered by an outboard motor of variable power P. The graph below shows the variation with speed v of the power P for a particular load.



For a steady speed of 2.0 m s^{-1}:

a) use the graph to determine the power of the boat's engine

b) calculate the frictional (resistive) force acting on the boat. (3 marks)

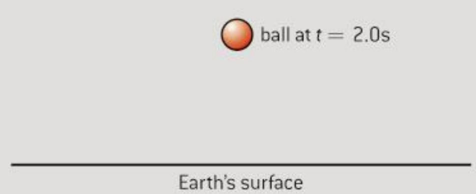
8 *(IB)*

The graph shows the variation with time t of the speed v of a ball of mass 0.50 kg that has been released from rest above the Earth's surface.

The force of air resistance is *not* negligible.

a) State, without any calculations, how the graph could be used to determine the distance fallen.

b) (i) Copy the diagram and draw and label arrows to represent the forces on the ball at 2.0 s.



(ii) Use the graph to show that the acceleration of the ball at 2.0 s is approximately 4 m s^{-2}.

(iii) Calculate the magnitude of the force of air resistance on the ball at 2.0 s.

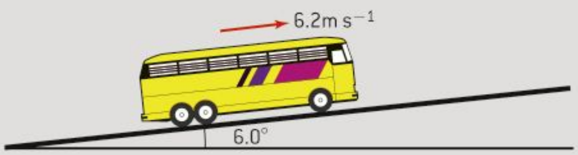
(iv) State and explain whether the air resistance on the ball at $t = 5.0$ s is smaller than, equal to, or greater than the air resistance at $t = 2.0$ s.

c) After 10 s the ball has fallen 190 m.

(i) Show that the sum of the potential and kinetic energies of the ball has decreased by about 800 J. (14 marks)

9 *(IB)*

A bus is travelling at a constant speed of 6.2 m s^{-1} along a section of road that is inclined at an angle of 6.0° to the horizontal.



a) (i) Draw a labelled sketch to represent the forces acting on the bus.

(ii) State the value of net force acting on the bus.

b) The total output power of the engine of the bus is 70 kW and the efficiency of the engine is 35%.

Calculate the input power to the engine.

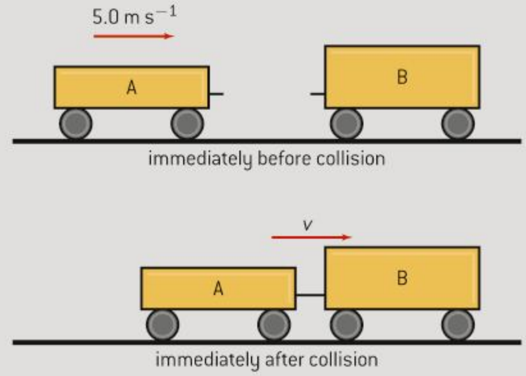
c) The mass of the bus is 8.5×10^3 kg.

Determine the rate of increase of gravitational potential energy of the bus.

d) Using your answer to c (and the data in b), estimate the magnitude of the resistive forces acting on the bus. (12 marks)

10 *(IB)*

Railway truck A moves along a horizontal track and collides with a stationary truck B. The two join together in the collision. Immediately before the collision, truck A has a speed of 5.0 m s^{-1}. Immediately after collision, the speed of the trucks is v.



The mass of truck A is 800 kg and the mass of truck B is 1200 kg.

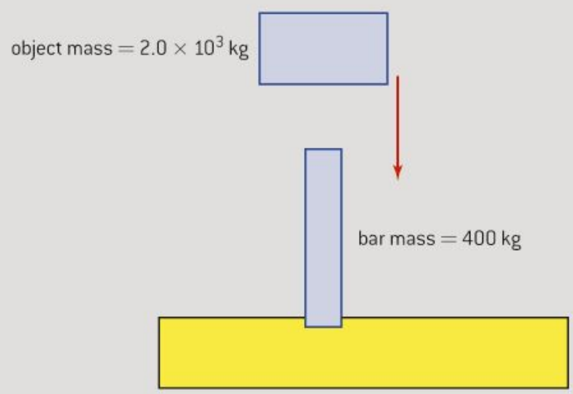
a) (i) Calculate v.

(ii) Calculate the total kinetic energy lost during the collision.

b) Suggest where the lost kinetic energy has gone. (6 marks)

11 *(IB)*

Large metal bars are driven into the ground using a heavy falling object.



The falling object has a mass 2000 kg and the metal bar has a mass of 400 kg.

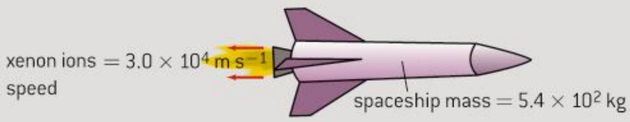
The object strikes the bar at a speed of 6.0 m s^{-1}. It comes to rest on the bar without bouncing. As a result of the collision, the bar is driven into the ground to a depth of 0.75 m.

a) Determine the speed of the bar immediately after the object strikes it.

b) Determine the average frictional force exerted by the ground on the bar. (7 marks)

12 *(IB)*

An engine for a spacecraft uses solar power to ionize and then accelerate atoms of xenon. After acceleration, the ions are ejected from the spaceship with a speed of $3.0 \times 10^4\,\mathrm{m\,s^{-1}}$.

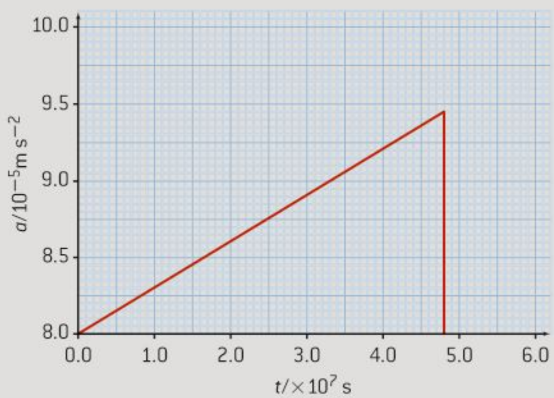


The mass of one ion of xenon is 2.2×10^{-25} kg

a) The original mass of the fuel is 81 kg. Determine how long the fuel will last if the engine ejects 77×10^{18} xenon ions every second.

b) The mass of the spaceship is 5.4×10^2 kg. Determine the initial acceleration of the spaceship.

The graph below shows the variation with time t of the acceleration a of the spaceship. The solar propulsion engine is switched on at time $t = 0$ when the speed of the spaceship is $1.2 \times 10^3\ \mathrm{m\,s^{-1}}$.

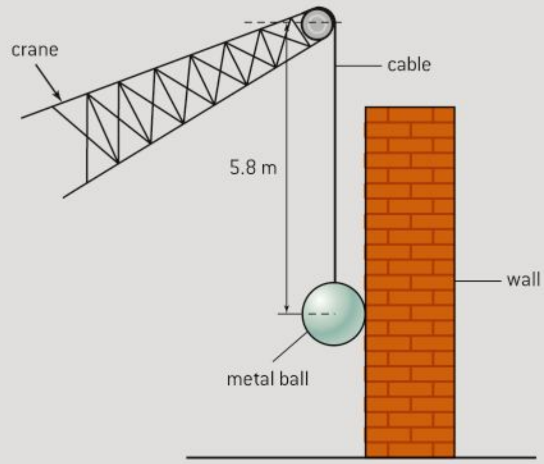


c) Explain why the acceleration of the spaceship increases with time.

d) Using data from the graph, calculate the speed of the spaceship at the time when the xenon fuel has all been used. (15 marks)

13 *(IB)*

A large metal ball is hung from a crane by means of a cable of length 5.8 m as shown below.



To knock a wall down, the metal ball of mass 350 kg is pulled away from the wall and then released. The crane does not move. The graph below shows the variation with time t of the speed v of the ball after release.

The ball makes contact with the wall when the cable from the crane is vertical.

a) For the ball just before it hits the wall use the graph, to estimate the tension in the cable. The acceleration of free fall is $9.8\,\mathrm{m\,s^{-2}}$.

b) Determine the distance moved by the ball after coming into contact with the wall.

c) Calculate the total change in momentum of the ball during the collision of the ball with the wall. (7 marks)